Inferring Symbolic Automata

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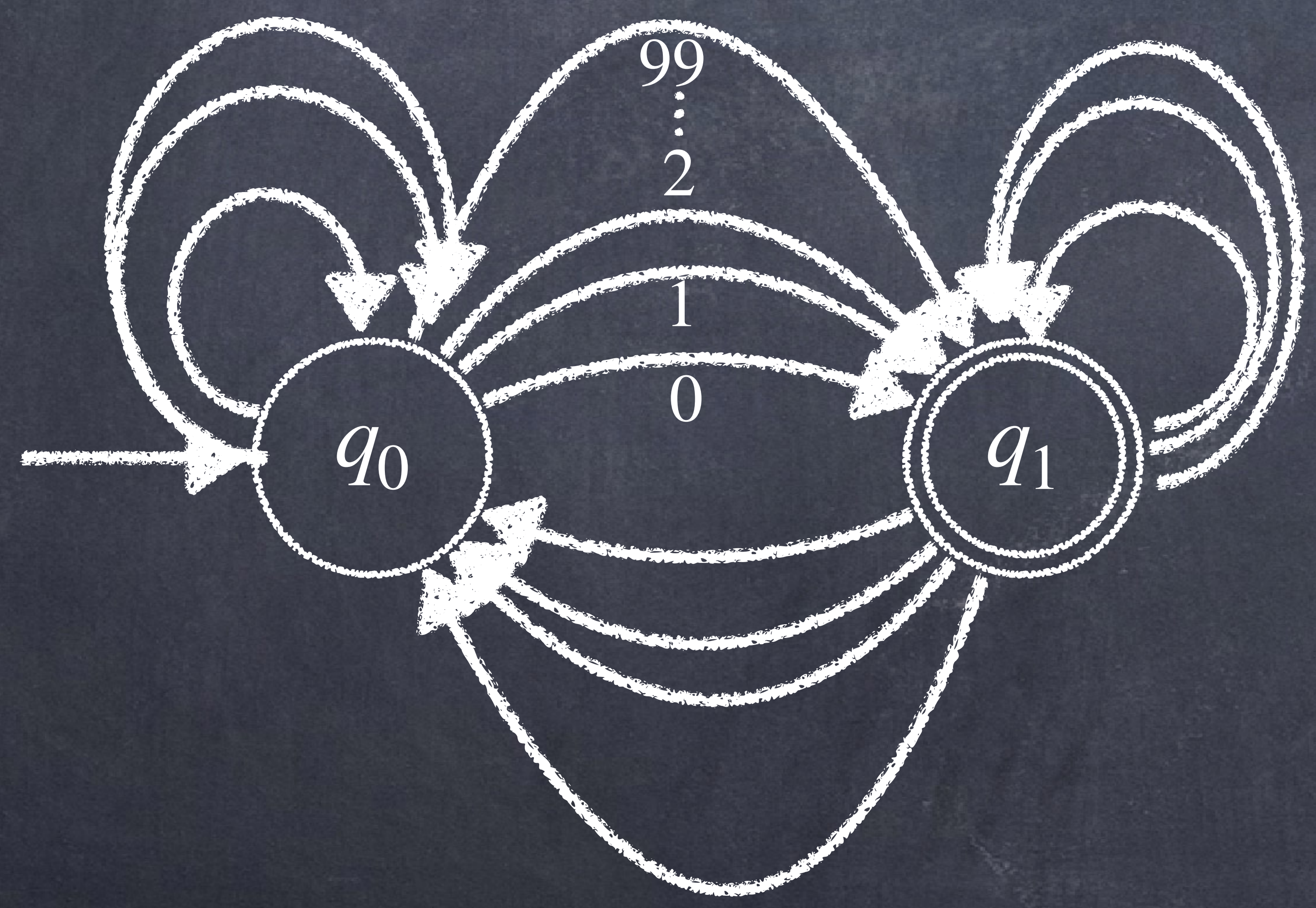
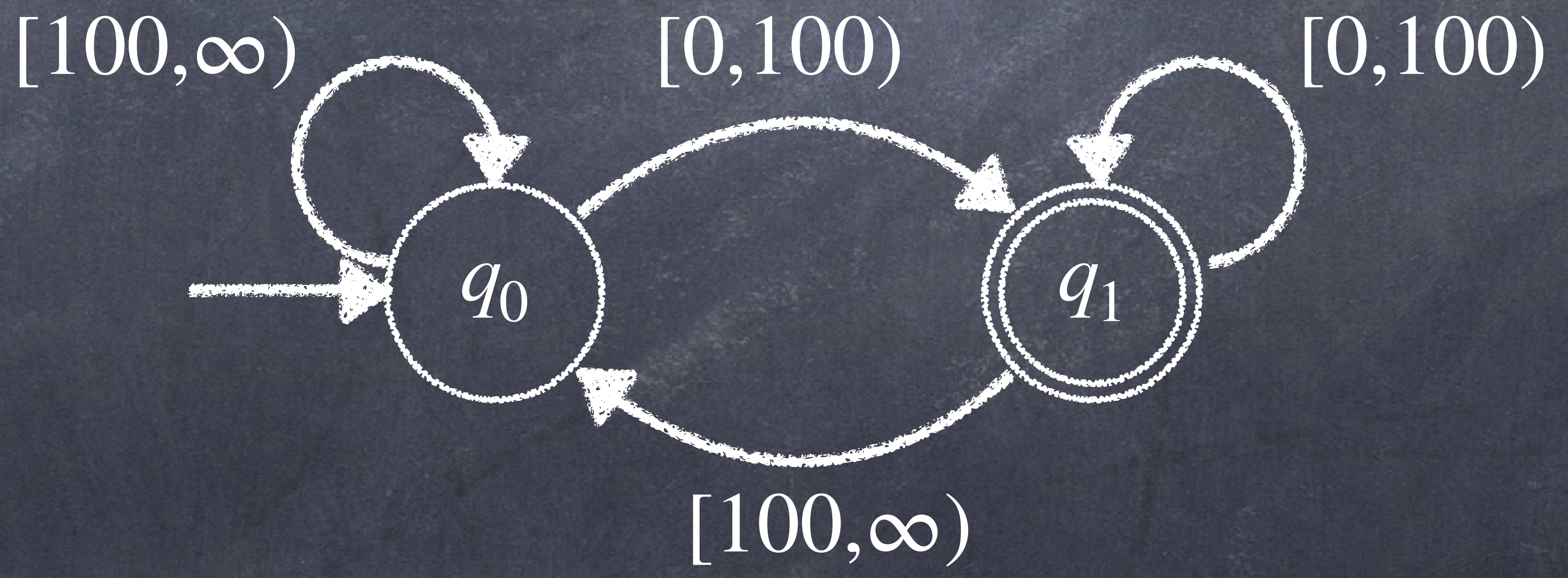
Joint work with Dana Fisman and Sandra Zilles

Symbolic Automata - SFAs

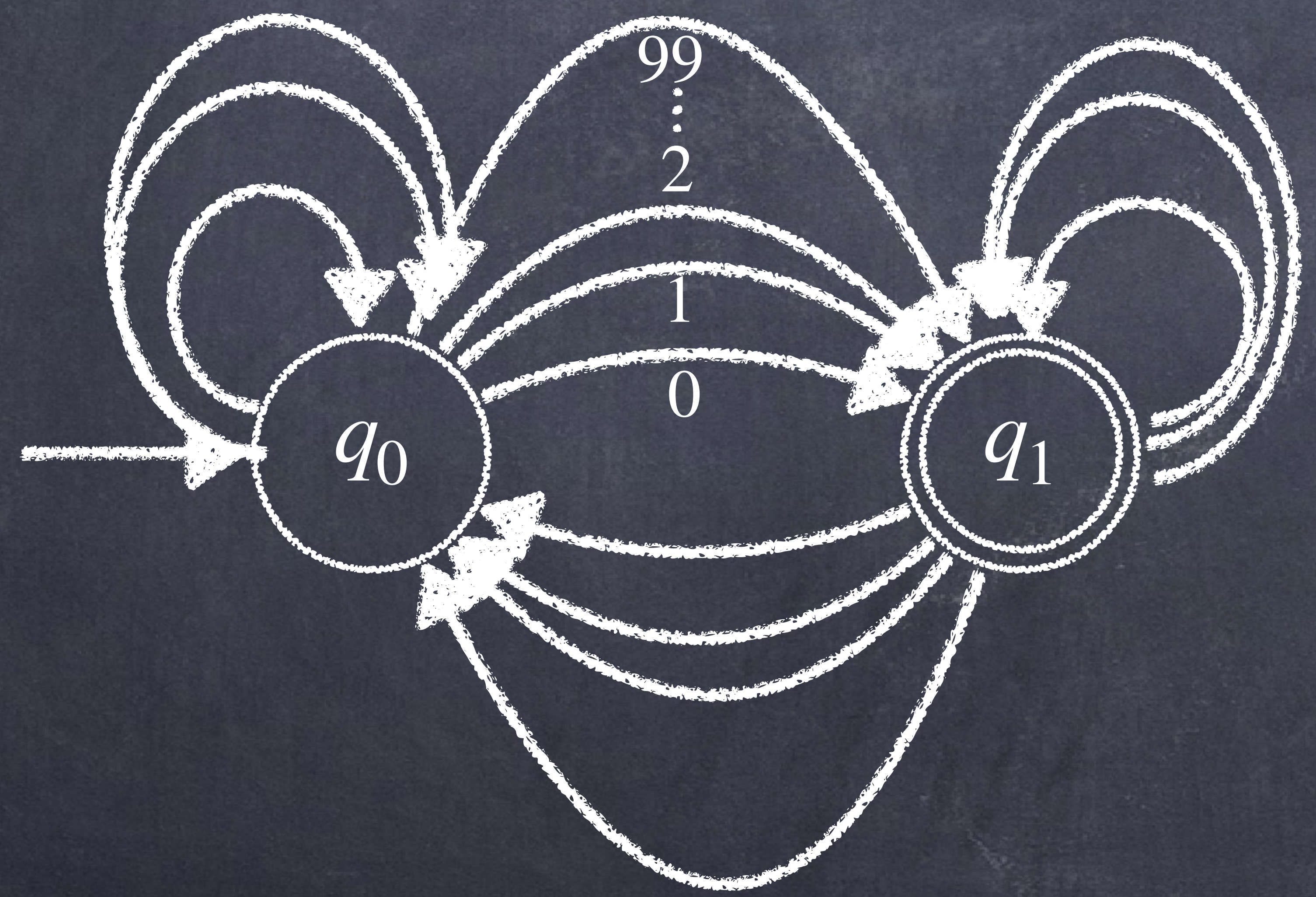
- Finite state automata
- Defined w.r.t. a Boolean algebra
- Transitions are over predicates

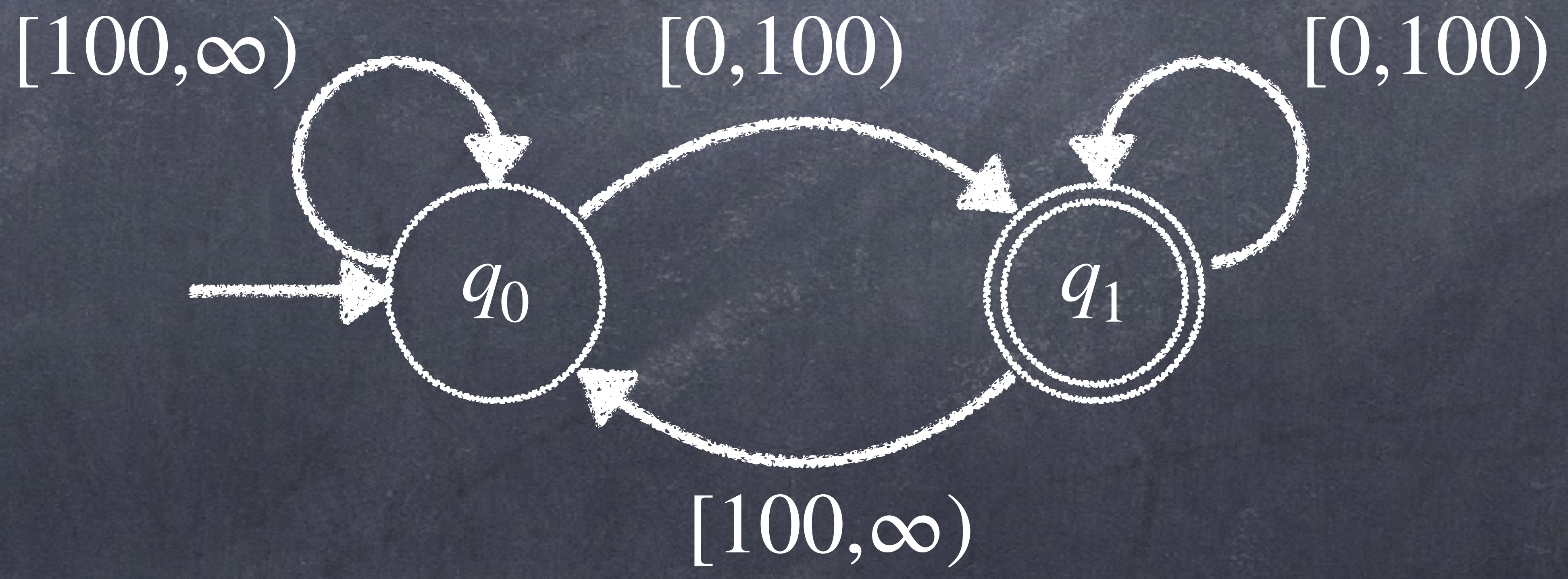




Symbolic Automata – SFAs

- Finite state automata
- Defined w.r.t. a Boolean algebra
- Transitions are over predicates
- Concise
- Reason about infinite domains





Monotonic Algebras

- Predicates correspond to a total order over the domain elements
- $[\psi] = \{d \mid a \leq d \leq b\}$
- Monotonic: Interval algebras (over $\mathbb{N}, \mathbb{Z}, \mathbb{R}$)

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Propositional algebra

- Predicates are Boolean combinations of atomic propositions
- $(p_1 \wedge p_2) \vee (\neg p_1 \wedge p_3)$
- Not monotonic!

Automata Learning

- Active learning - L* style learning [Angluin 1987]
- Passive learning

Automata Learning

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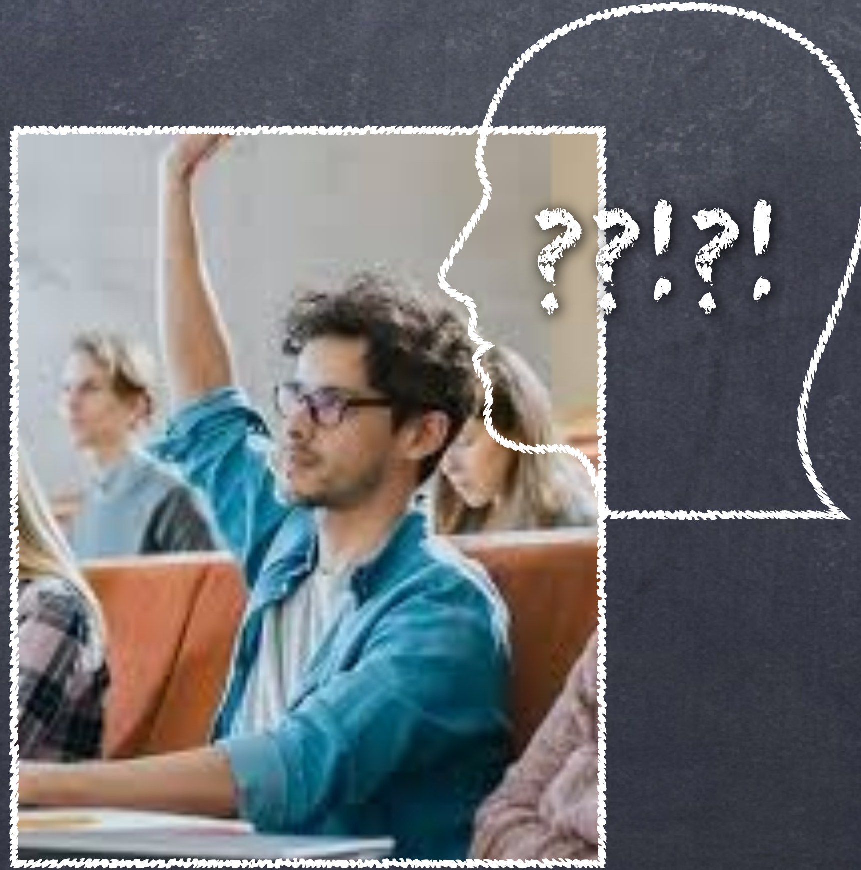
$\langle w_1, \bot \rangle$

$\langle w_2, \top \rangle$

$\vdots$

$\langle w_n, \bot \rangle$



Learnability of a Class of languages via Representation $\mathscr{R}$

- Different representations of languages

- E.g. regular languages – DFAs, NFAs

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- A class of languages $\mathscr{L}$ is learnable via representation in $\mathscr{R}$ if there is an algorithm ALG such that we can apply ALG to learn a representation in $\mathscr{R}$ for every language in $\mathscr{L}$

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- E.g. regular languages – DFAs, NFAs
- E.g. L* algorithm for regular languages via representation in DFAs

Active Learning of SFAs

- Positive results
  - Learning of SFAs over monotonic algebras using membership and equivalence queries
    - [Maler & Mens 2014], [Maler & Mens 2017]
    - [Chubachi, Diptarama, Yoshinaka, Shinohara 2017]
  - MAT* algorithm for learning SFAs
    - [Argyros & D'Antoni 2018]

Active Learning of SFAs

- First negative result

Active Learning of SFAs

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- Necessary condition:
- We can **polynomially learn SFAs** over a Boolean algebra $\mathscr{A}$ using membership and equivalence queries only if we can **polynomially learn the predicates** of $\mathscr{A}$ using membership and equivalence queries

Active Learning of SFAs

- First negative result
- Necessary condition:
- We can polynomially learn SFAs over a Boolean algebra $\mathscr{A}$ using membership and equivalence queries only if we can polynomially learn the predicates of $\mathscr{A}$ using membership and equivalence queries

-> SFAs over the propositional algebra are not polynomially learnable

Passive Learning

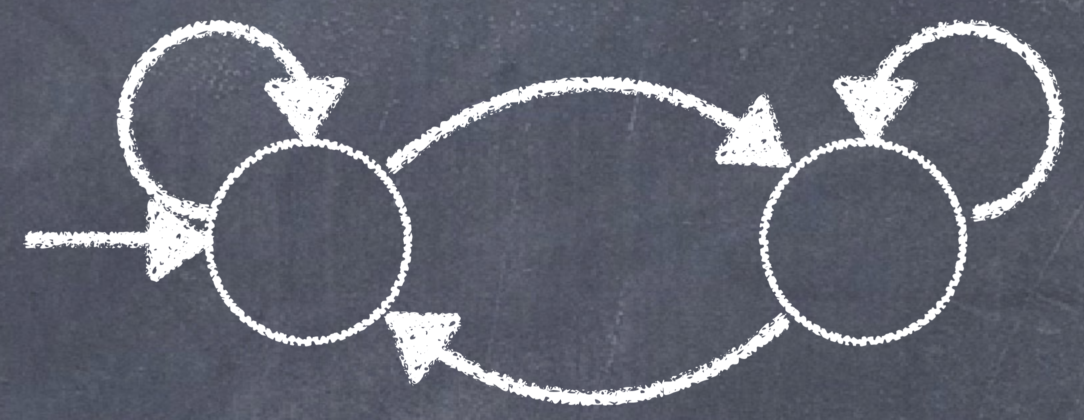
set
$\mathcal{S}'$

Infer

Identification in the Limit Using Polynomial Time and Data

set $\mathcal{S}$

← Characteristic



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[Gold 1978, de la Higuera 1997]

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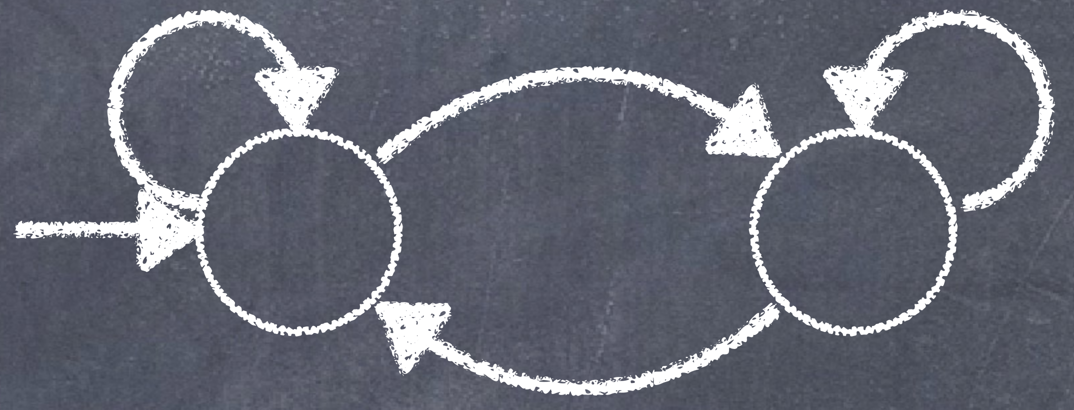

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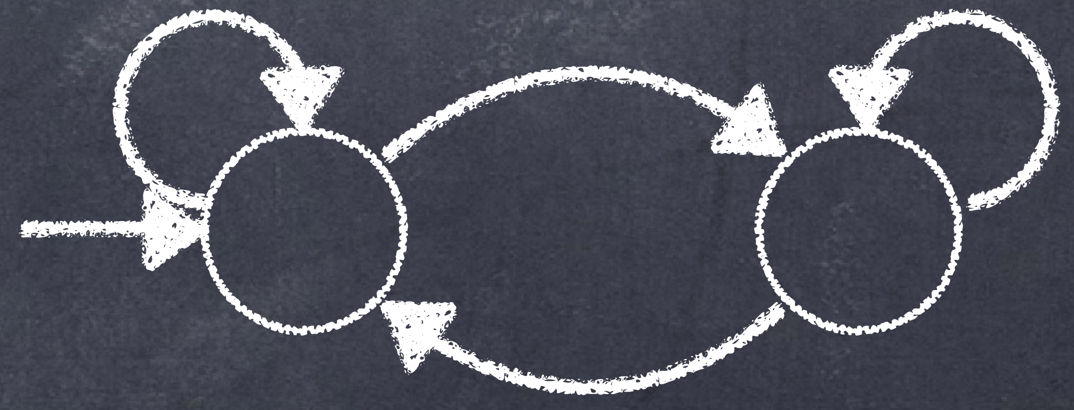

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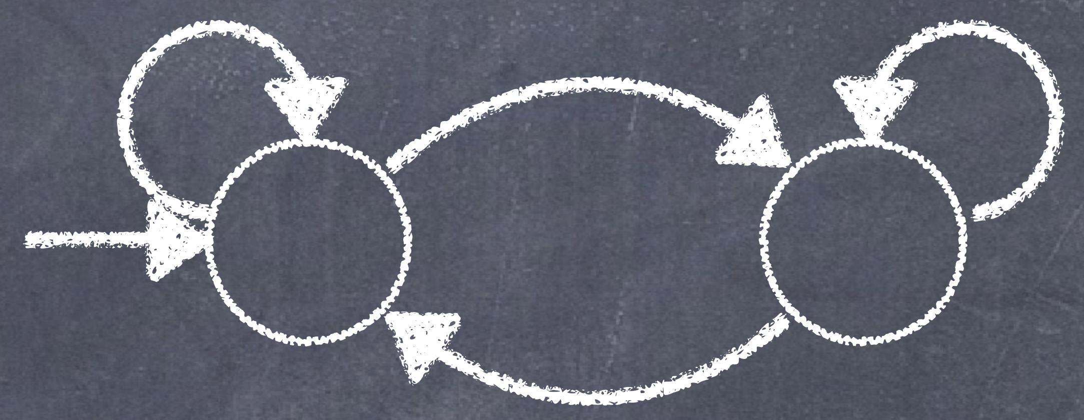

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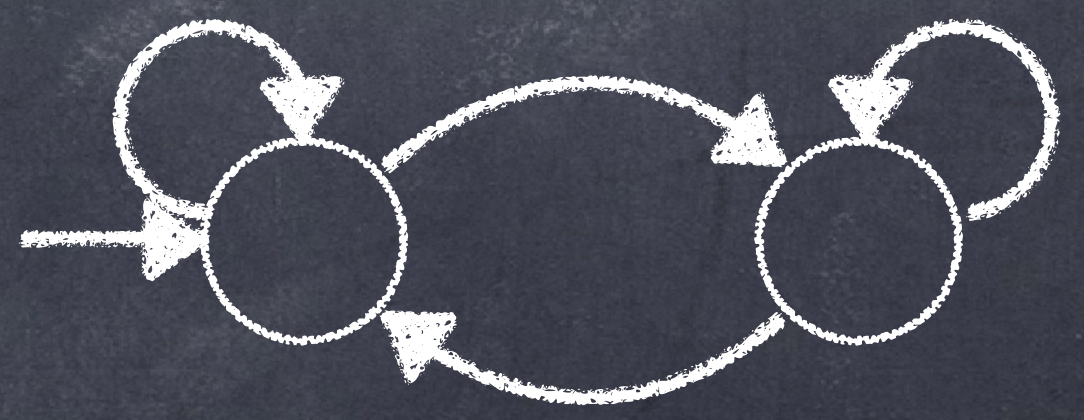

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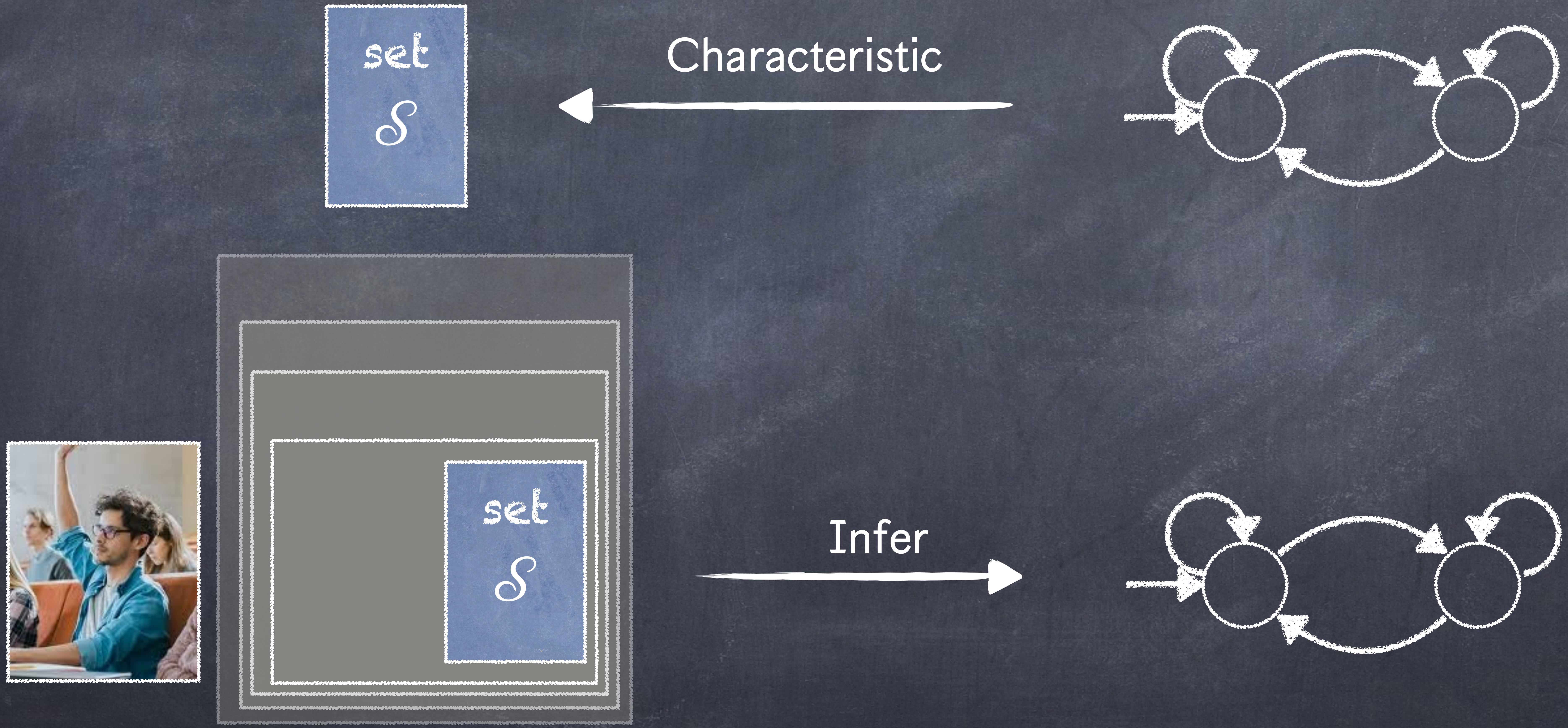

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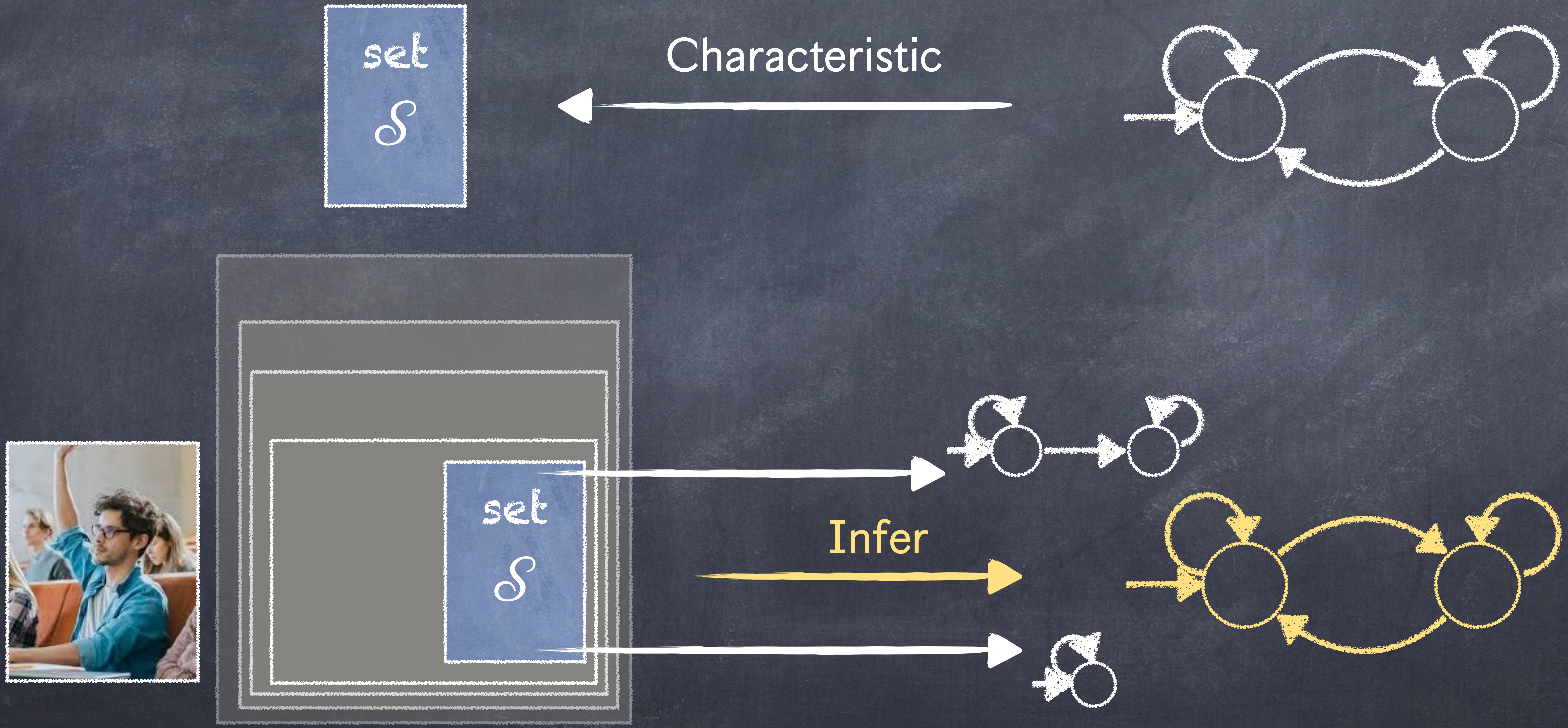

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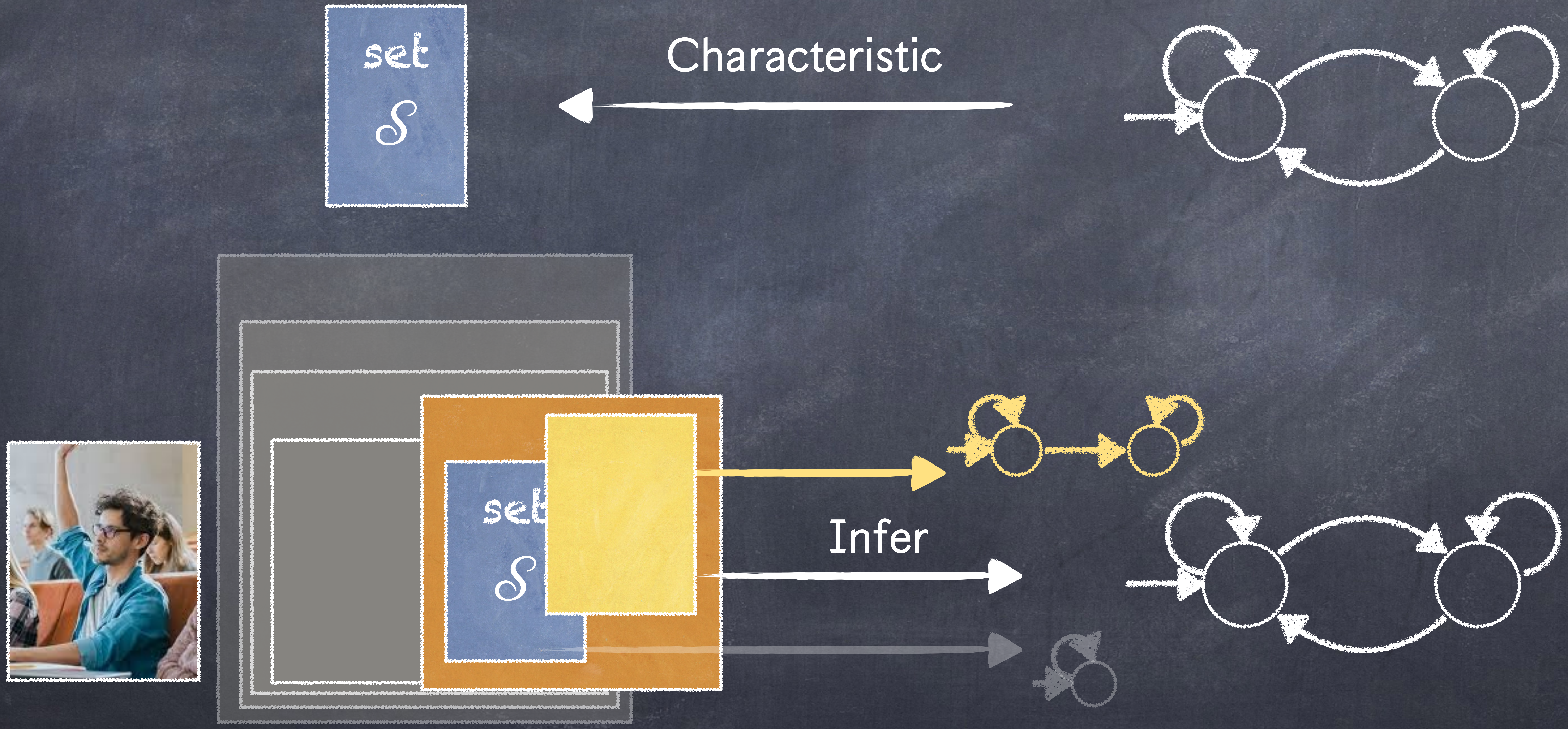

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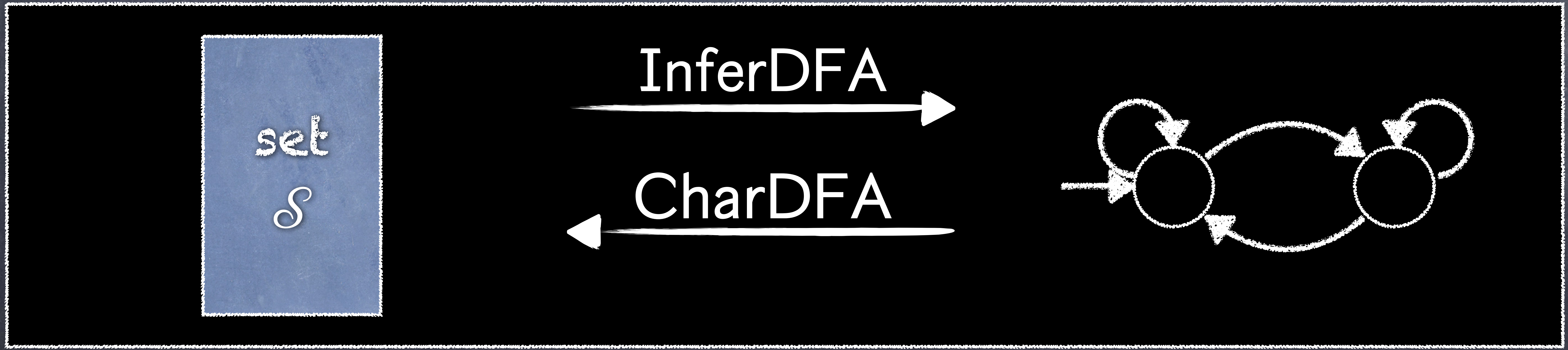
Identification in the Limit for DFAs



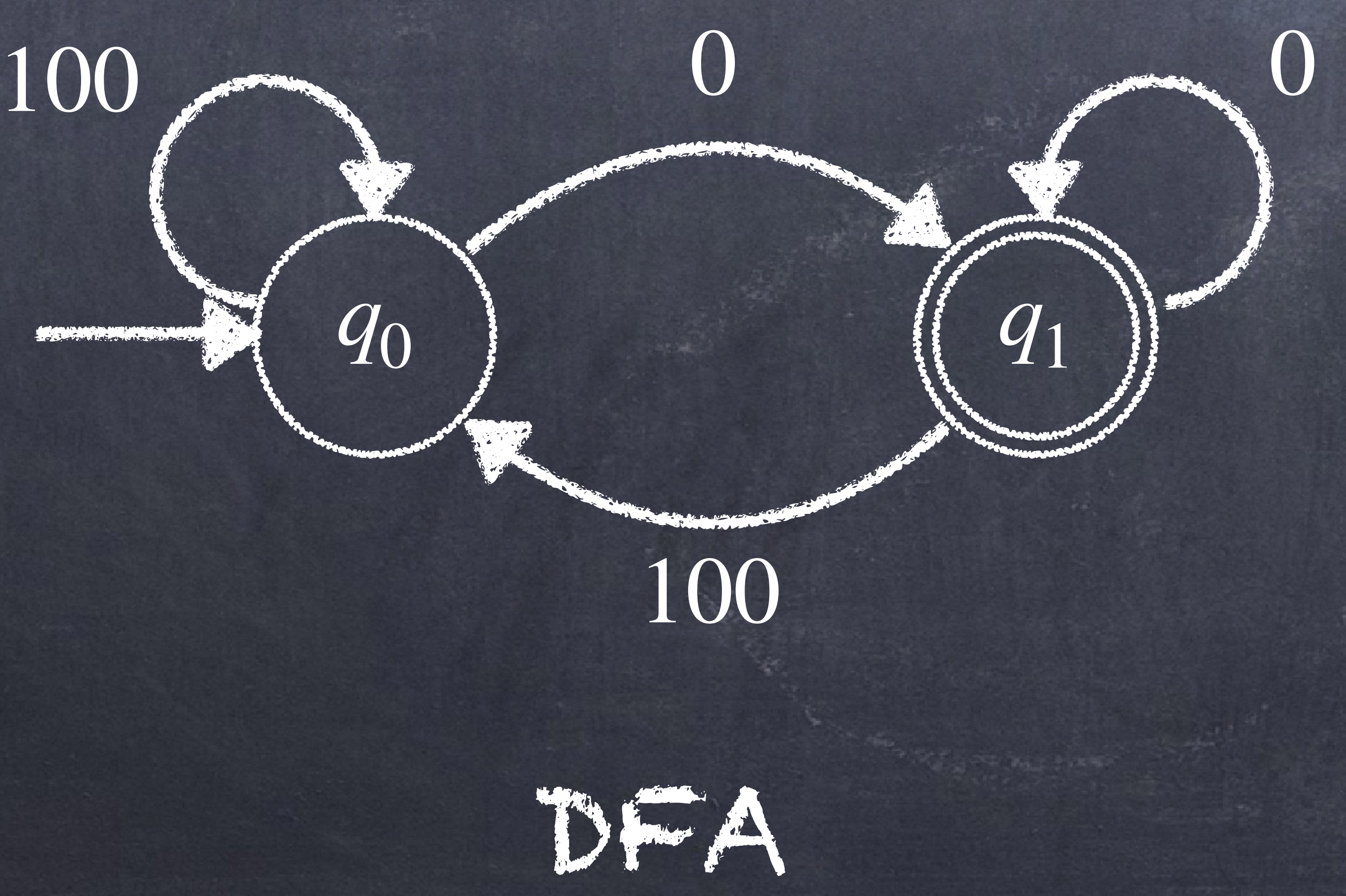
Concrete Sample set

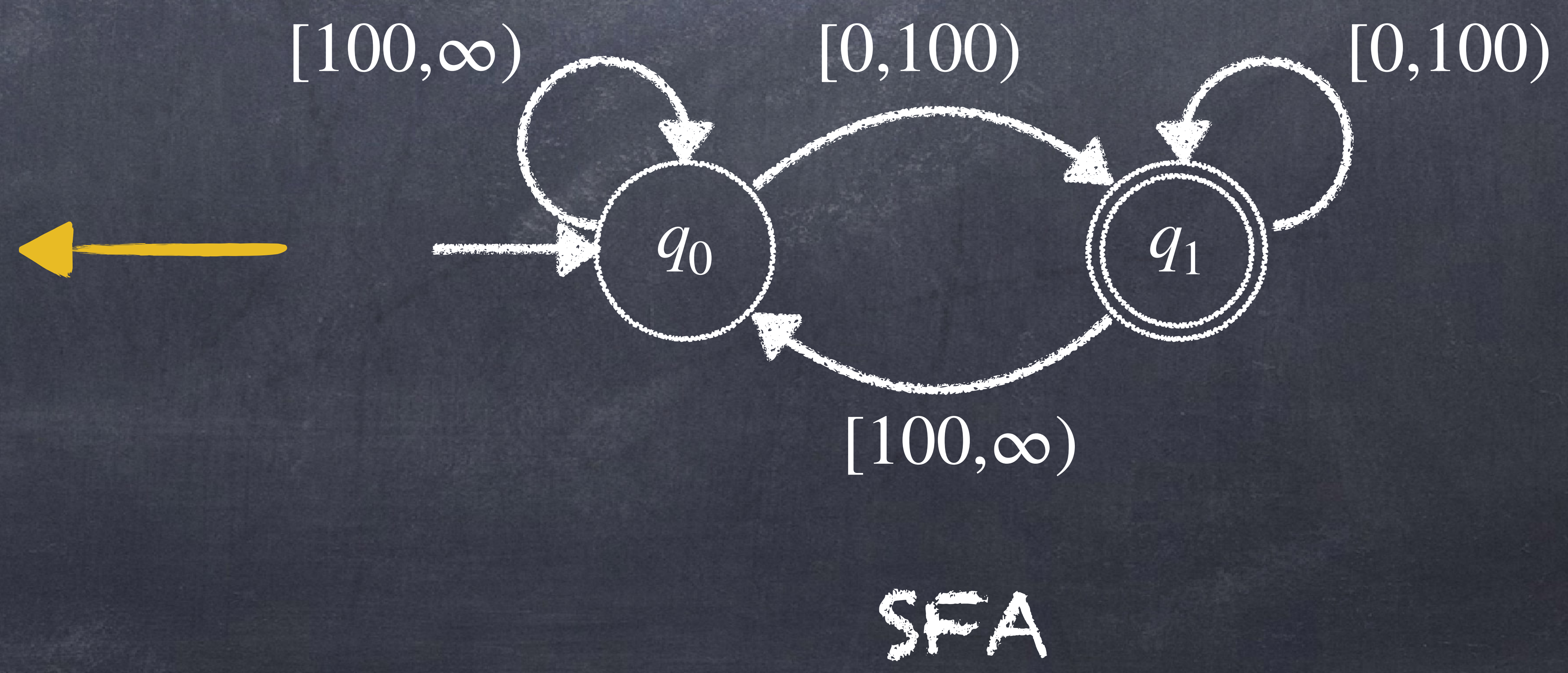
DFA

[Oncina&García 1992]

Identification in the Limit for SFAs - CharSFA

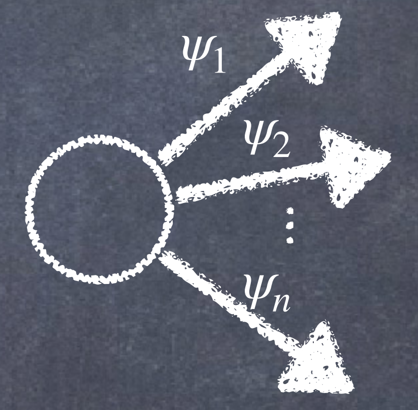
- Learning with respect to the concrete alphabet
- Creating a set of concrete words





Identification in the Limit for SFAs - CharSFA

- Learning with respect to the concrete alphabet
- Creating a set of concrete words
- concretize$(\langle \psi_1, \ldots, \psi_n \rangle) = \langle \Gamma_1, \ldots, \Gamma_n \rangle$



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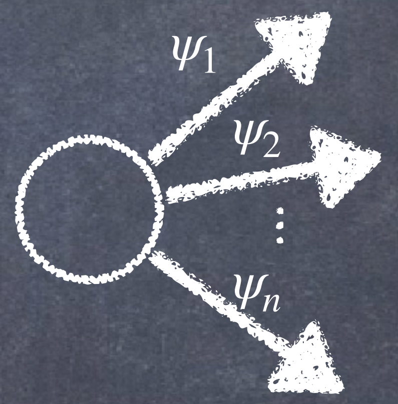
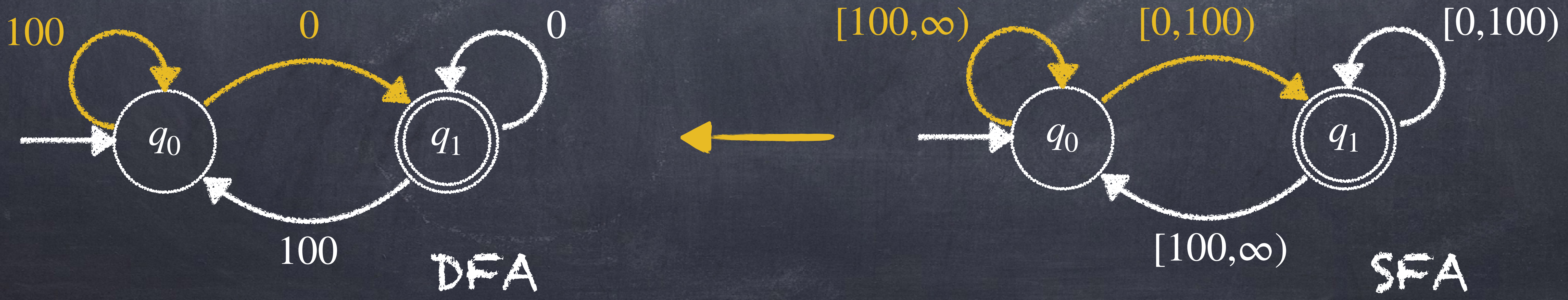


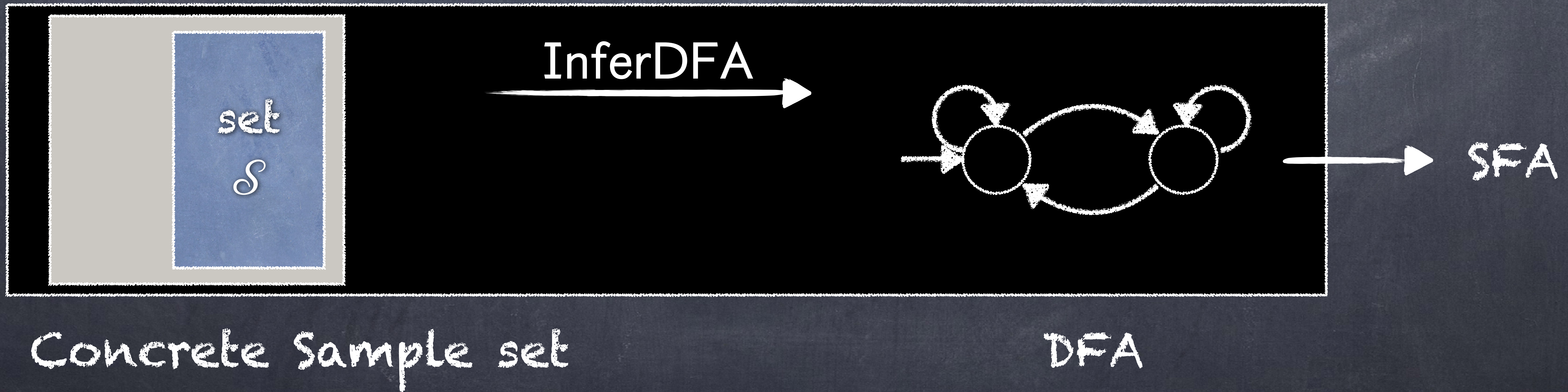
- concretize($\langle [0,100), [100,\infty) \rangle$) $= \langle \{0\}, \{100\} \rangle$



Identification in the Limit for SFAs - CharSFA

- Creating a set of concrete words

Identification in the Limit for SFAs - InferSFA



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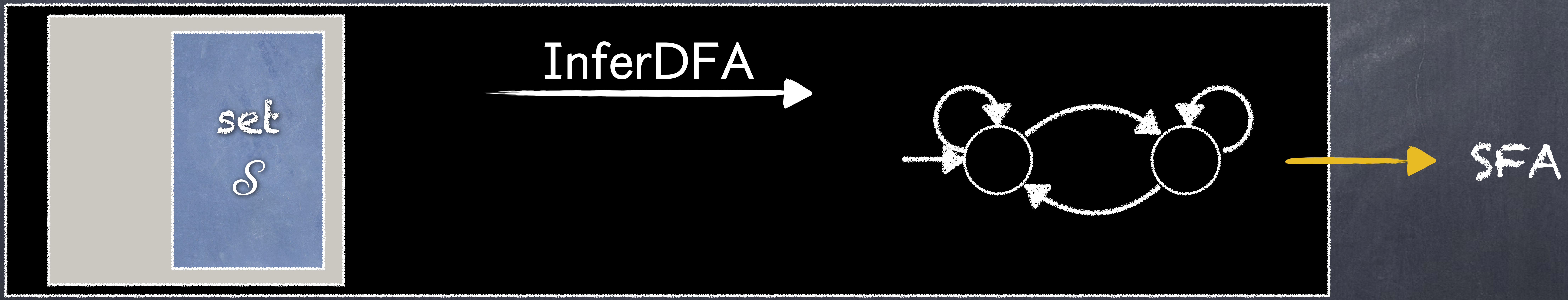
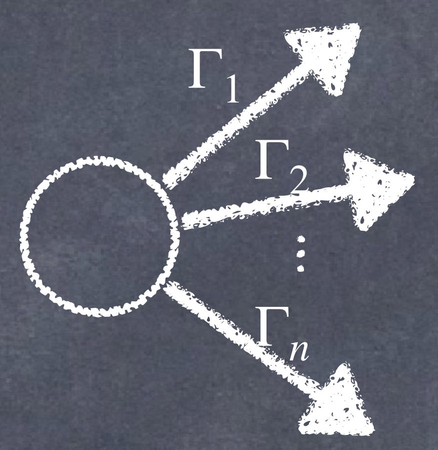


Concrete Sample set

DFA Generalize

Identification in the Limit for SFAs - InferSFA

- generalize$(\langle \Gamma_1, \ldots, \Gamma_n \rangle) = \langle \psi_1, \ldots \psi_n \rangle$



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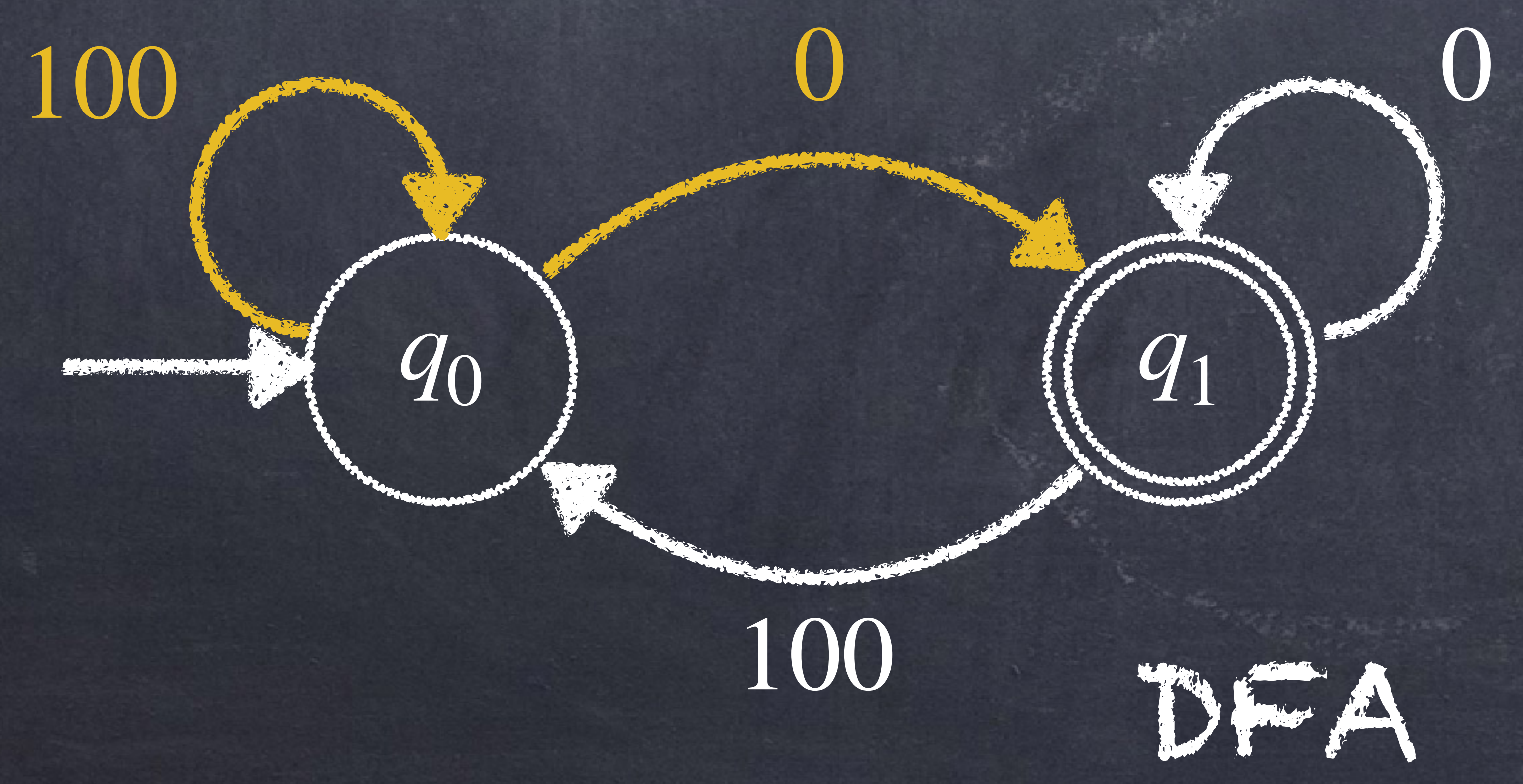

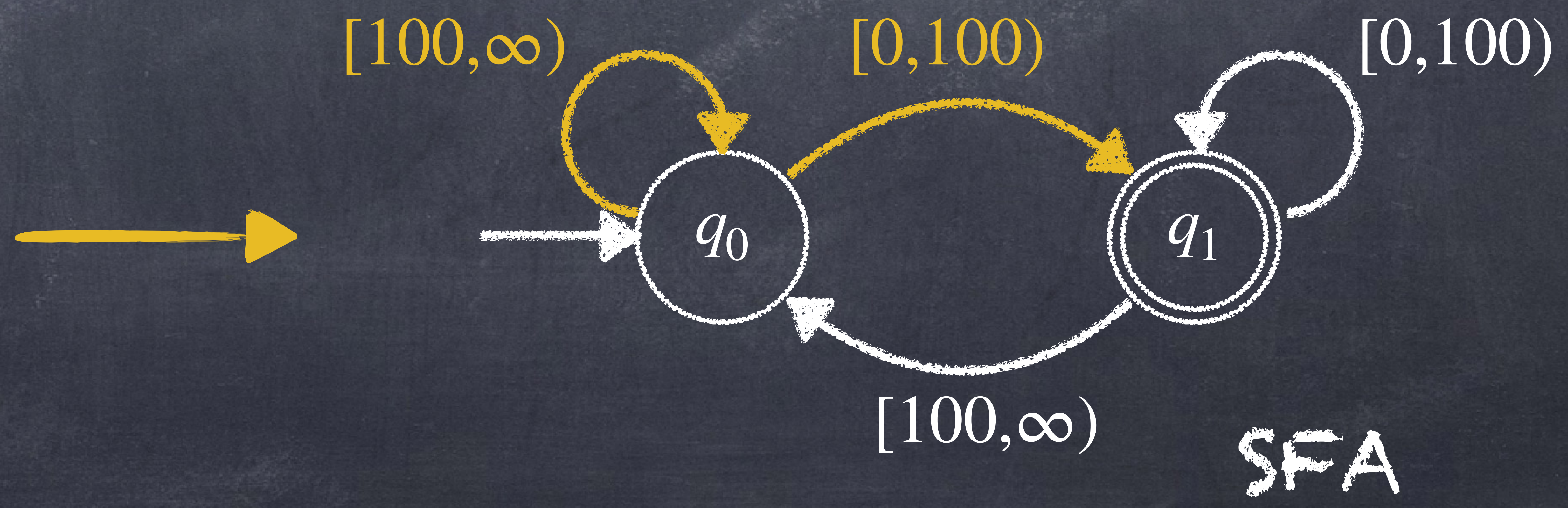

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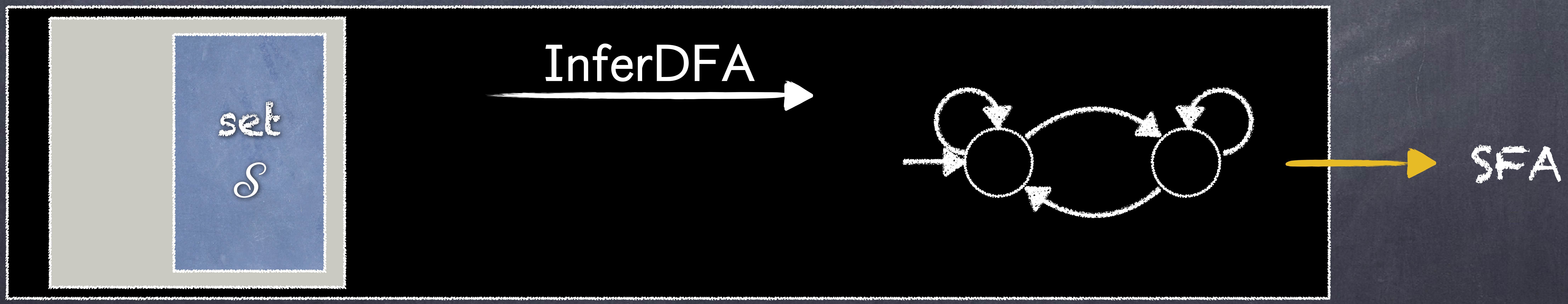

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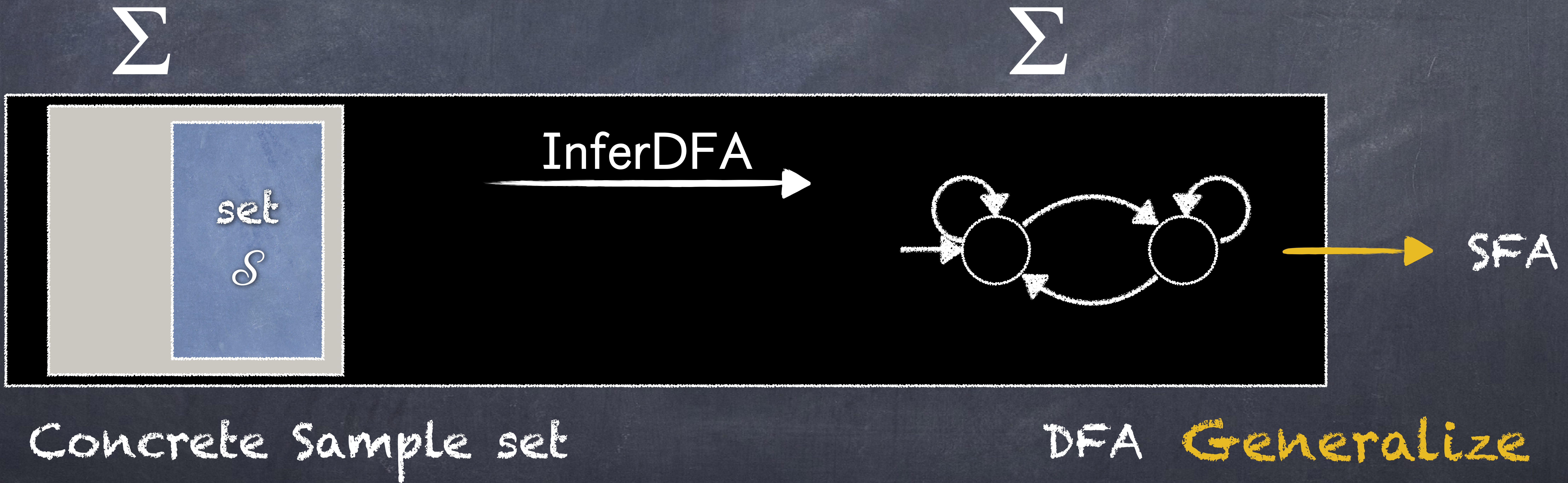

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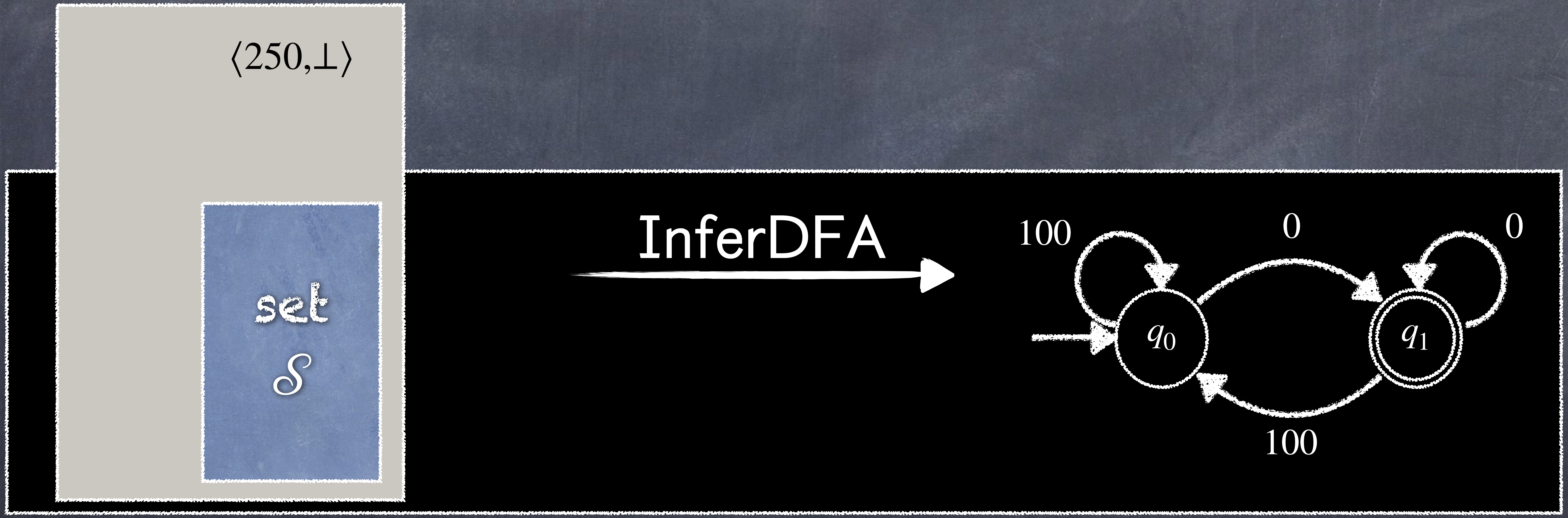

Concrete Sample set

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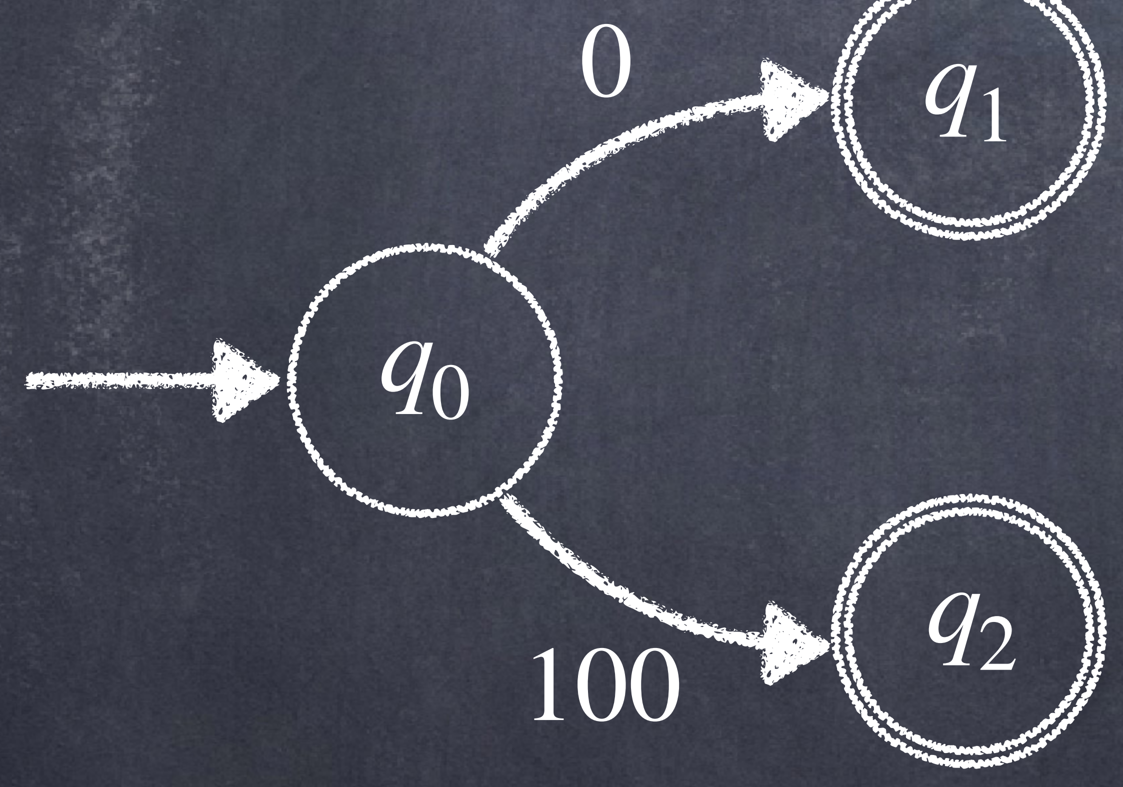
Identification in the Limit for SFAs - InferSFA

- Additional alphabet adds confusion



$\langle \epsilon, \perp \rangle, \langle 0, \top \rangle, \langle 100, \top \rangle$

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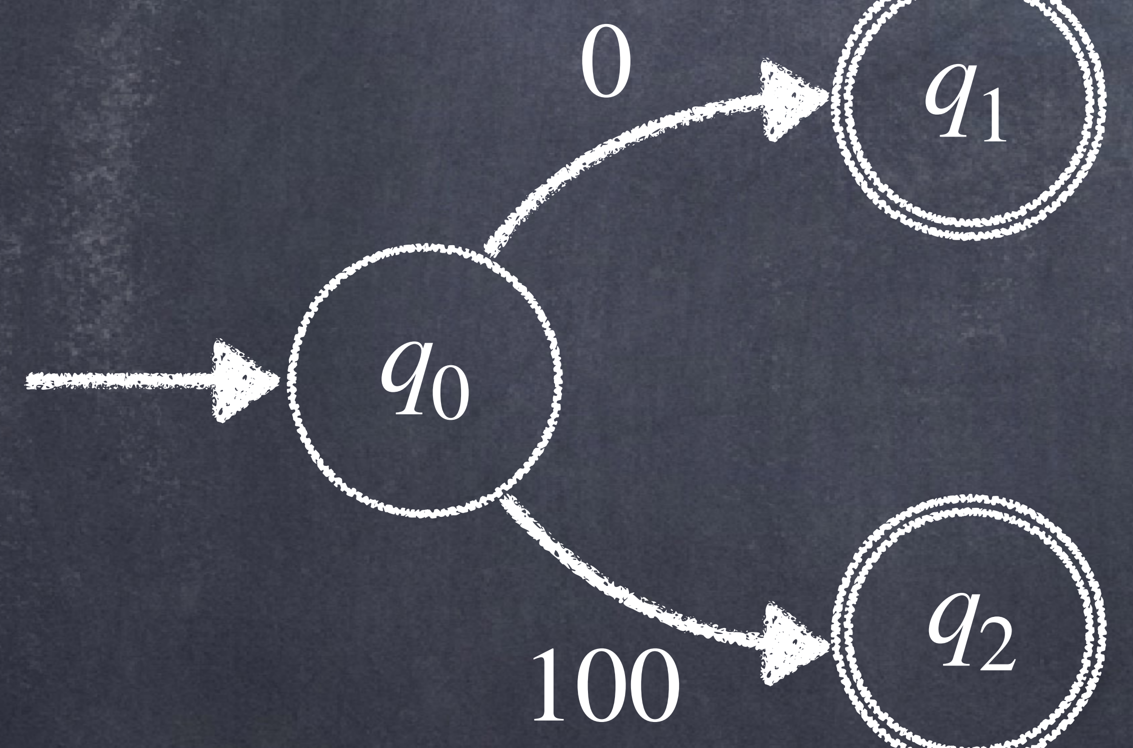

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$\langle 0 \cdot 0, \top \rangle, \langle 100 \cdot 0, \bot \rangle$

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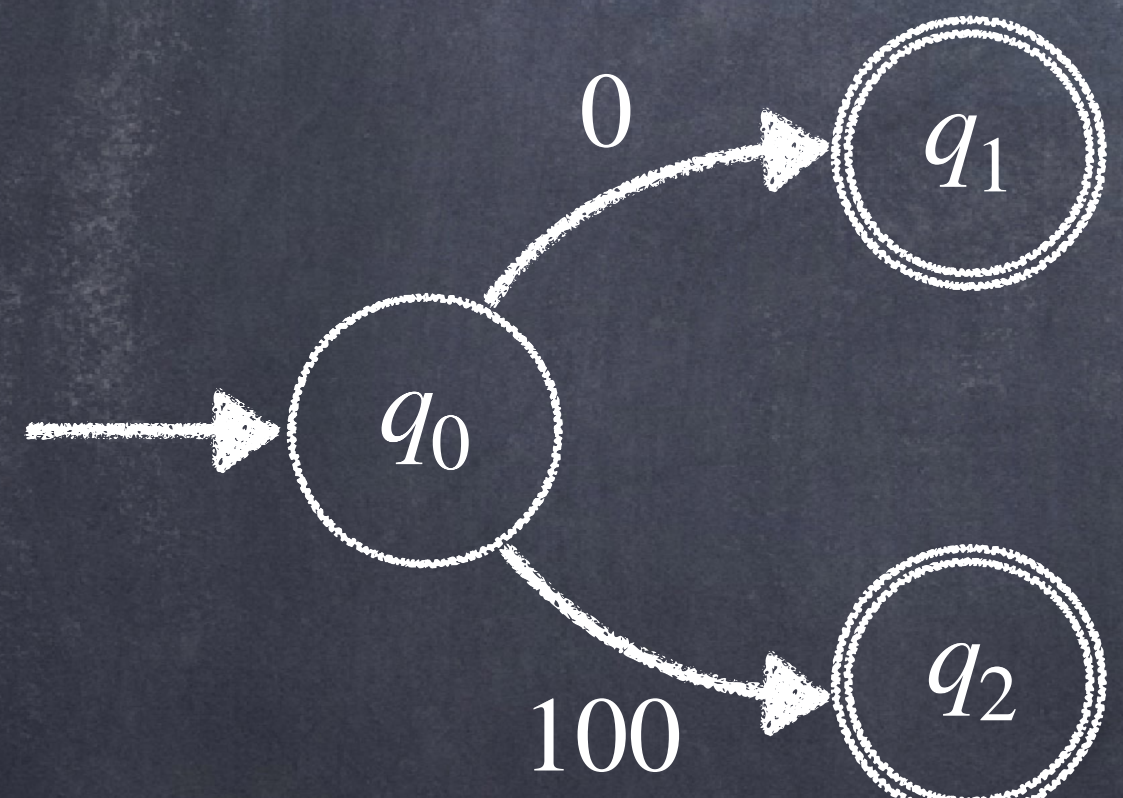

$\langle \epsilon, \bot \rangle, \langle 0, \top \rangle, \langle 100, \top \rangle$

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$\langle 150, \top \rangle$

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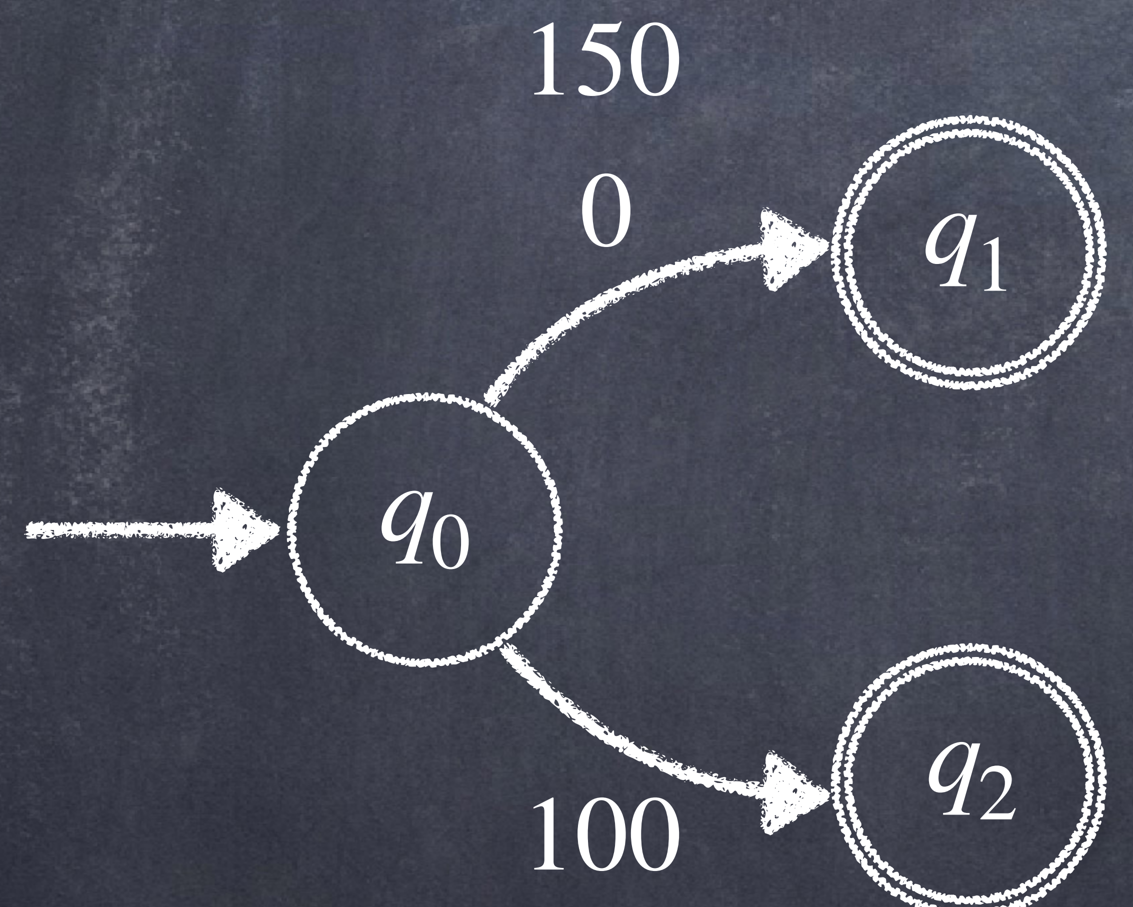

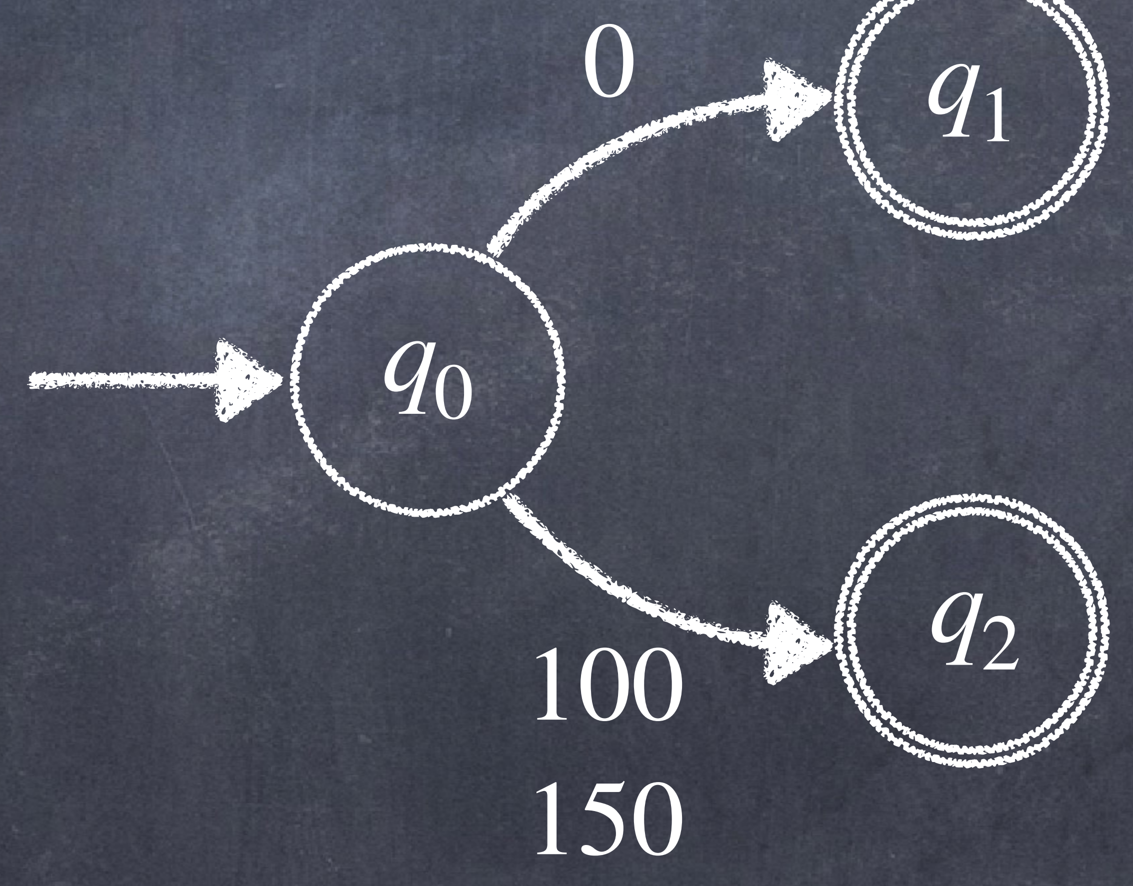

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$\langle 150, \top \rangle$

Identification in the Limit for SFAs – InferSFA

$\langle 250, \bot \rangle$

$\langle 75, \top \rangle$

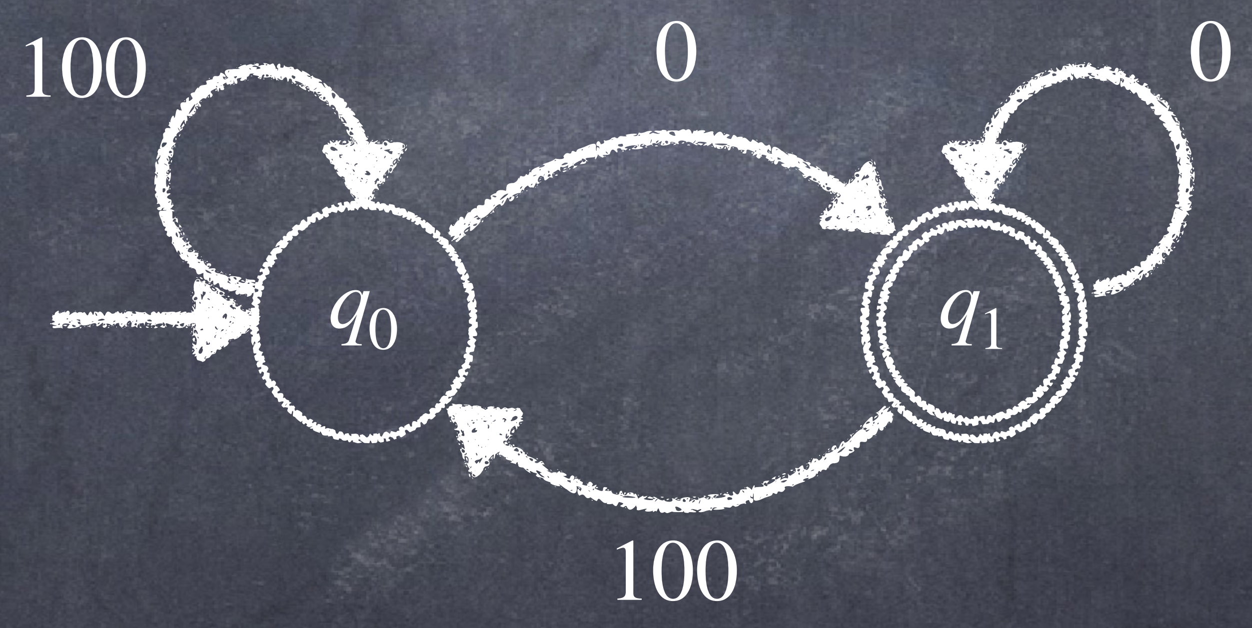
$\langle 100 \cdot 0, \top \rangle$

$\langle 100, \bot \rangle$

$\langle 0, \top \rangle$

Decontaminate

Concrete Sample set



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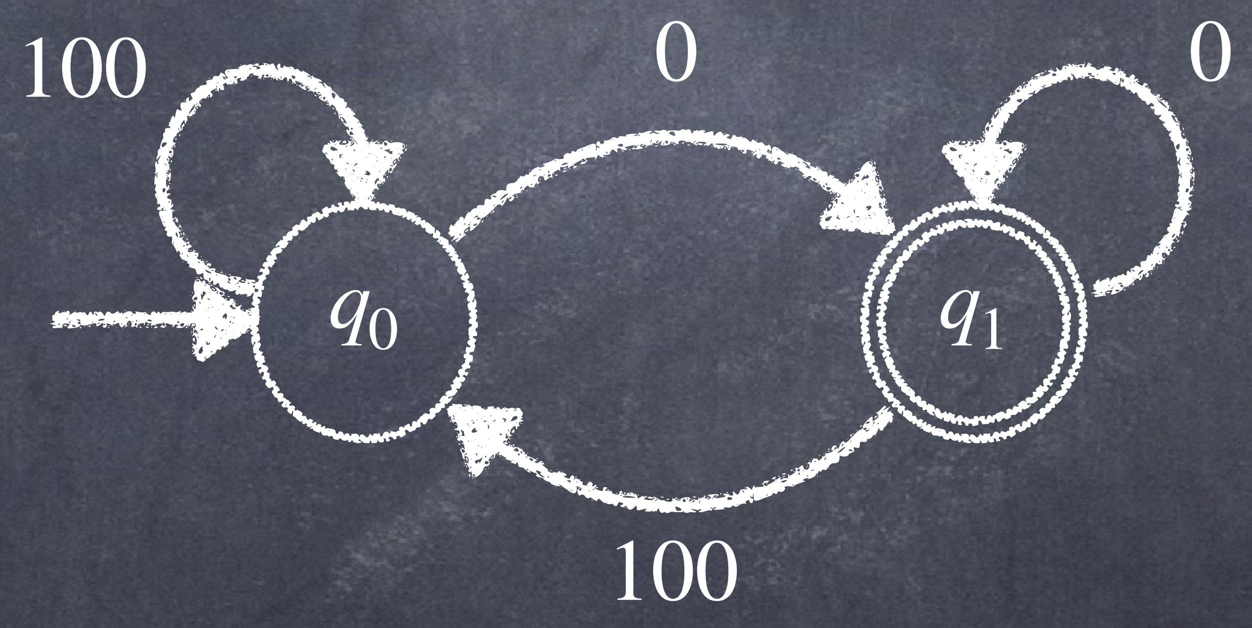

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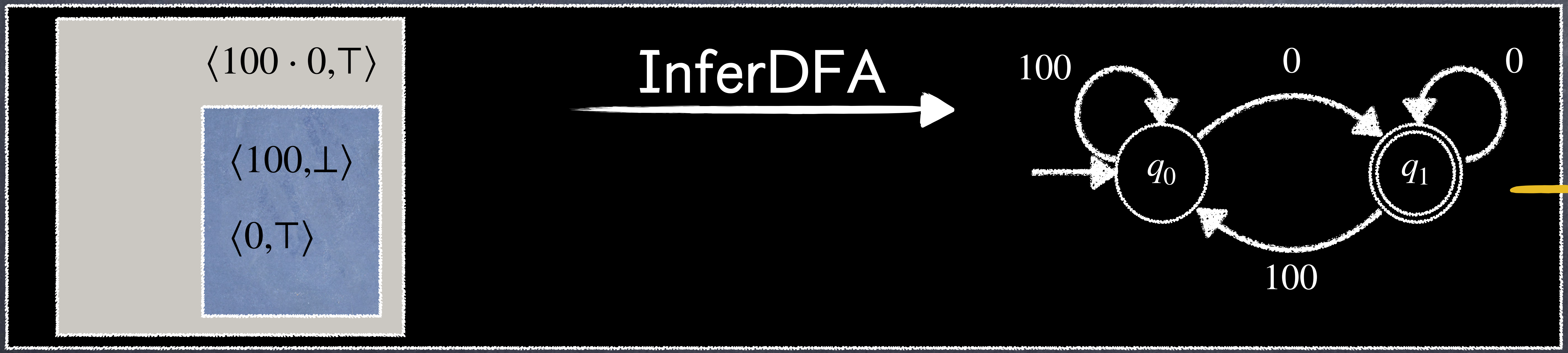

SFA

Concrete Sample set

DFA Generalize

The Whole Process

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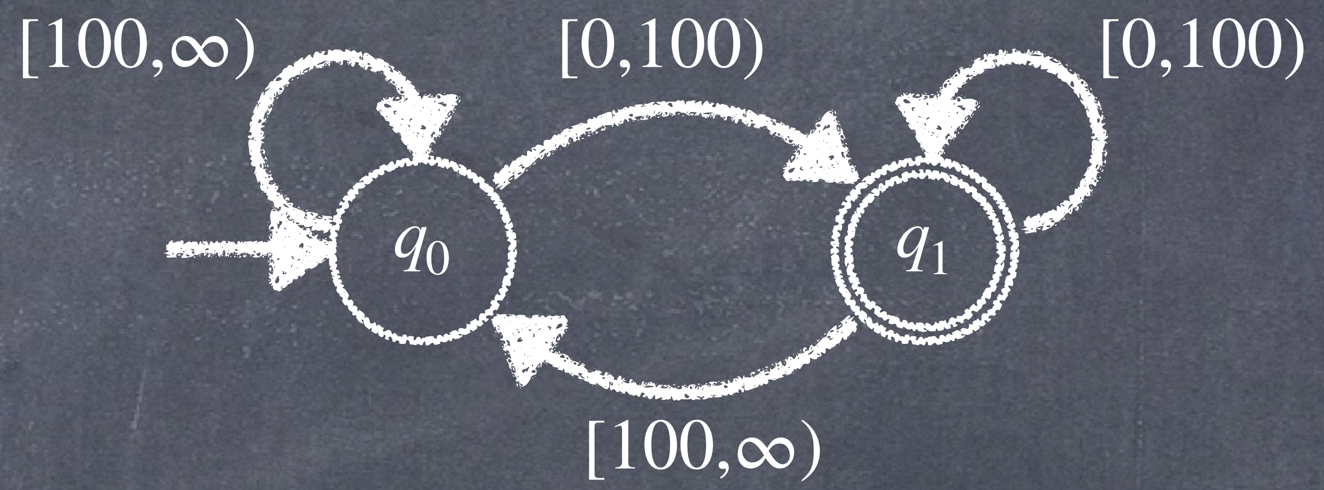

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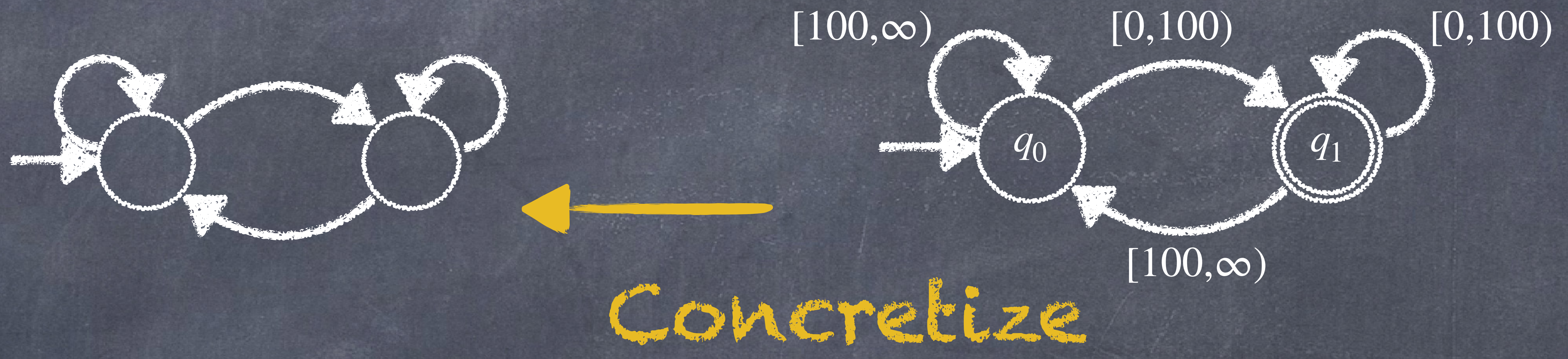

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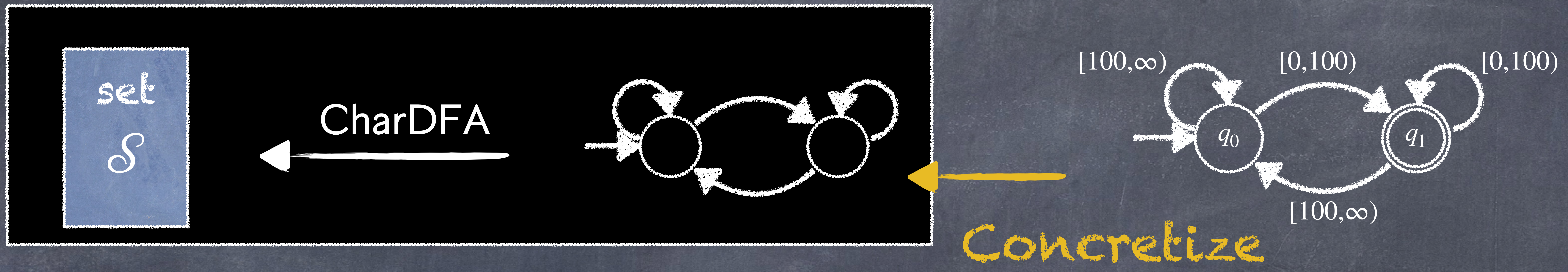

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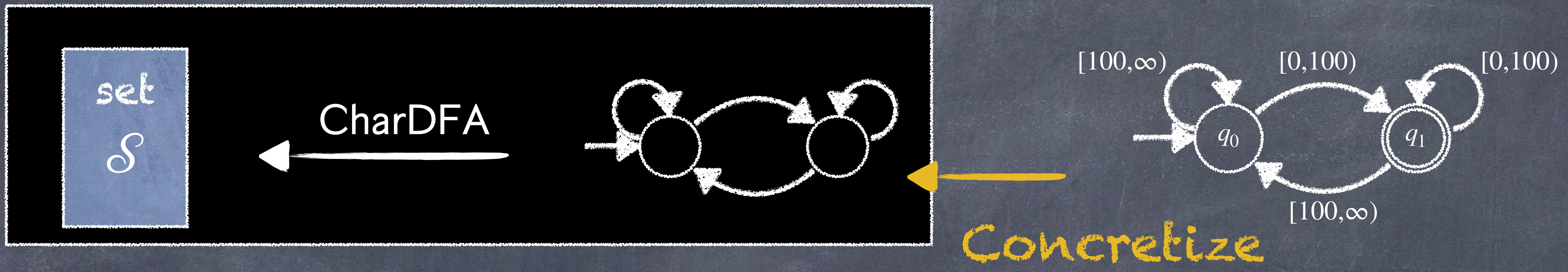

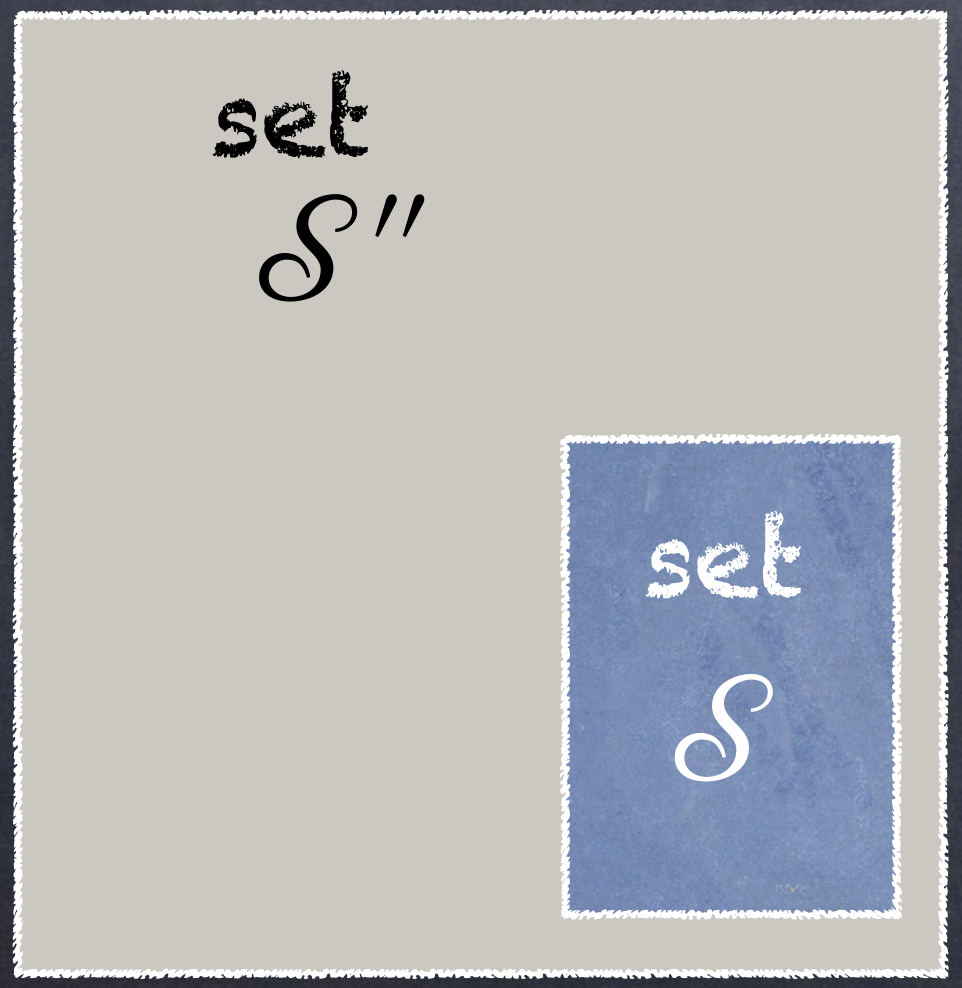

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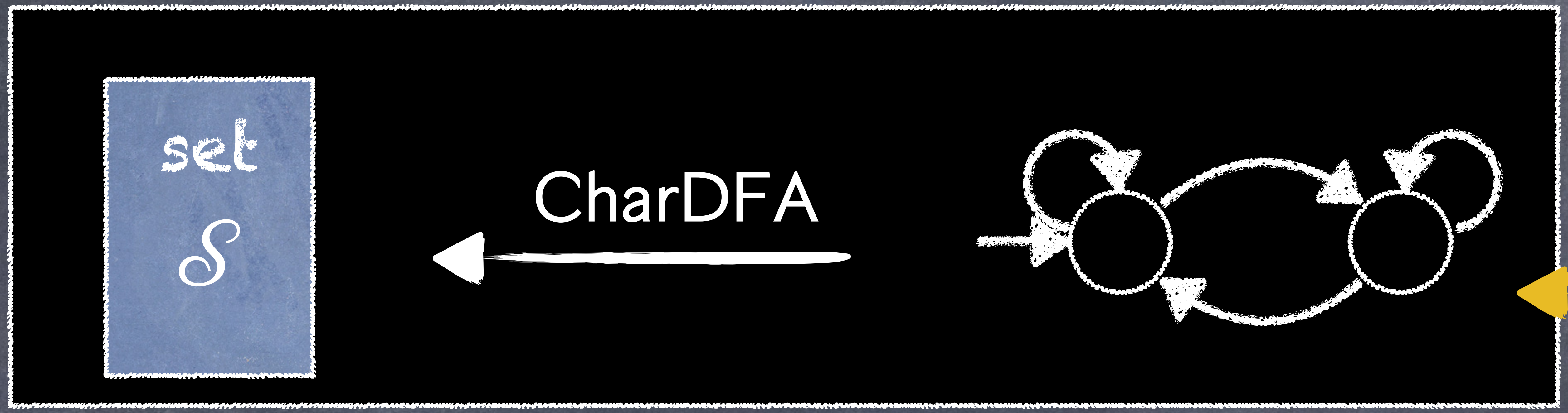
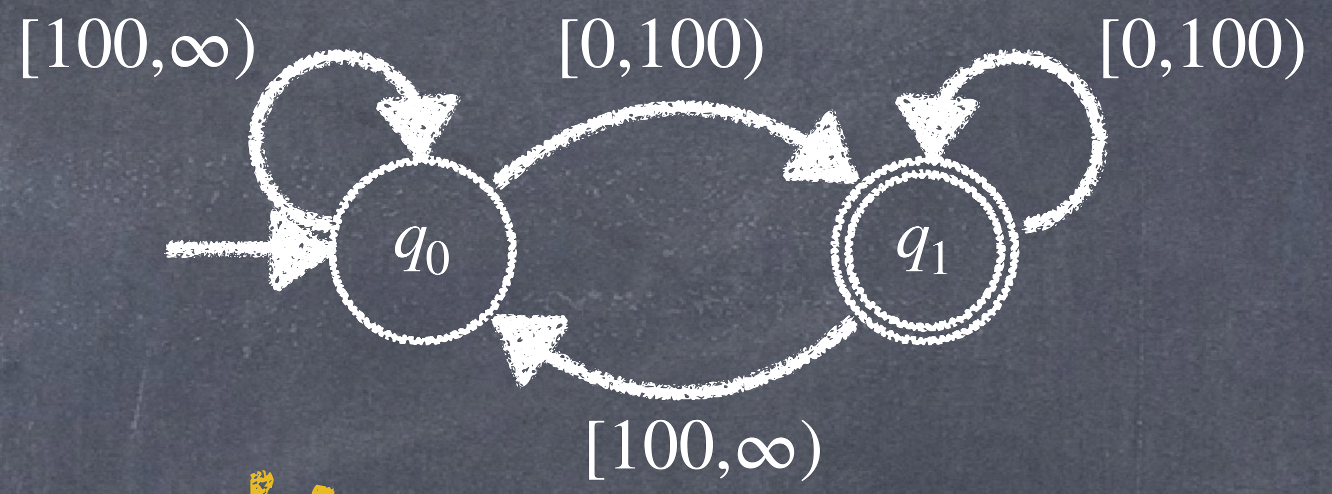
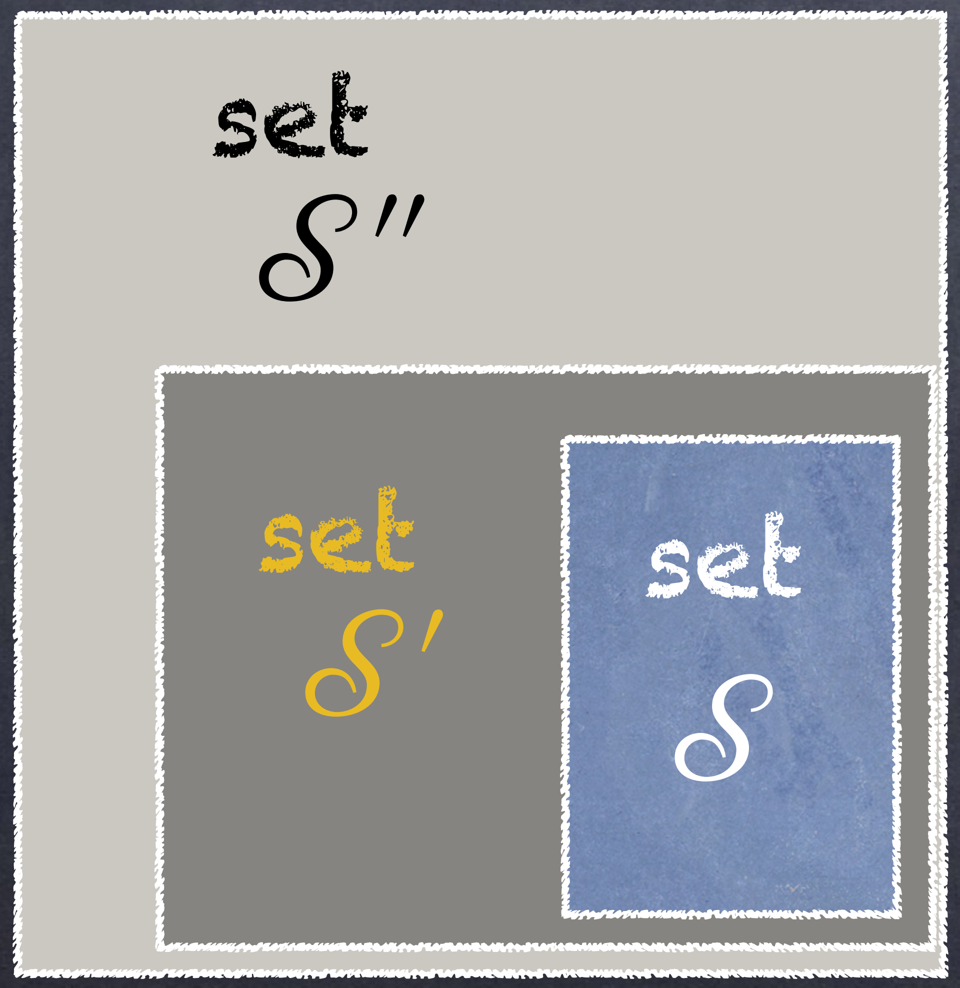




Concretize

Decontaminate



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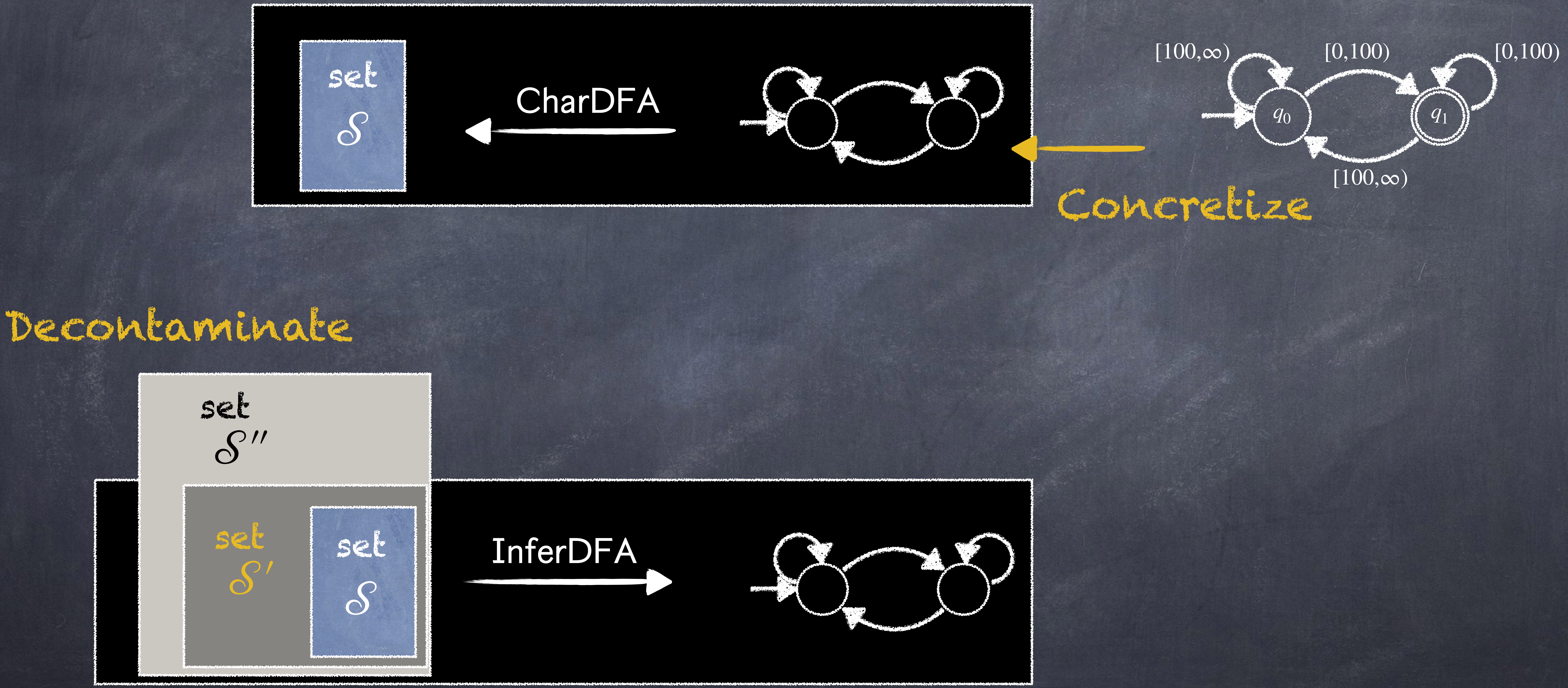

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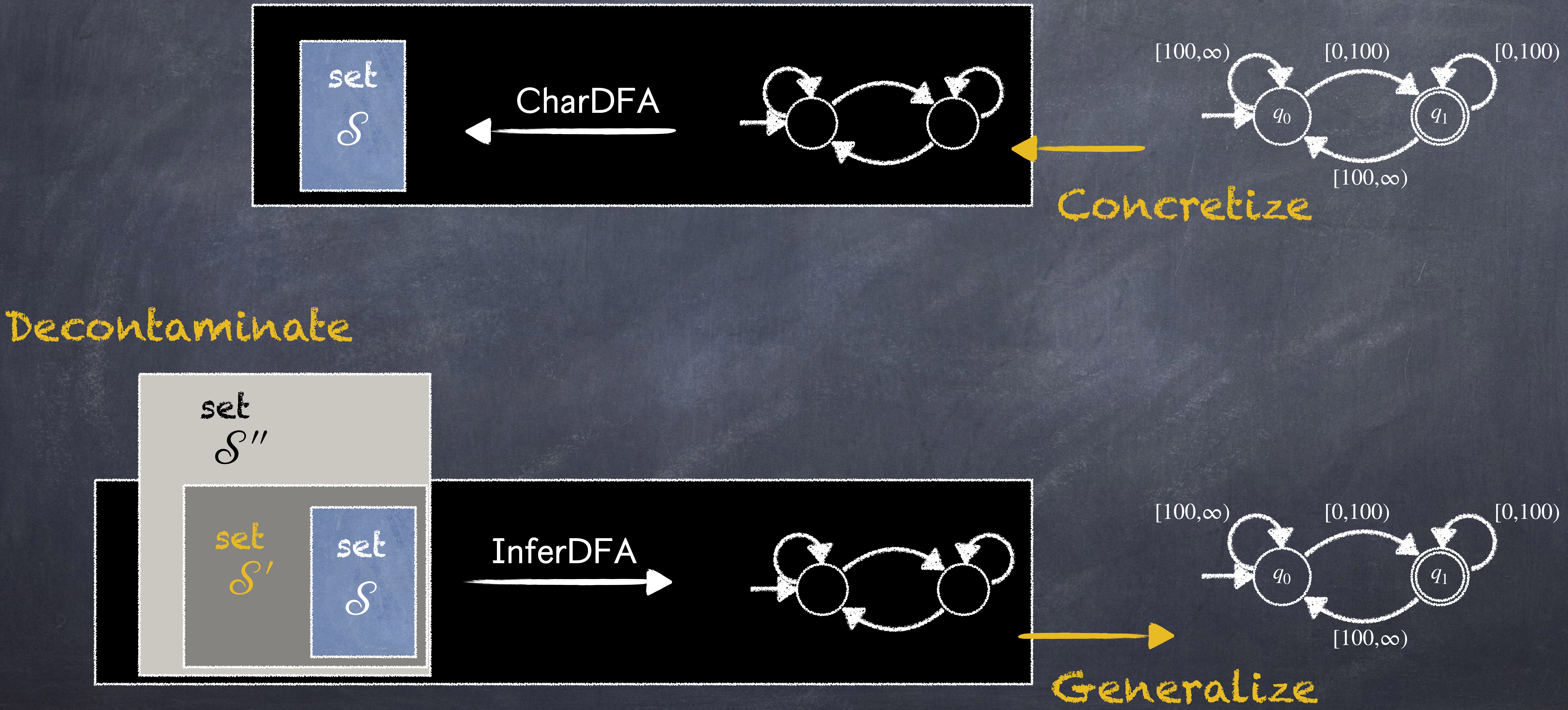
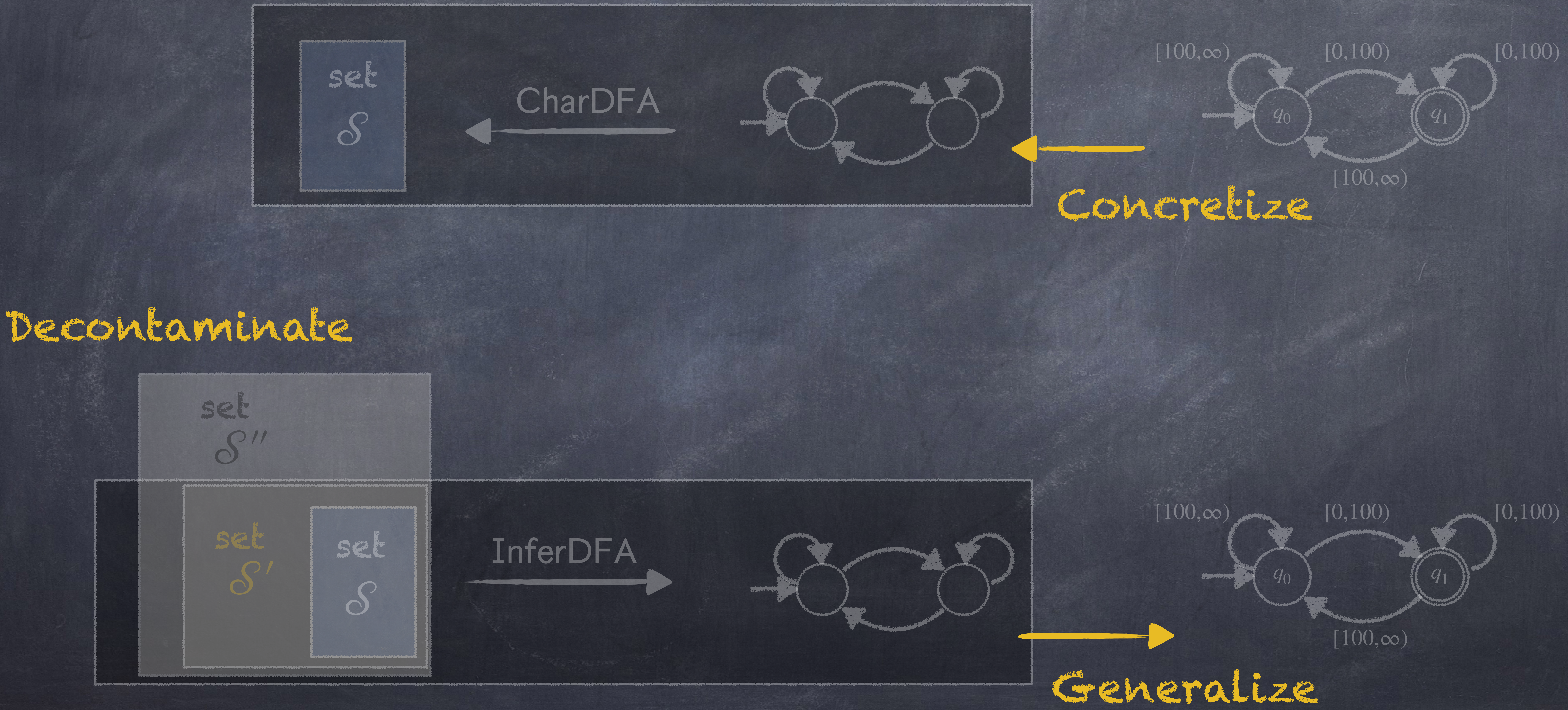
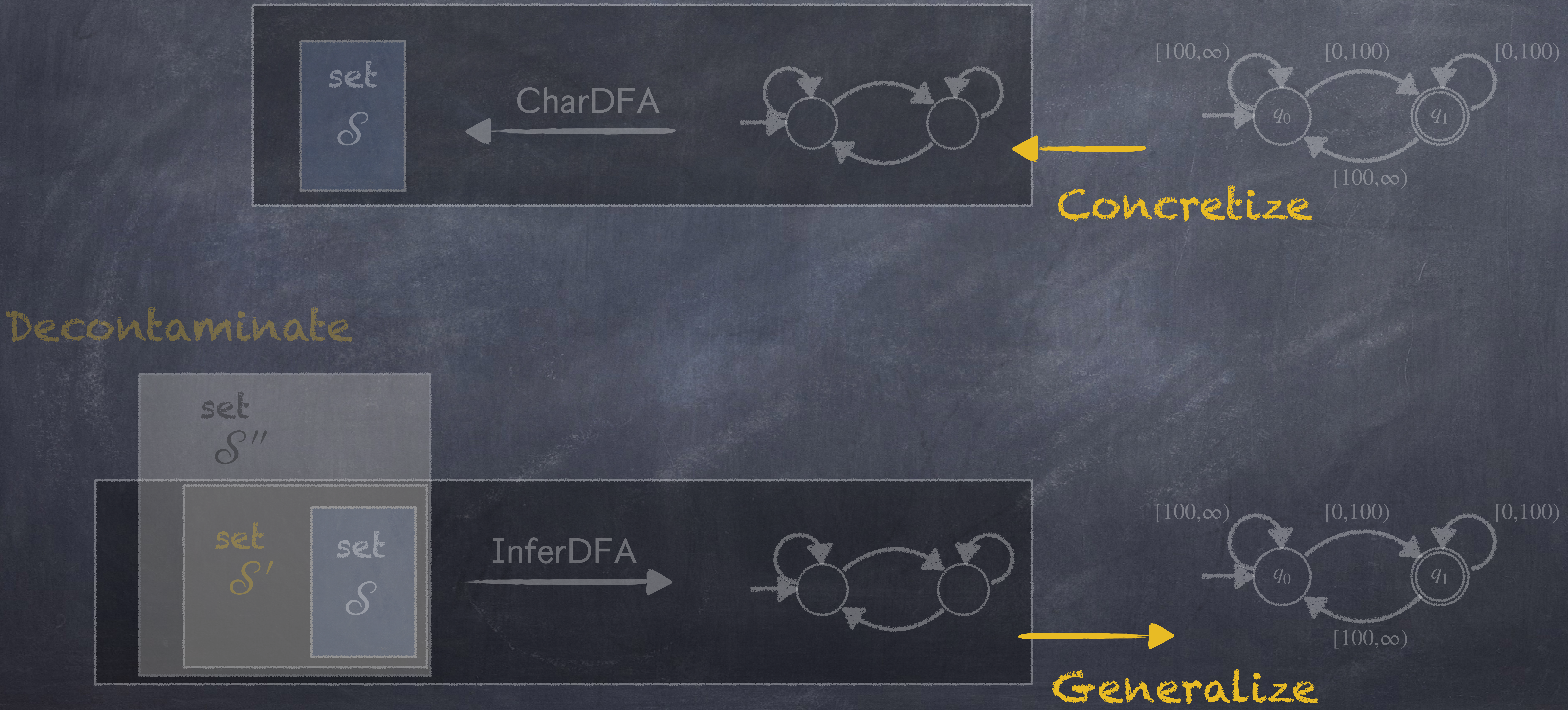


Sufficient Condition



Sufficient Condition

Necessary Condition



Necessary Condition

Summary

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  - Necessary condition
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THANK YOU!

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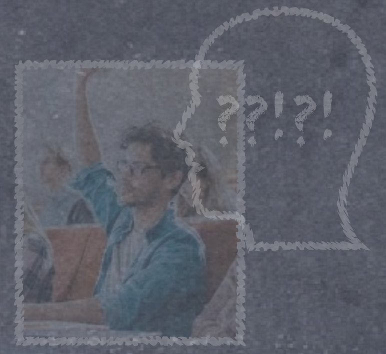
Questions?

- Passive Learning



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