Symbolic Bounded Synthesis

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CAV 2010 – July 18, 2010

Synthesis of reactive systems - overview

Problem description

Given ...

- a set of input atomic propositions AP_I,
- a set of output atomic propositions AP_O,
- a temporal logic formula ψ over $\mathsf{AP}_I \uplus \mathsf{AP}_O$

... does there exist a Mealy/Moore automaton reading AP_I and outputting AP_O that satisfies ψ?

Properties of this problem

Church's problem is known to be 2EXPTIME-complete for LTL specifications.

LTL synthesis in practice

Approaches

- Several approaches exist (e.g., generalized reactivity(1) synthesis [PPS06], "classical" parity game solving, etc.)
- Here, we are concerned with **bounded synthesis** [SF07], a Safraless approach for LTL synthesis [KV05].

Criteria for the evaluation of these approaches

- Expressivity
- Scalability
  - suitability for *typical* specifications
  - amenable to symbolic implementations

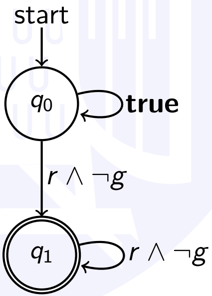
A Safraless approach for LTL synthesis [KV05]

Basic Approach

1. Convert $\neg\psi$ to a non-deterministic Büchi word automaton $\mathcal{A}$
2. Dualize $\mathcal{A}$ to a universal co-Büchi word automaton (UCW) $\mathcal{A}'$
3. Check the universal co-Büchi tree automaton (UCT) obtained from $\mathcal{A}'$ for emptiness

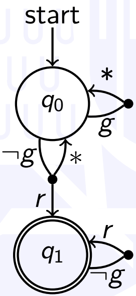
Basic idea: **Universality makes the world simpler**

Bounded synthesis [SF07]



NBW for the negated specification /
UCW for the specification

Bounded synthesis [SF07]

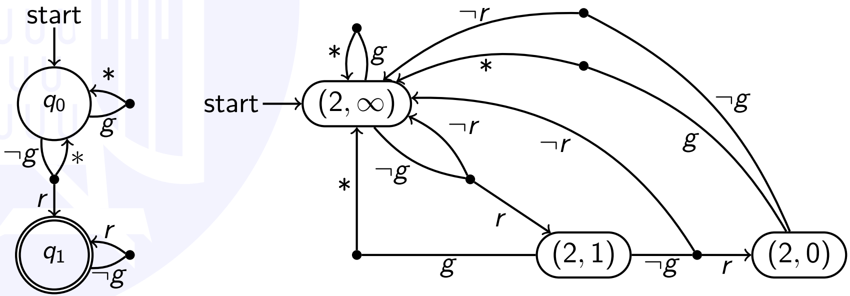


Corresponding UCT

Bounded synthesis [SF07]

Central idea

- For every finite-state system satisfying ψ, there exists an upper bound on the number of visits to rejecting UCT states
- **Bound** that number!
- Then, synthesis can be done by solving a safety game.



On the efficiency of the technique

Properties of the game structure

- Number of states: roughly $(b+1)^{|Q|}$ – huge!
- Structure is amenable to symbolic implementations

A symbolic approach from last year's CAV [FJR09]

Antichains can efficiently represent **frontier sets** during the game solving process.

Basic idea: sets of winning states are closed under counter increasals, e.g., if state $(2, 0)$ is winning for the system player, then so is state $(2, 1)$.

Binary decision diagrams

Interestingly, they seem to be unconsidered so far. **In this work, we show how to solve the challenges of applying them in practice**.

BDDs for bounded synthesis

Points **for** BDDs

- Good for *tracking components that run in parallel*:
  - games/automata for the specification conjuncts
  - evolution of the counters

Points **against** BDDs

- Counters in BDDs are evil! [Weg00, SL99, BMPY97, TV07]

The question raised and answered in this paper:

How can we reduce the number of counters such that the BDD-approach to Safraless/bounded synthesis is feasible in practice?

Structure of the remainder of the talk

The steps for reducing the number of counters

- Splitting the specification into safety/non-safety properties and composing them to a synthesis game
- Getting rid of some counters in the resulting synthesis game

Experiments & Outlook

- Comparison of our prototype against Lily/Acacia

Splitting a specification into safety and non-safety prop's

The shape of a "typical specification"

$$(a_1 \wedge a_2 \wedge \ldots \wedge a_n) \rightarrow (g_1 \wedge g_2 \wedge \ldots \wedge g_m)$$

Decomposing the specification

- Assumptions $a_1, \ldots, a_n$
- Guarantees $g_1, \ldots, g_m$
- Both assumptions and guarantees typically contain safety formulas.

Intuition for splitting the specification

Safety properties do not need counters.

Splitting a simple conjunction

$$\mathsf{G}a \wedge \mathsf{G}(b \rightarrow \mathsf{X}c) \wedge \mathsf{GF}d$$

Splitting a simple conjunction

$$\underbrace{\mathsf{G}a \wedge \mathsf{G}(b \rightarrow \mathsf{X}c)}_{\text{safety}} \wedge \underbrace{\mathsf{GF}d}_{\text{non-safety}}$$

Splitting a simple conjunction

$$\underbrace{\mathsf{G}a \wedge \mathsf{G}(b \rightarrow \mathsf{X}c)}_{\text{DST}} \wedge \underbrace{\mathsf{GF}d}_{\text{UCT}}$$

Splitting a simple conjunction

$$\underbrace{\mathsf{G}a \wedge \mathsf{G}(b \rightarrow \mathsf{X}c)} \wedge \underbrace{\mathsf{GF}d}$$

DST

Safety game

$\mathcal{G}_1$

UCT

Splitting a simple conjunction

$$\underbrace{\mathsf{G}a \wedge \mathsf{G}(b \rightarrow \mathsf{X}c)} \wedge \underbrace{\mathsf{GF}d}$$

DST

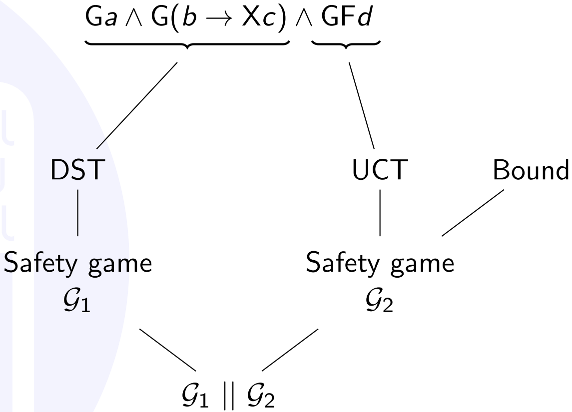
Safety game $\mathcal{G}_1$

UCT

Bound

Safety game $\mathcal{G}_2$

Splitting a simple conjunction



Winning condition

The system player wins $\mathcal{G}_1 \,||\, \mathcal{G}_2$ iff she wins $\mathcal{G}_1$ and $\mathcal{G}_2$ at the same time.

Splitting an assumptions→guarantees specification (1/2)

$$\mathsf{AP}_I = \{a, b\}, \mathsf{AP}_O = \{c, d\}$$

$$(\mathsf{G}a \wedge \mathsf{GF}b) \rightarrow (\mathsf{G}c \wedge \mathsf{GF}d)$$

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Safety game $\mathcal{G}_1$

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DST

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$$\mathsf{AP}_I = \{a, b\}, \mathsf{AP}_O = \{c, d, \textcolor{red}{\mathit{safe}}\}$$

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DST

Safety game $\mathcal{G}_1$

DST

Safety game $\mathcal{G}_2$, won if *safe* always represents whether the I/O so far is still accepted by the DST

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UCT

Bound

Safety game $\mathcal{G}_3$

Splitting an assumptions→guarantees specification (1/2)

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$(\underbrace{\mathsf{G}a}_{} \wedge \underbrace{\mathsf{GF}b}_{}) \rightarrow (\underbrace{\mathsf{G}c}_{} \wedge \underbrace{\mathsf{GF}d}_{})$

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DST

DST

UCT

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Safety game $\mathcal{G}_1$

Safety game $\mathcal{G}_2$, won if *safe* always represents whether the I/O so far is still accepted by the DST

Safety game $\mathcal{G}_3$

$\mathcal{G}_1 \mid\mid \mathcal{G}_2 \mid\mid \mathcal{G}_3$

Splitting an assumptions→guarantees specification (2/2)

Winning condition

The system player wins $\mathcal{G}_1 \,||\, \mathcal{G}_2 \,||\, \mathcal{G}_3$ iff she loses $\mathcal{G}_1$ or she wins $\mathcal{G}_2$ and $\mathcal{G}_3$ at the same time.

The role of *safe*

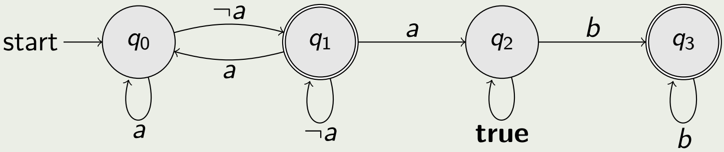
The AP *safe* connects the non-safety and safety guarantee parts. This is important for soundness. Example:

$$(\mathsf{G}a \wedge \mathsf{GF}\neg a) \rightarrow (\mathsf{G}c \wedge \mathsf{G}\neg c)$$

Getting rid of additional counters in $\mathcal{G}_3$

Example specification/Example UCT

$$\mathsf{FG}a \wedge \mathsf{G}((\neg a \wedge \mathsf{X}a) \rightarrow \mathsf{XXGF}\neg b)$$



States of type $(*, \infty, *, \infty)$ in the safety game for $b = 3$

$(3,\infty,\infty,\infty)$	$(2,\infty,\infty,\infty)$	$(1,\infty,\infty,\infty)$	$(0,\infty,\infty,\infty)$
	$(2,\infty,2,\infty)$	$(1,\infty,2,\infty)$	$(0,\infty,2,\infty)$
	$(2,\infty,1,\infty)$	$(1,\infty,1,\infty)$	$(0,\infty,1,\infty)$
	$(2,\infty,0,\infty)$	$(1,\infty,0,\infty)$	$(0,\infty,0,\infty)$

Getting rid of additional counters in $\mathcal{G}_3$

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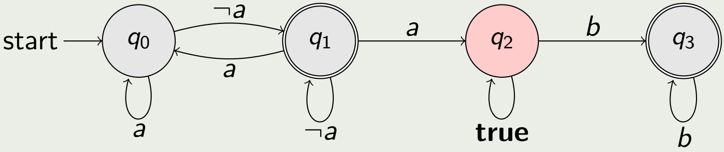

States of type $(*, \infty, *, \infty)$ in the safety game for $b = 3$

$(3, \infty, \infty, \infty)$ $(2, \infty, \infty, \infty)$ $(1, \infty, \infty, \infty)$ $(0, \infty, \infty, \infty)$

$(2, \infty, 3, \infty)$ $(1, \infty, 3, \infty)$ $(0, \infty, 3, \infty)$

Experimental results

A prototype implementation of the BDD-based approach

- Tools/Libraries used:
  - The `cudd` BDD library
  - The `ltl2ba` LTL$\rightarrow$Büchi converter
  - For verifying the results: `NuSMV`
- Written in C++
- Available at `http://react.cs.uni-saarland.de/tools/unbeast`

General workflow

- Read the specification and solve the synthesis problem for increasing bounds until the game is winning.
- Also run the tool with negated specification and swapped input/output at the same time (to detect unrealisability)

Performance comparison (1/3)

The 23 examples from `Lily`

Speed comparison on an AMD Opteron 2.6 Ghz computer (2 GB of memory available, 1h time limit):

- `Lily`: 54.35 seconds
- `Acacia`: 53.71 seconds
- `Unbeast`: 19.41 seconds

Performance comparison (2/3)

The scalable example from the `Acacia` paper

# Clients:	1	2	3	4	5	6	7
Unbeast:	0.3 s	0.7 s	0.6 s	1.9 s	0.9 s	4.6 s	3.0 s
Acacia:	0.9 s	2.0 s	4.0 s	9.8 s	47.3 s	506.5 s	m/o

# Clients:	10	14	15	20	21	22
Unbeast:	651.5 s	491.0 s	t/o	1909.0 s	t/o	t/o
Acacia:	m/o	m/o	m/o	m/o	m/o	m/o

Performance comparison (3/3) - The load balancer

Tool	Setting / # Clients	2	3	4	5	6	7	8	9
Unbeast	1	0.6	0.6	0.2	1.3	0.2	0.3	0.2	0.3
Acacia		0.3	0.4	0.6	0.9	1.5	2.7	5.3	12.1
Unbeast	$1 \wedge 2$	0.4	0.3	0.6	0.6	0.7	0.6	0.6	0.7
Acacia		0.3	0.3	0.4	0.4	0.6	0.9	1.6	3.1
Unbeast	$1 \wedge 2 \wedge 3$	0.5	0.5	0.5	0.5	0.7	1.0	6.9	73.9
Acacia		19.2	475.6	t/o	t/o	t/o	m/o	m/o	t/o
Unbeast	$1 \wedge 2 \wedge 4$	0.3	0.4	0.9	65.5	104.6	990.3	t/o	t/o
Acacia		0.6	1.3	8.7	277.9	m/o	m/o	m/o	t/o
Unbeast	$1 \wedge 2 \wedge 4 \wedge 5$	0.2	0.7	t/o	t/o	t/o	t/o	t/o	t/o
Acacia		163.4	t/o	t/o	t/o	m/o	m/o	m/o	t/o
Unbeast	$6 \rightarrow 1 \wedge 2 \wedge 4 \wedge 5$	0.2	0.7	3244.1	t/o	t/o	t/o	t/o	t/o
Acacia		175.3	t/o	t/o	t/o	m/o	m/o	t/o	t/o
Unbeast	$6 \wedge 7 \rightarrow 1 \wedge 2 \wedge 4 \wedge 5$	0.5	1.1	t/o	t/o	t/o	t/o	t/o	t/o
Acacia		190.7	m/o	t/o	t/o	t/o	t/o	t/o	t/o
Unbeast	$6 \wedge 7 \rightarrow 1 \wedge 2 \wedge 5 \wedge 8$	0.3	0.6	2.4	20.7	368.6	t/o	t/o	t/o
Acacia		7.5	69.0	357.4	m/o	t/o	t/o	t/o	t/o
Unbeast	$6 \wedge 7 \rightarrow 1 \wedge 2 \wedge 5 \wedge 8 \wedge 9$	0.3	0.2	0.3	1.0	16.8	449.1	t/o	t/o
Acacia		48.8	2133.5	t/o	m/o	t/o	t/o	t/o	t/o
Unbeast	$6 \wedge 7 \wedge 10 \rightarrow 1 \wedge 2 \wedge 5 \wedge 8 \wedge 9$	0.4	0.8	118.7	t/o	t/o	t/o	t/o	t/o
Acacia		26.9	295.8	m/o	t/o	t/o	t/o	t/o	t/o

Conclusion

The contributions of this paper

- Showing that BDDs have potential for synthesis from full LTL
- Providing optimisation techniques for this case
- Describing a new scalable benchmark for synthesis from LTL specifications

Details of the paper left out

- Efficient encoding of safety specifications into games
- Extracting winning strategies from the game in a symbolic way
- Dealing with unrealisability checking
- Counter encoding in BDDs
- Swapping input and output for shorter specifications
- Putting labels onto the edges of the (co-)Büchi automata

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