

Actual Causality in Reactive Systems

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CISPA
HELMHOLTZ CENTER FOR
INFORMATION SECURITY



Joint work with

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Explaining Hyperproperty Violations

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Hadar Frenkel¹, Christopher Hahn¹, Tom Horak³, Niklas Metzger¹,
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Abstract. Hyperproperties relate multiple computation traces to each other. Model checkers for hyperproperties thus return, in case a system model violates the specification, a set of traces as a counterexample. Fixing the erroneous relations between traces in the system that led to the counterexample is a difficult manual effort that highly benefits from additional explanations. In this paper, we present an explanation method for counterexamples to hyperproperties described in the specification logic HyperLTL. We extend Halpern and Pearl's definition of actual causality to sets of traces witnessing the violation of a HyperLTL formula, which allows us to identify the events that caused the violation. We report on the implementation of our method and show that it significantly improves on previous approaches for analyzing counterexamples returned by HyperLTL model checkers.

Temporal Causality in Reactive Systems

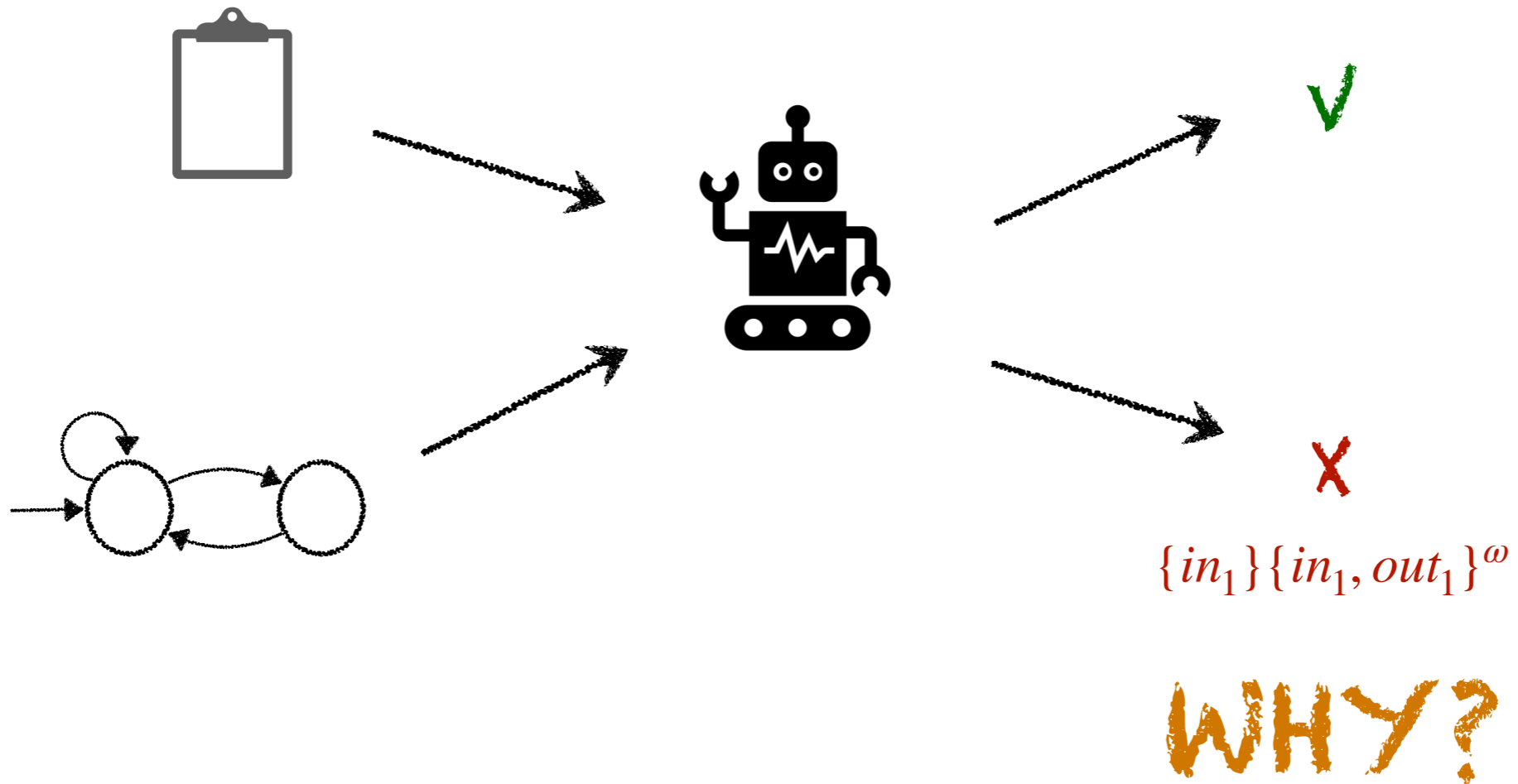
Norine Coenen¹, Bernd Finkbeiner¹, Hadar Frenkel¹,
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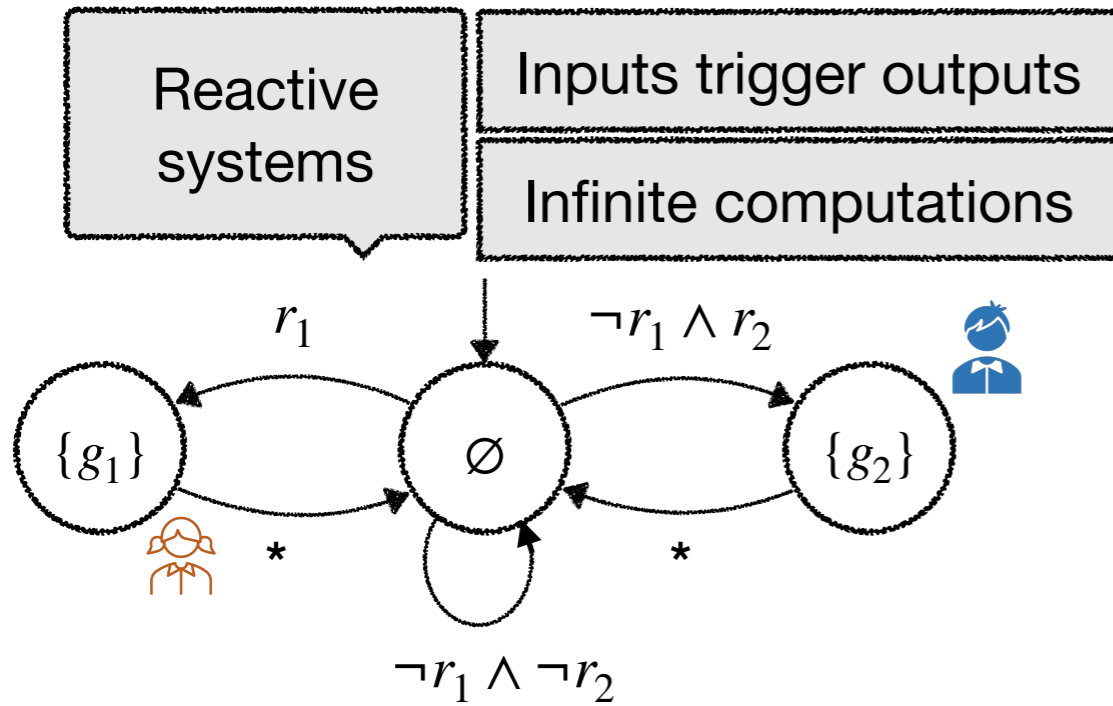
² Stanford University, Stanford, USA
hahn@cs.stanford.edu

Abstract. Counterfactual reasoning is an approach to infer what causes an observed effect by analyzing the hypothetical scenarios where a suspected cause is not present. The seminal works of Halpern and Pearl have provided a workable definition of counterfactual causality for finite settings. In this paper, we propose an approach to check causality that is tailored to reactive systems, i.e., systems that interact with their environment over a possibly infinite duration. We define causes and effects as trace properties which characterize the input and observed output behavior, respectively. We then instantiate our definitions for ω -regular properties and give automata-based constructions for our approach. Checking that an ω -regular property qualifies as a cause can then be encoded as a hyperproperty model-checking problem.

Model Checking

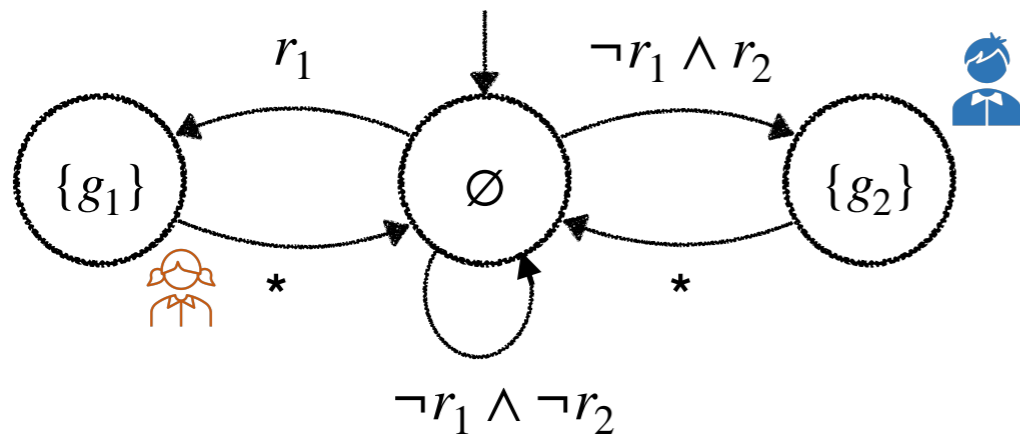


Model Checking



Model Checking

Causes over input sequences
Analyse the system dynamics



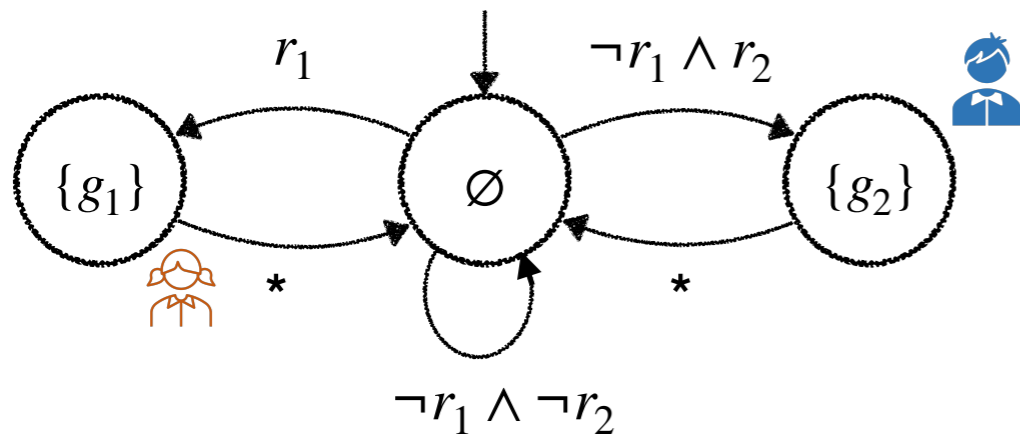
Is “always r_1 ”
the cause for “always not g_2 ”?

$$\pi \left(\begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline \end{array} \left(\begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline g_1 & \end{array} \begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline \end{array} \right)^\omega \right)$$

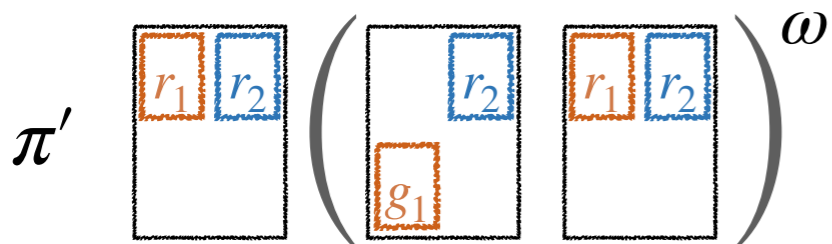
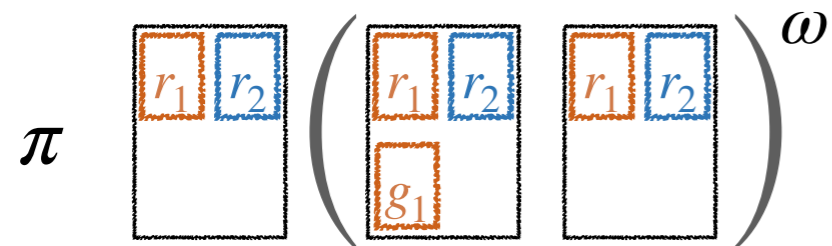


“eventually g_2 ”

Causality as a Hyperproperty



Is “always r_1 ”
the cause for “always not g_2 ”?



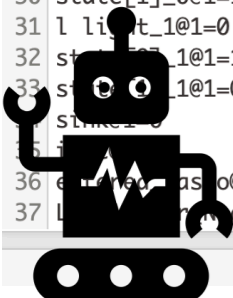
	φ	
	φ	
	φ	
	φ	

$\forall \pi \exists \pi' \varphi$

Compare π with the
counterfactual trace π'

Actual Causality in Reactive Systems

```
Outputfiles
Generated Aiger  Generated Dot  Counter Example
1 in_0@0=0
2 in_1@0=1
3 I:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@0=1
4 I:remember_state@0=0
5 l light_0@0=0
6 state[0]_0@0=0
7 state[1]_0@0=0
8 l light_1@0=0
9 state[0]_1@0=0
10 state[1]_1@0=0
11 sink@0=0
12 init@0=0
13 entered_lasso@0=0
14 L:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@0=0
15 l0_copy@0=0
16 l1_copy@0=0
17 l2_copy@0=0
18 l3_copy@0=0
19 l4_copy@0=0
20 l5_copy@0=0
21 l6_copy@0=0
22 l7_copy@0=0
23 L_MH:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@0=0
24 in_0@1=0
25 in_1@1=0
26 I:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@1=0
27 I:remember_state@1=0
28 l light_0@1=1
29 state[0]_0@1=0
30 state[1]_0@1=1
31 l light_1@1=0
32 state[0]_1@1=1
33 state[1]_1@1=0
34 sink@1=0
35 entered_lasso@1=0
36 L:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@1=1
37 L_MH:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@1=1
```



MCHyper

1

$\{\langle p, \pi, 0 \rangle\}$

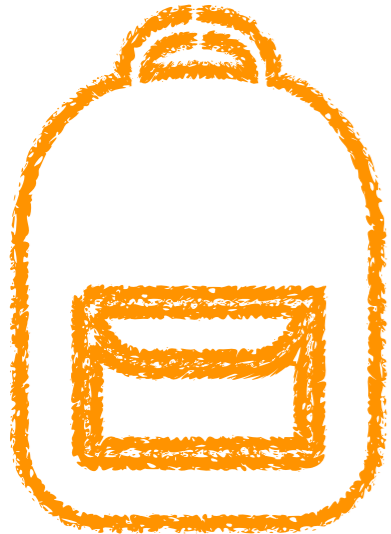
Specific events on the trace that cause the violation

2

“infinitely often p”

Trace properties

Explainability — analysis of the counterexample
Applicability — repair



Hyperproperties

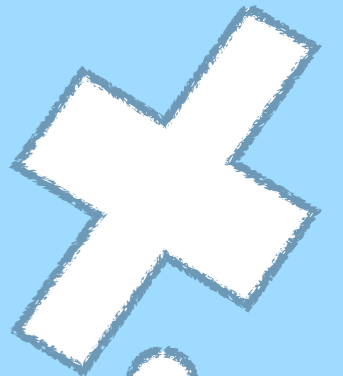
Halpern & Pearl
Causality

Causality in
reactive
systems

Causes as
sets of events

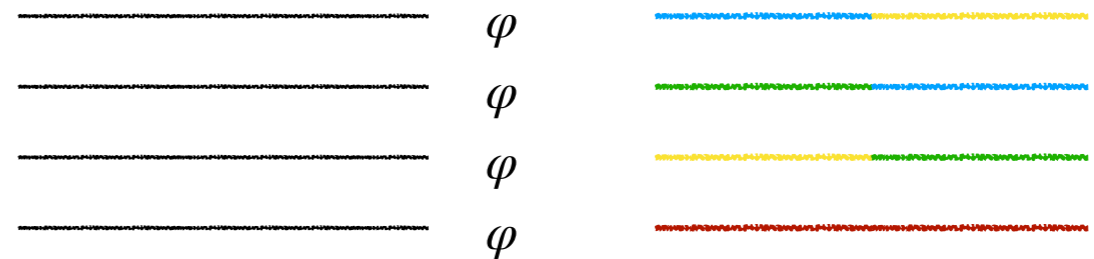
Causes as
temporal
properties

Causality as a
Hyperproperty



Hyperproperties

- Extend trace properties (e.g., in LTL) to system properties
- Reason about sets of traces



$$\forall \pi \exists \pi' \varphi$$

Linear Temporal Logic – LTL

$\square p$ – p holds at every timepoint

$\diamond p$ – p eventually holds

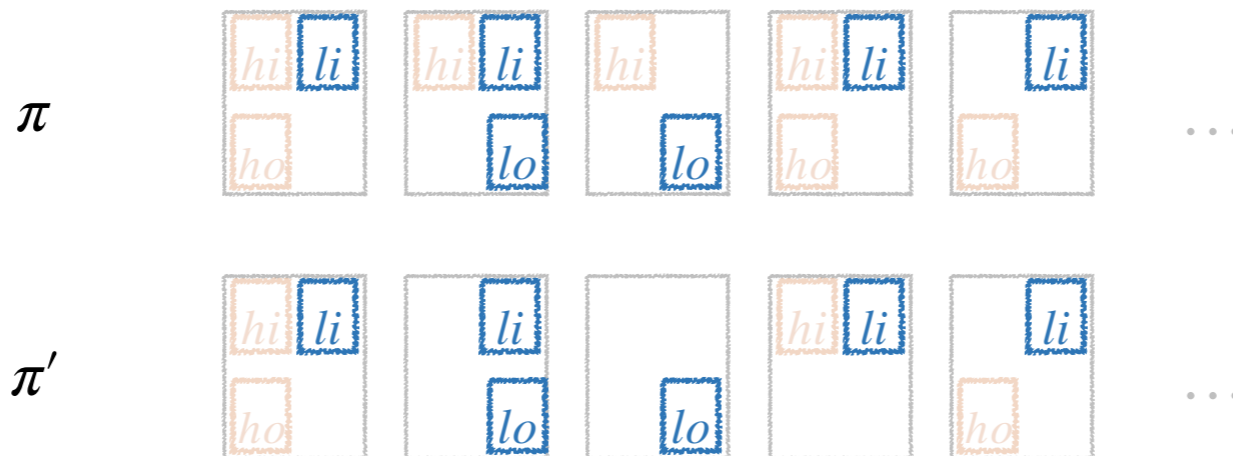
$\bigcirc p$ – p holds at the next timepoint

Hyperproperties

- Extend trace properties (e.g., in LTL) to system properties
- Reason about sets of traces

HyperLTL – extending LTL with trace quantification

- Observational determinism: $\forall \pi \forall \pi' \square (li_{\pi} \leftrightarrow li_{\pi'}) \rightarrow \square (lo_{\pi} \leftrightarrow lo_{\pi'})$



Hyperproperties

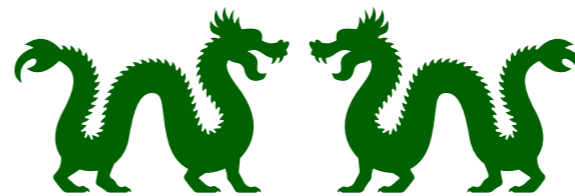
- Extend trace properties (e.g., in LTL) to system properties
- Reason about sets of traces



Information-flow
properties
[Observational
determinism]



Robustness



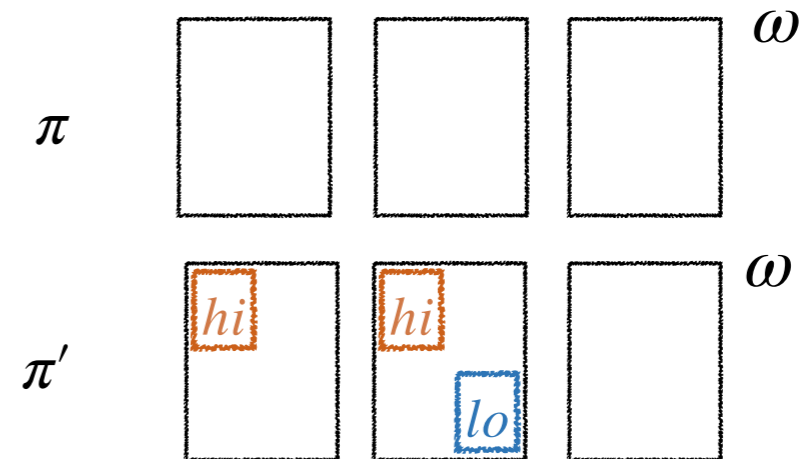
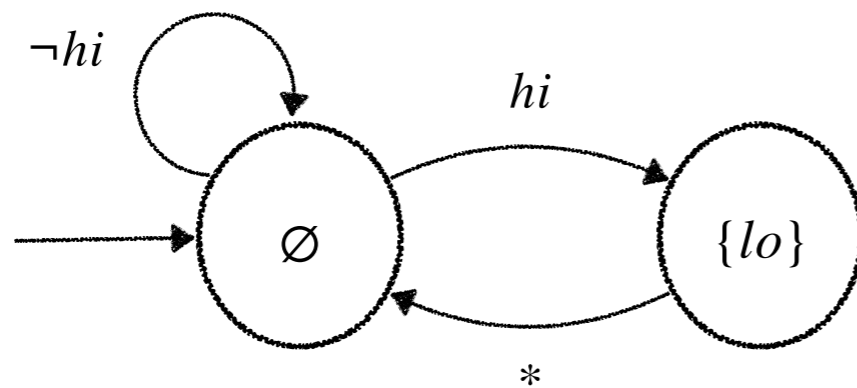
Symmetry



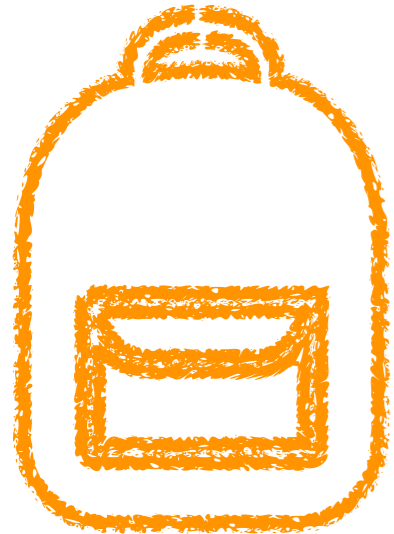
Causality

Explaining Hyperproperty Violations

$$\forall \pi \forall \pi' \square (li_{\pi} \leftrightarrow li_{\pi'}) \rightarrow \square (lo_{\pi} \leftrightarrow lo_{\pi'})$$



WHY?



Hyperproperties

Halpern & Pearl
Causality

Joseph Y. Halpern, Judea Pearl:

Causes and Explanations: A Structural-Model Approach: Part 1: Causes. UAI 2001: 194-202

Causes and Explanations: A Structural-Model Approach. Part I: Causes

Author(s): Joseph Y. Halpern and Judea Pearl

Source: *The British Journal for the Philosophy of Science*, Dec., 2005, Vol. 56, No. 4 (Dec., 2005), pp. 843-887

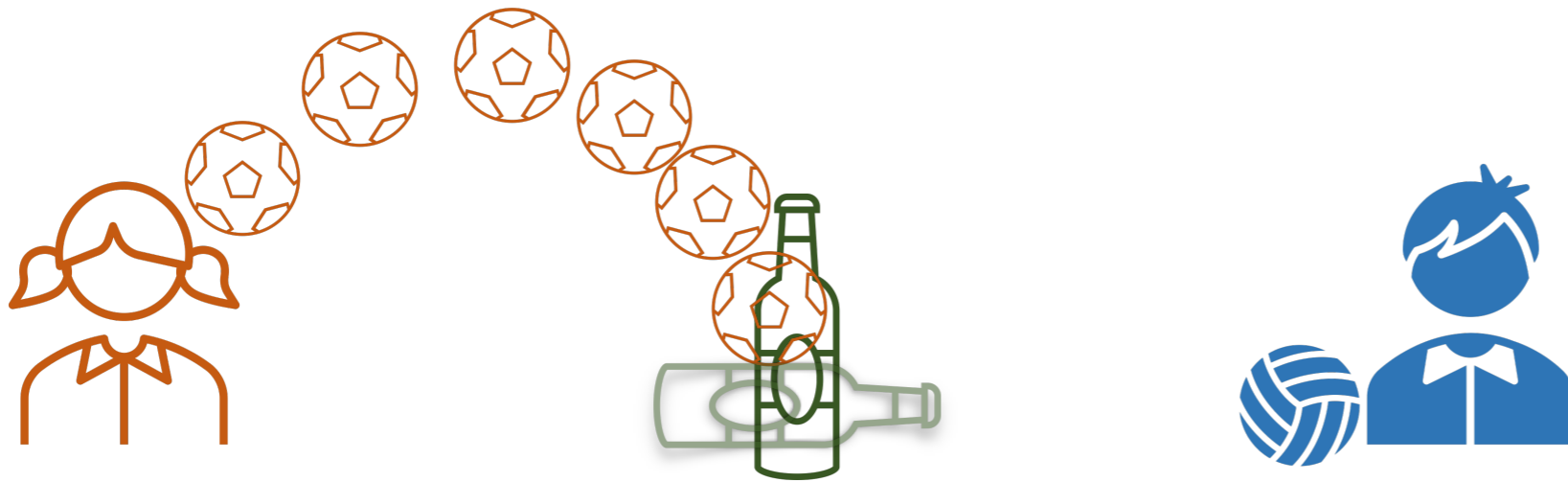
Proceedings of the Twenty-Fourth International Joint Conference on Artificial Intelligence (IJCAI 2015)

A Modification of the Halpern-Pearl Definition of Causality

Joseph Y. Halpern*
Cornell University

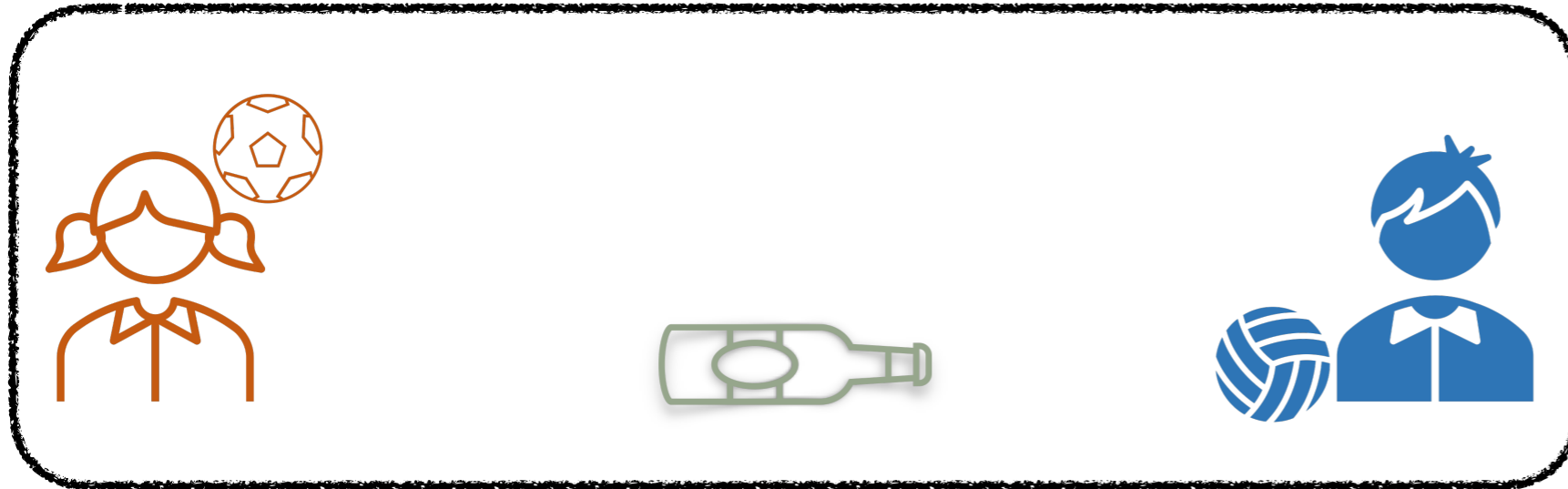
Causality in
reactive
systems

Actual Causality



Actual Causality

Actual world



Counterfactual world

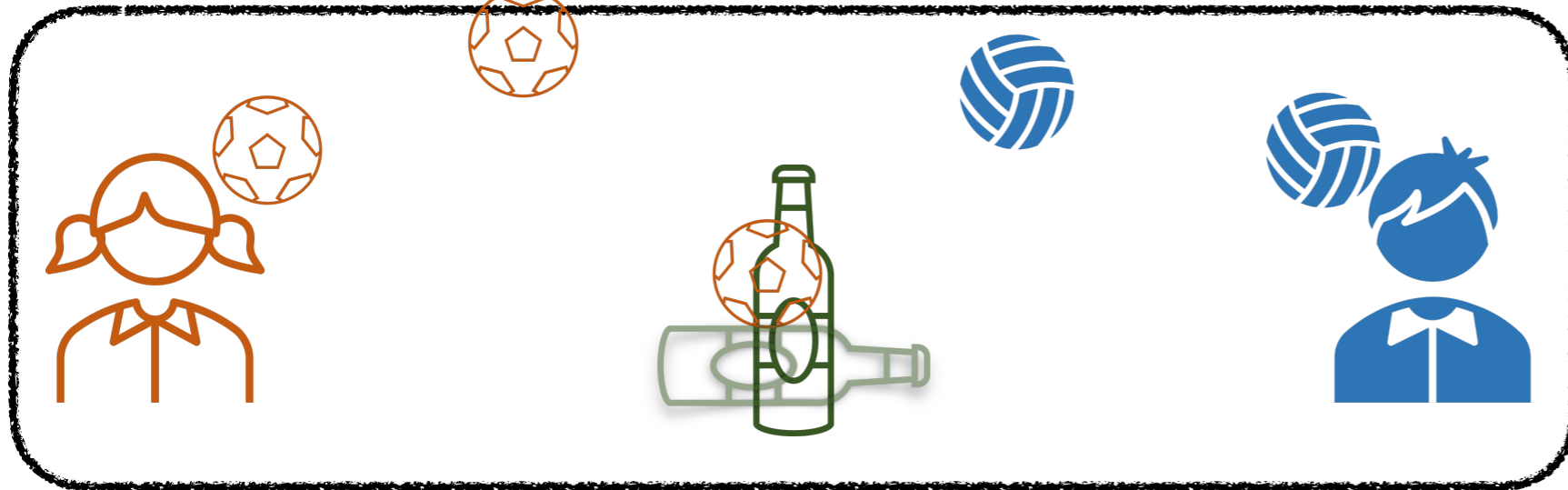


Hyperproperties
Relate multiple
system executions



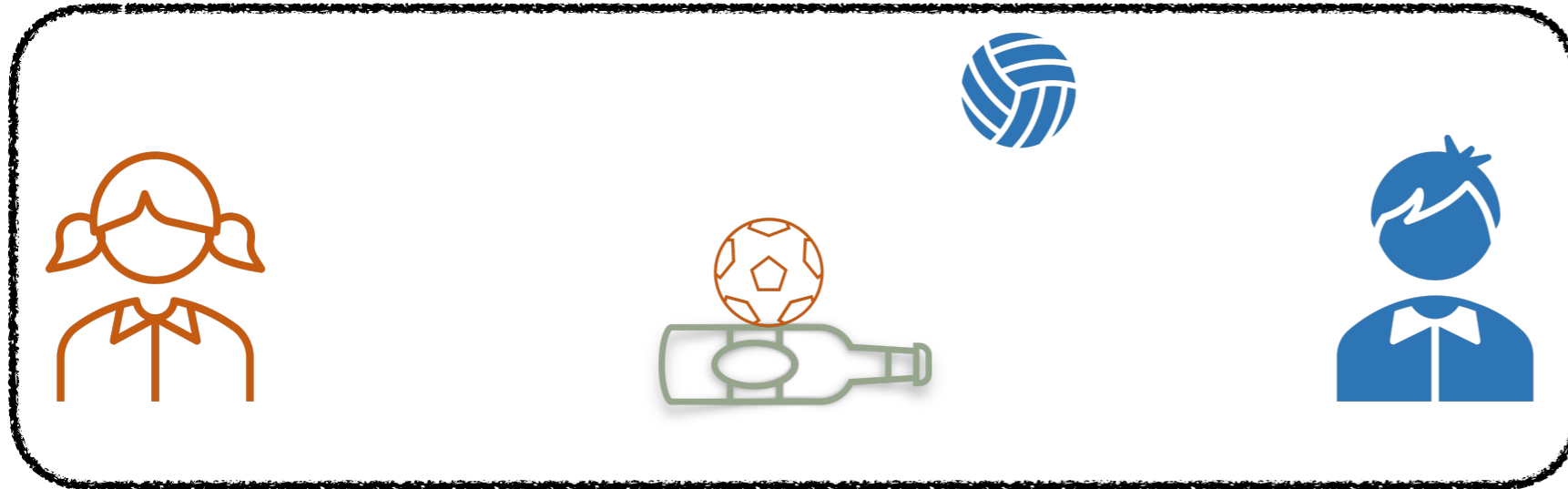
Actual Causality

Actual world

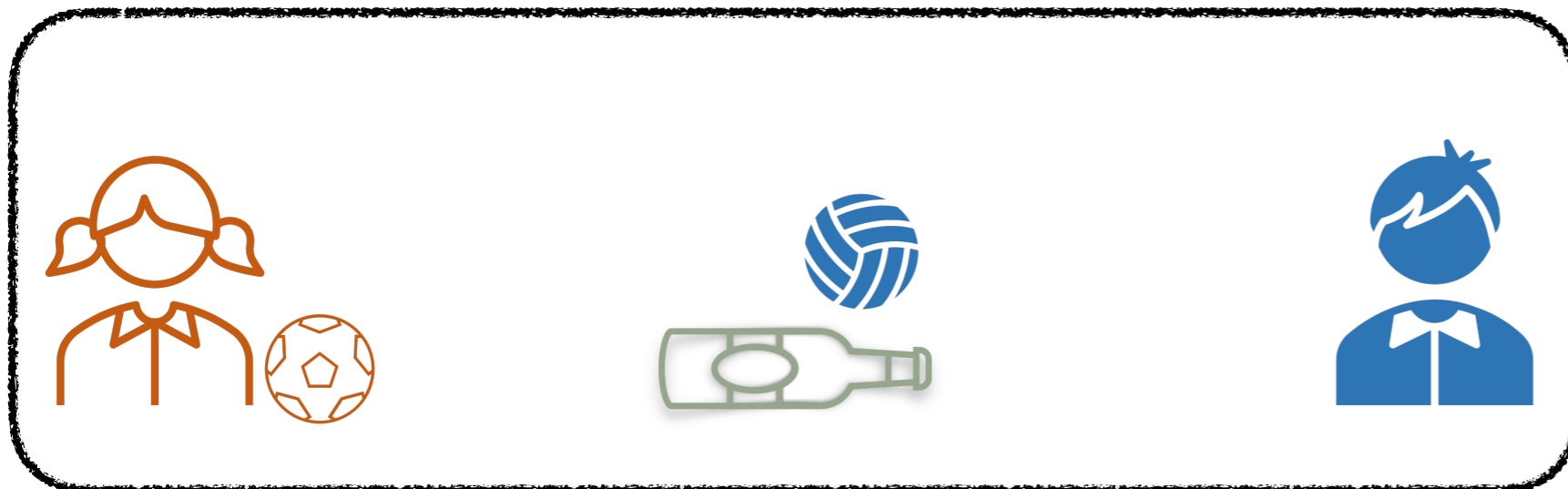


Actual Causality

Actual world



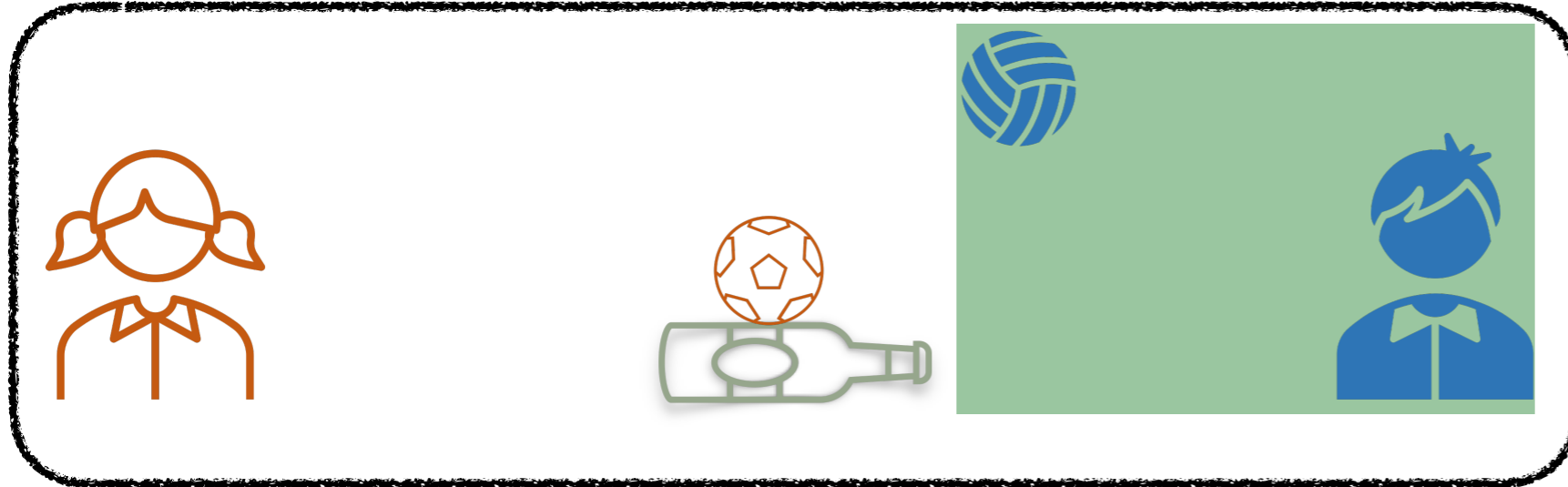
Counterfactual world



Billy's Ball breaks the bottle!

Actual Causality Contingencies

Actual world



preemption
of causes

Counterfactual world + contingency



Actual Causality



AC1: the cause appears in the actual world



AC2: for every counterfactual world there exists a contingency where **effect** does not hold



AC3: this is a minimal cause

Actual Causality



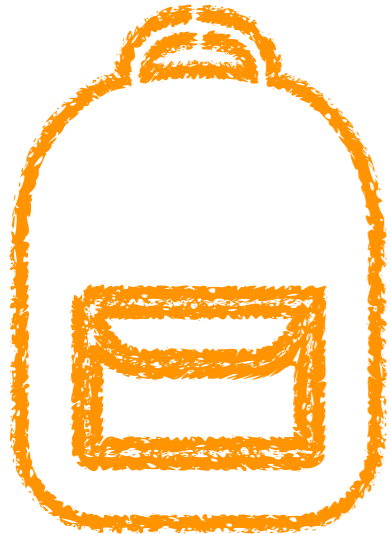
SAT: the cause appears in the actual world



CF: for every counterfactual world there exists a contingency where **effect** does not hold



MIN: this is a minimal cause



Hyperproperties

**Halpern & Pearl
Causality**

**Causality in
reactive
systems**

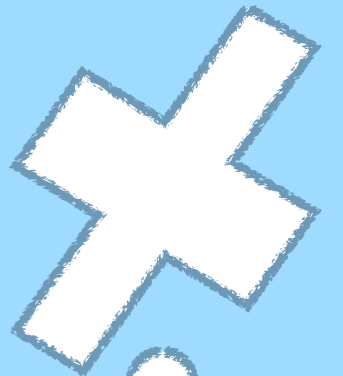
**Causes as
sets of events**

*Explaining
Counterexamples
Using Causality.*
Beer, Ben-David,
Chockler, Orni, and
Trefler. (CAV 2009).

*Causality Checking
for Complex System
Models.* Leitner-
Fischer, Leue.
(VMCAI 2013)

**Causes as
temporal
properties**

**Causality as a
Hyperproperty**



Actual Causality for Hyperproperties



SAT



CF : for every counterfactual world there exists a contingency where **effect** does not hold



MIN

Actual Causality for Hyperproperties

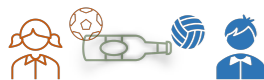
\forall^* prefix:

$$\psi = \forall \pi_1 \forall \pi_2 \exists \pi'_1 \exists \pi'_2 . \varphi$$

$$\neg \psi = \exists \pi_1 \exists \pi_2 \forall \pi'_1 \forall \pi'_2 . \neg \varphi$$



- Effect: a violation of a **Hyperproperty** ψ



- Actual World: a set Γ of counterexample traces

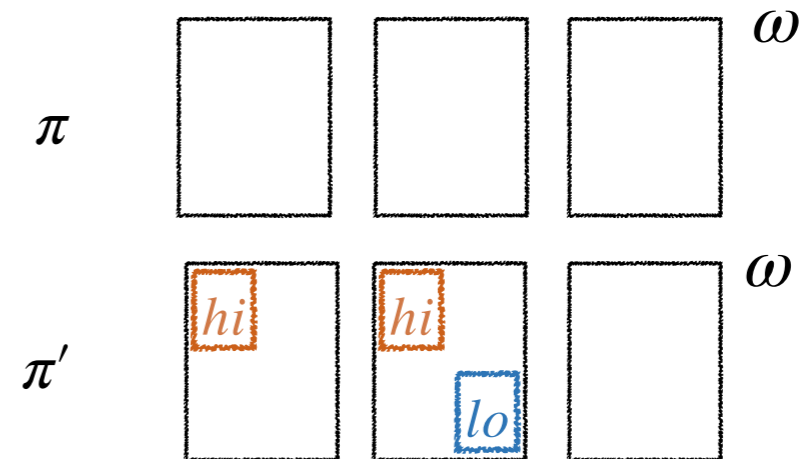
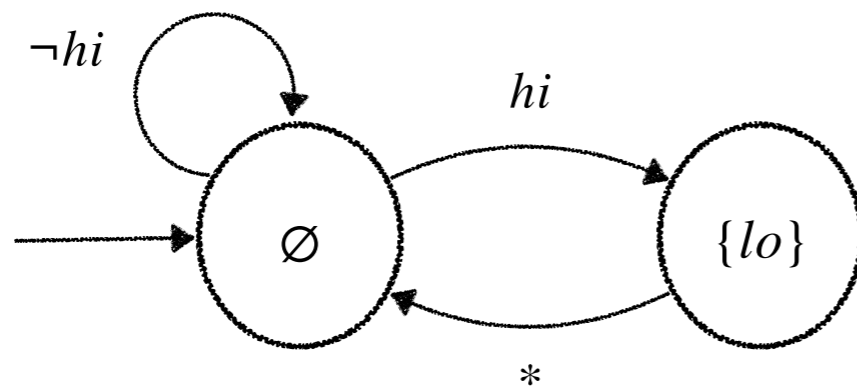


- Cause: set of events on the set of traces

Lasso-shaped

Explaining Hyperproperty Violations

$$\forall \pi \forall \pi' \square (li_{\pi} \leftrightarrow li_{\pi'}) \rightarrow \square (lo_{\pi} \leftrightarrow lo_{\pi'})$$

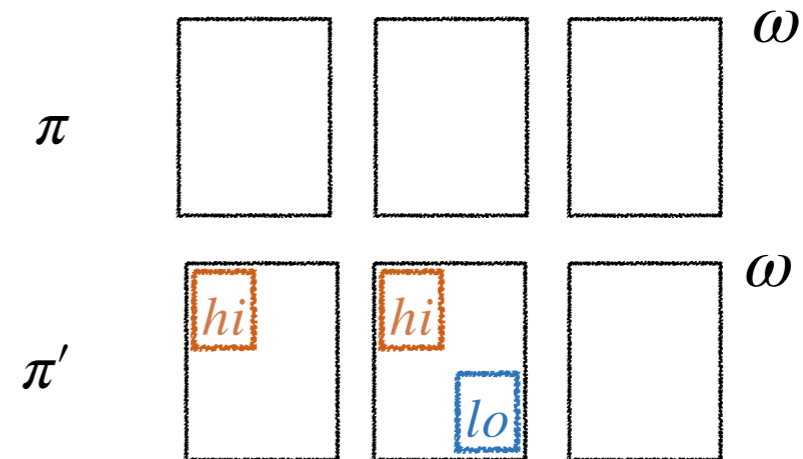
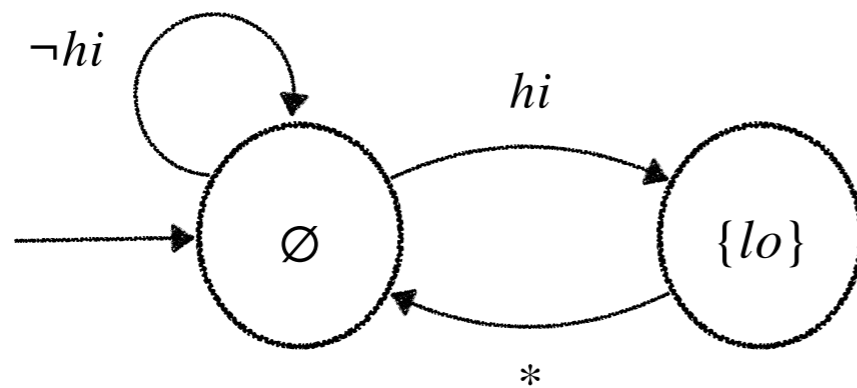


WHY?

Explaining Hyperproperty Violations

CF: \forall counterfactual \exists contingency s.t. φ holds

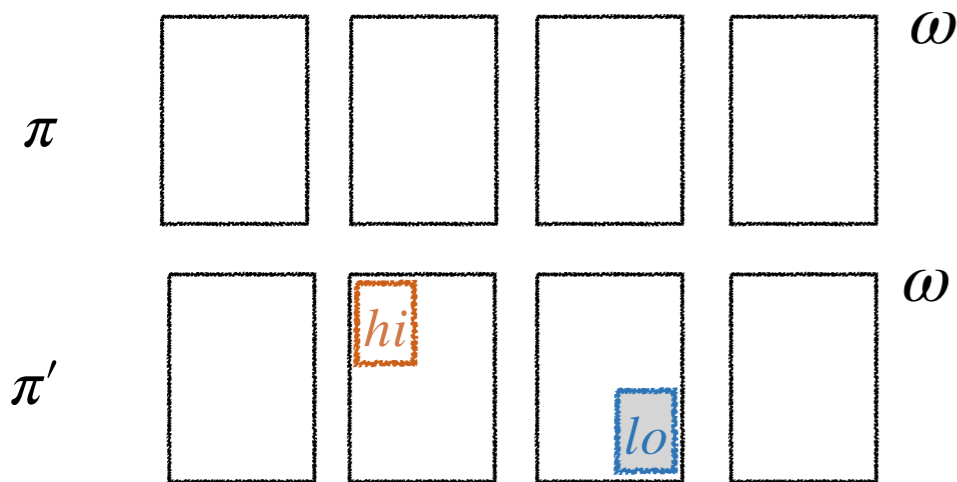
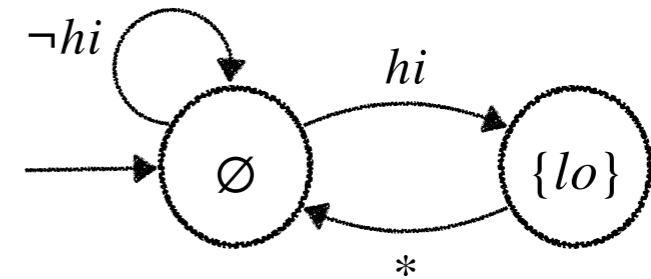
$$\forall \pi \forall \pi' \boxed{(li_{\pi} \leftrightarrow li_{\pi'}) \rightarrow \boxed{(lo_{\pi} \leftrightarrow lo_{\pi'})}^{\varphi}}$$



Explaining Hyperproperty Violations

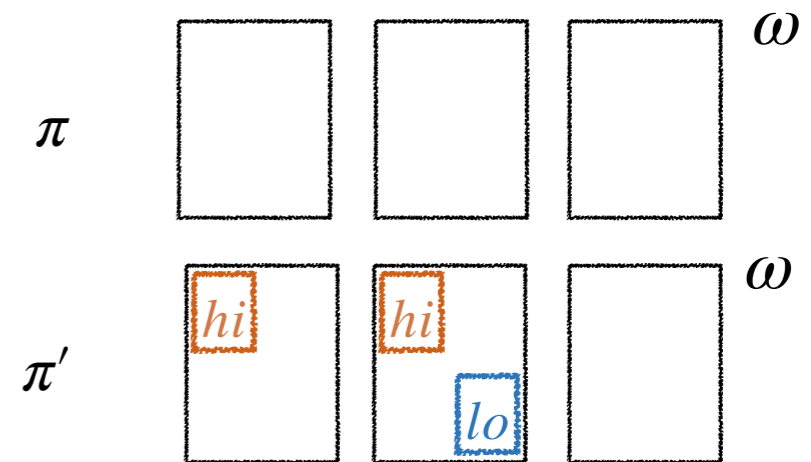
CF: \forall counterfactual \exists contingency s.t. φ holds

$$\forall \pi \forall \pi' \square (li_{\pi} \leftrightarrow li_{\pi'}) \rightarrow \square (lo_{\pi} \leftrightarrow lo_{\pi'})$$



$intervene(\Gamma, C, \emptyset)$

Flip all events in C



$$C = \{ \langle hi, 0, \pi' \rangle \}$$

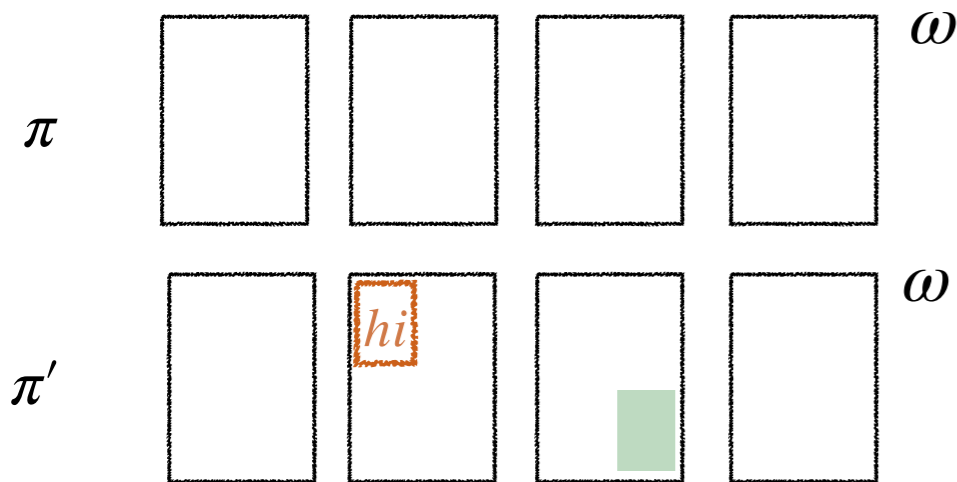
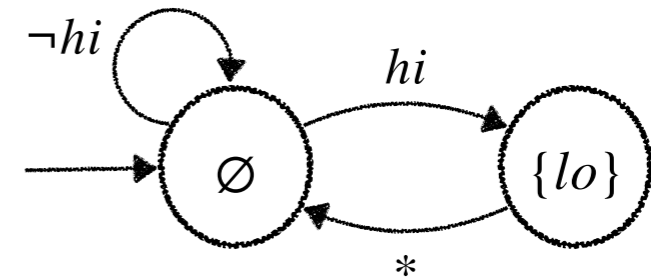
$$\Gamma = (\pi, \pi')$$

Cause - set of events

Explaining Hyperproperty Violations

CF: \forall counterfactual \exists contingency s.t. φ holds

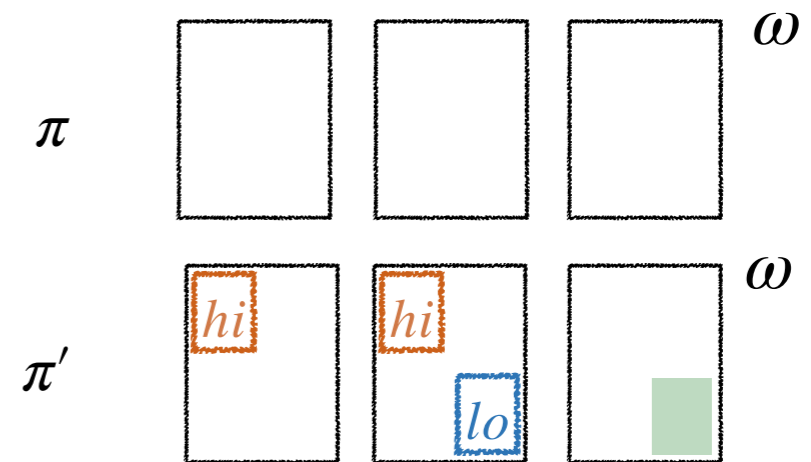
$$\forall \pi \forall \pi' \square (li_{\pi} \leftrightarrow li_{\pi'}) \rightarrow \square (lo_{\pi} \leftrightarrow lo_{\pi'}) \checkmark$$



$intervene(\Gamma, C, \{\langle lo, 2, \pi' \rangle\})$

Flip all events in C

Setting back to values of the original world

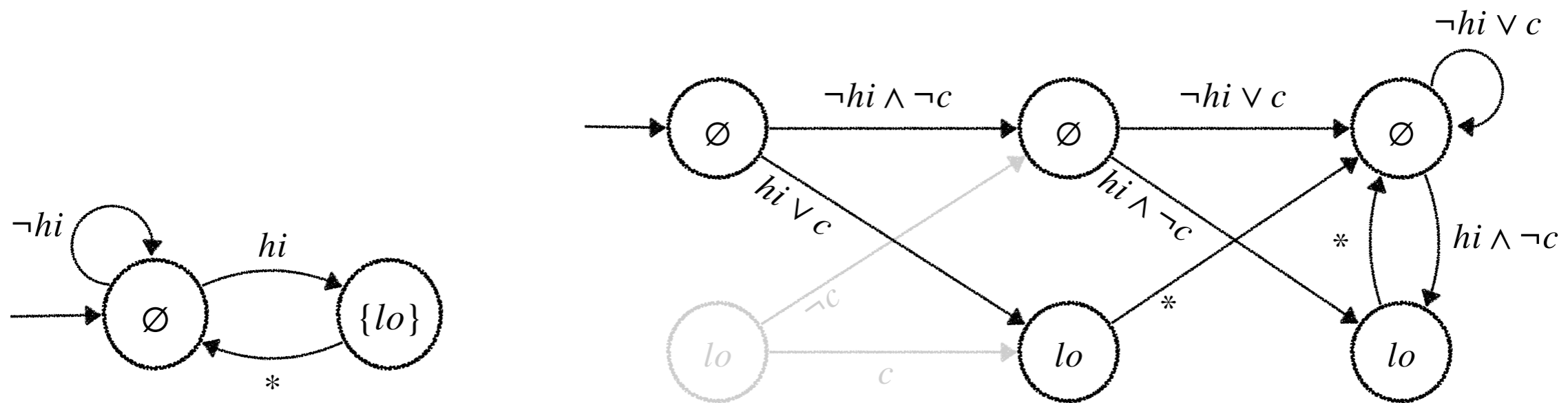


$C = \{\langle hi, 0, \pi' \rangle\}$

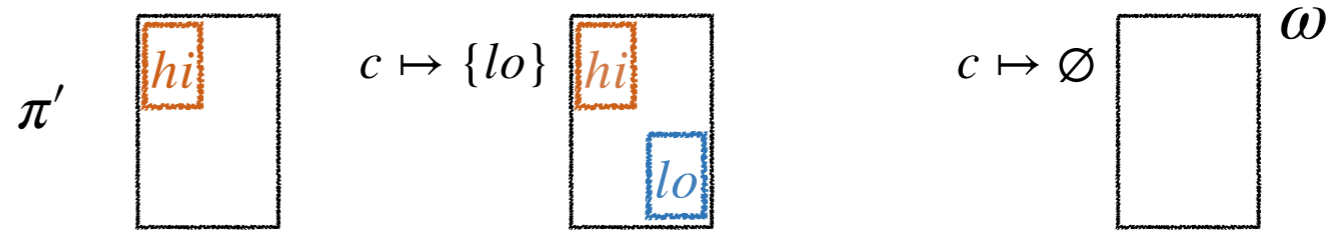
$\Gamma = (\pi, \pi')$

Cause - set of events

Computing Contingencies



an input c_o for each output o



Counterfactual automaton: additional inputs $[c]$ to set a contingency

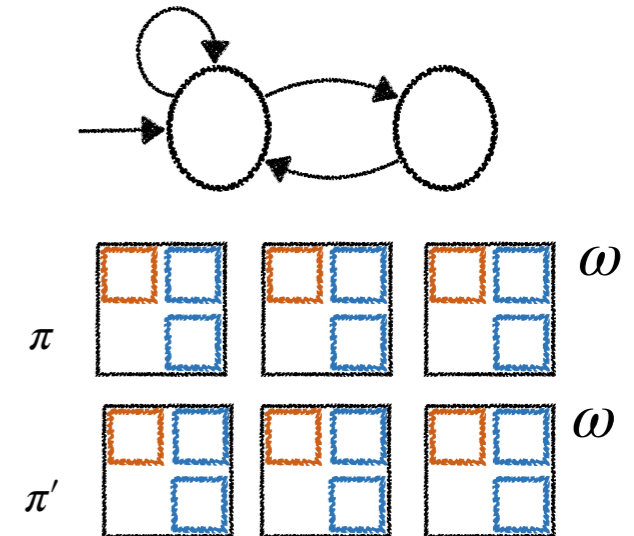
Actual Causality for Hyperproperties

Find C such that

SAT: $\Gamma \models C$

CF: \forall counterfactual \exists contingency s.t. φ holds

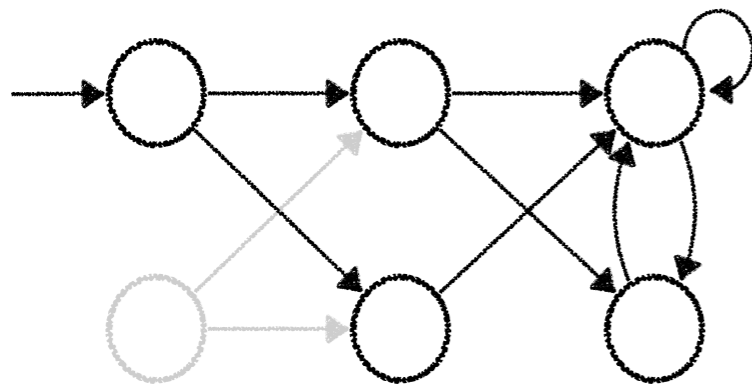
MIN: no subset of C satisfies SAT & CF



$$\forall \pi \forall \pi' \square (li_{\pi} \leftrightarrow li'_{\pi}) \rightarrow \square (lo_{\pi} \leftrightarrow lo'_{\pi})$$

Finding a cause as a hyperproperty

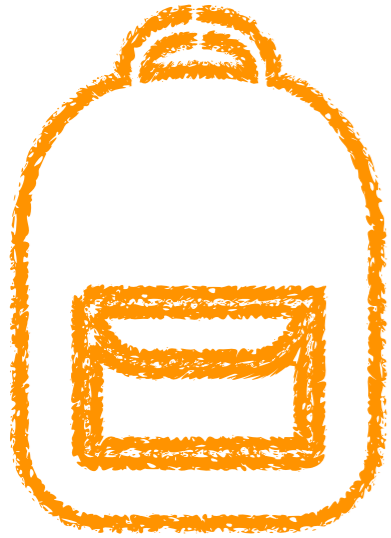
$$\exists \pi_1 \exists \pi_2 . \forall \pi'_1 \forall \pi'_2 . \psi_{cause}$$



HyperLTL
model
checking

Events on π_1, π_2 correspond to the
cause

π'_1, π'_2 represent other possible (not
minimal) causes



Hyperproperties

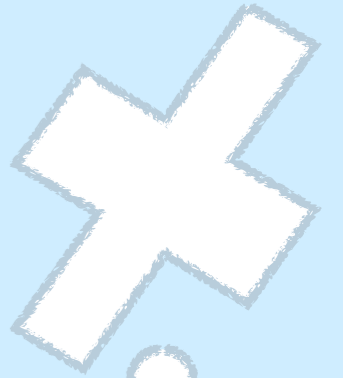
Halpern & Pearl
Causality

Causality in
reactive
systems

Causes as
sets of events

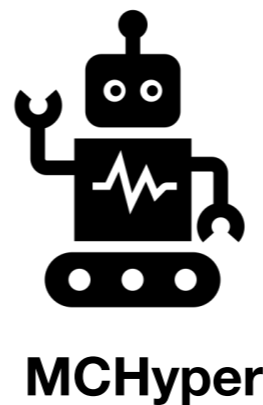
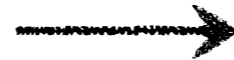
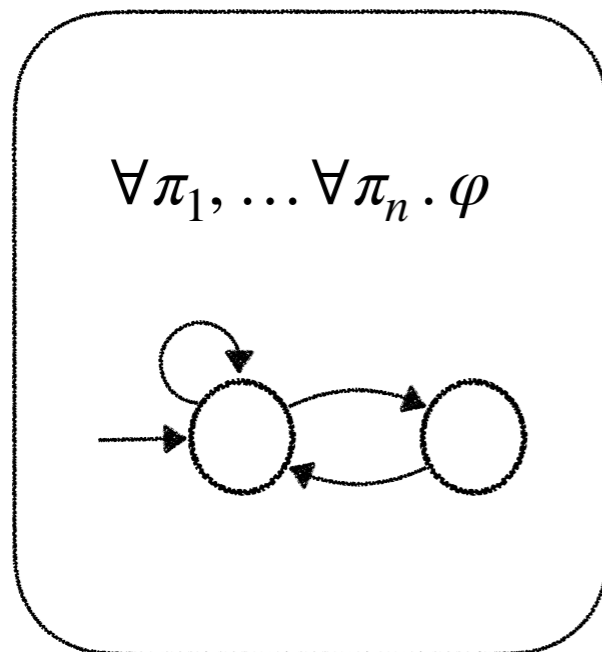
Causes as
temporal
properties

Causality as a
Hyperproperty



Computing Actual Causes

largest candidate cause C – SAT dependencies



MCHyper

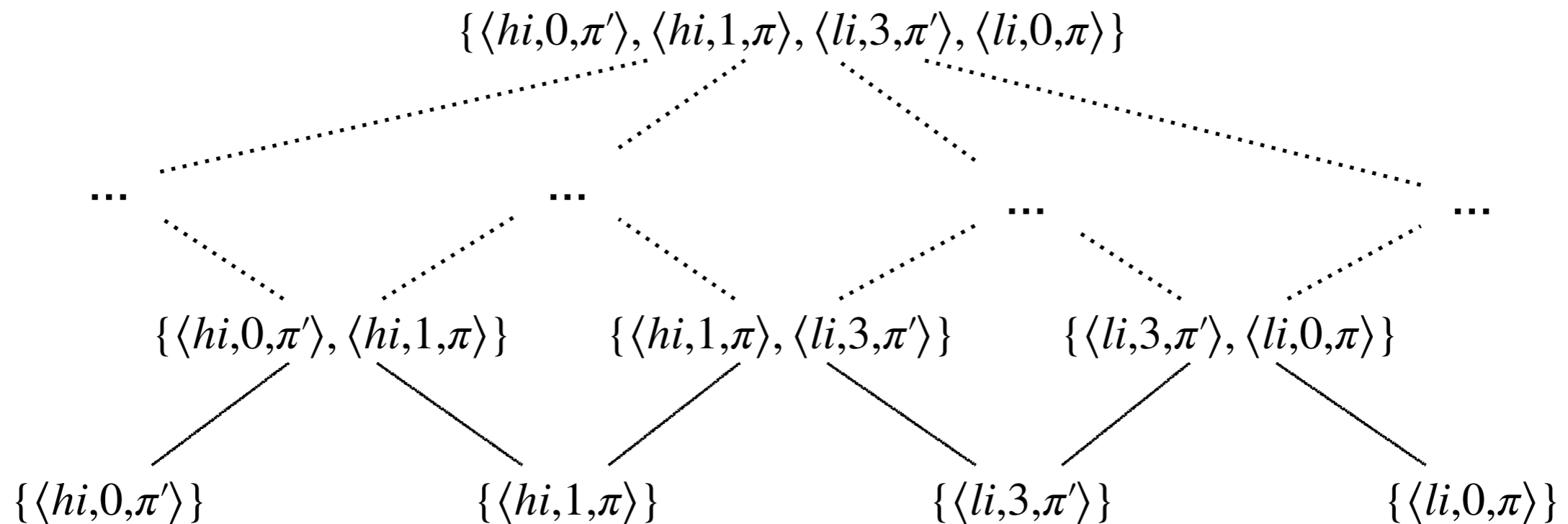


$\Gamma = \pi_1, \dots, \pi_n$

```
Outputfiles
Generated Alger  Generated Dot  Counter Example
1 in_0@0=0
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8 l light_1@0=0
9 state[0]_1@0=0
10 state[1]_1@0=0
11 sink@0=0
12 init@0=0
13 entered_lasso@0=0
14 L:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@0=0
15 l0_copy@0=0
16 l1_copy@0=0
17 l2_copy@0=0
18 l3_copy@0=0
19 l4_copy@0=0
20 l5_copy@0=0
21 l6_copy@0=0
22 l7_copy@0=0
23 L_MH:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@0=0
24 in_0@1=0
25 in_1@1=0
26 I:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@1=0
27 I:remember_state@1=0
28 l light_0@1=1
29 state[0]_0@1=0
30 state[1]_0@1=1
31 l light_1@1=0
32 state[0]_1@1=1
33 state[1]_1@1=0
34 sink@1=0
35 init@1=1
36 entered_lasso@1=0
37 L:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@1=1
```

Computing Actual Causes

largest candidate cause C – SAT dependencies



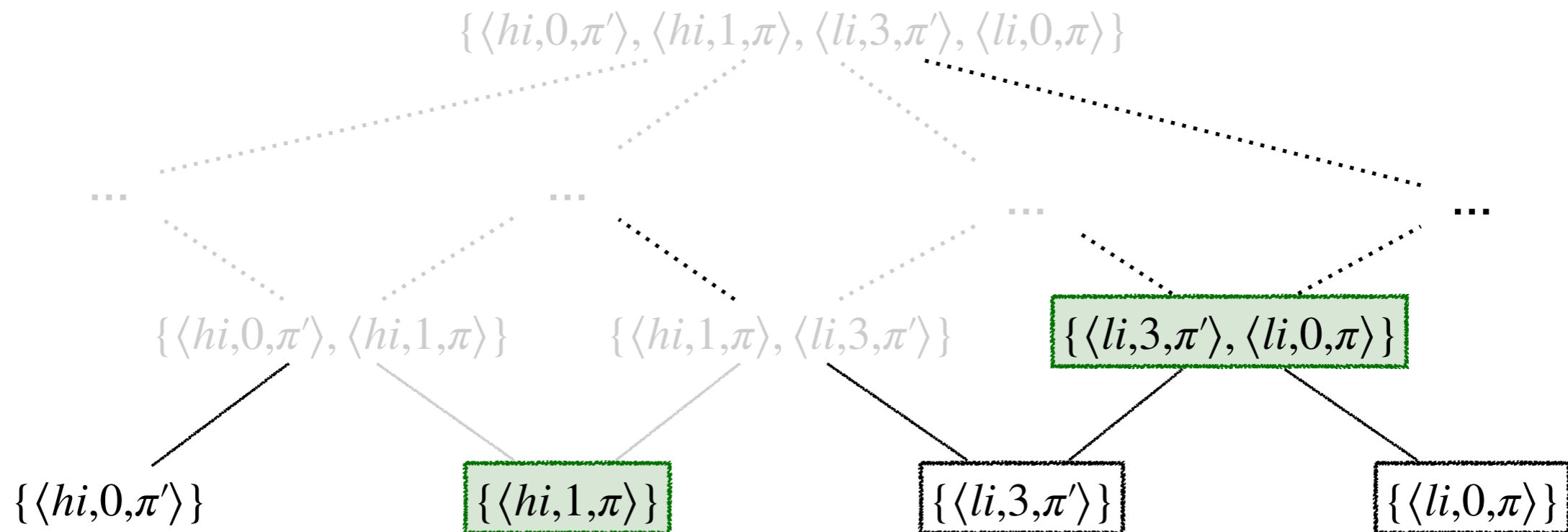
SAT: $\Gamma \models C$

CF: \forall counterfactual \exists contingency s.t. φ holds

MIN: no subset of C satisfies SAT & CF

Computing Actual Causes

largest candidate cause C – SAT dependencies



SAT: $\Gamma \models C$

CF: \forall counterfactual \exists contingency s.t. φ holds

MIN: no subset of C satisfies SAT & CF

Experiments



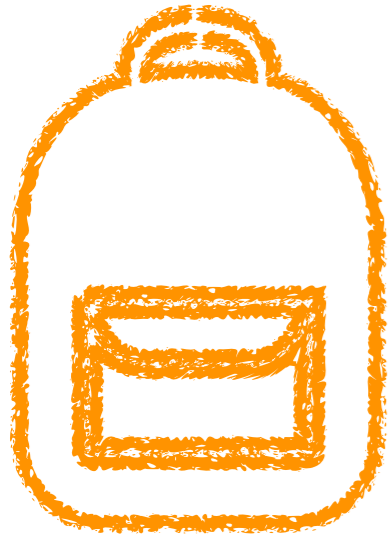
Instance	$ \Gamma $	$ \varphi $	$\#(\mathcal{C})$	time(ms)
Running example (paper)	10	9	2	55
Security in & out	35	19	8	798
Drone example 1	24	19	5	367
Drone example 2	18	36	3	256
Asymmetric arbiter '19	28	35	10	490
Asymmetric arbiter	72	35	24	1480

```

Outputfiles
Generated Alger  Generated Dot  Counter Example
1 in_0@0=0
2 in_1@0=1
3 I:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@0=1
4 I:remember_state@0=0
5 l light_0@0=0
6 state[0]_0@0=0
7 state[1]_0@0=0
8 l light_1@0=0
9 state[0]_1@0=0
10 state[1]_1@0=0
11 sink@0=0
12 init@0=0
13 entered_lasso@0=0
14 L:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@0=0
15 l0_copy@0=0
16 l1_copy@0=0
17 l2_copy@0=0
18 l3_copy@0=0
19 l4_copy@0=0
20 l5_copy@0=0
21 l6_copy@0=0
22 l7_copy@0=0
23 L_MH:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@0=0
24 in_0@1=0
25 in_1@1=0
26 I:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@1=0
27 I:remember_state@1=0
28 l light_0@1=1
29 state[0]_0@1=0
30 state[1]_0@1=1
31 l light_1@1=0
32 state[0]_1@1=1
33 state[1]_1@1=0
34 sink@1=0
35 init@1=1
36 entered_lasso@1=0
37 L:F(And(Or(Neg(light0))(Neg(light1)))(Or(light0)(light1)))...284@1=1
    
```



$\{ \langle hi, 0, \pi' \rangle \}$
 $\{ \langle \neg hi, 0, \pi \rangle \}$



Hyperproperties

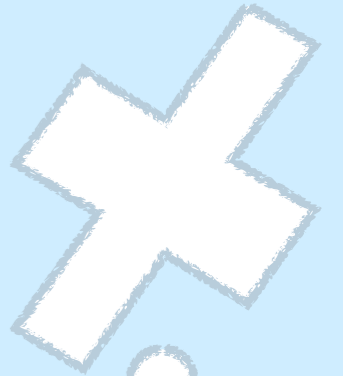
Halpern & Pearl
Causality

Causality in
reactive
systems

Causes as
sets of events

Causes as
temporal
properties

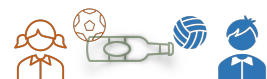
Causality as a
Hyperproperty



Causes as Trace Properties



- Effect: a violation of an ω -regular property ψ



- Actual World: a counterexample trace

Lasso-shaped



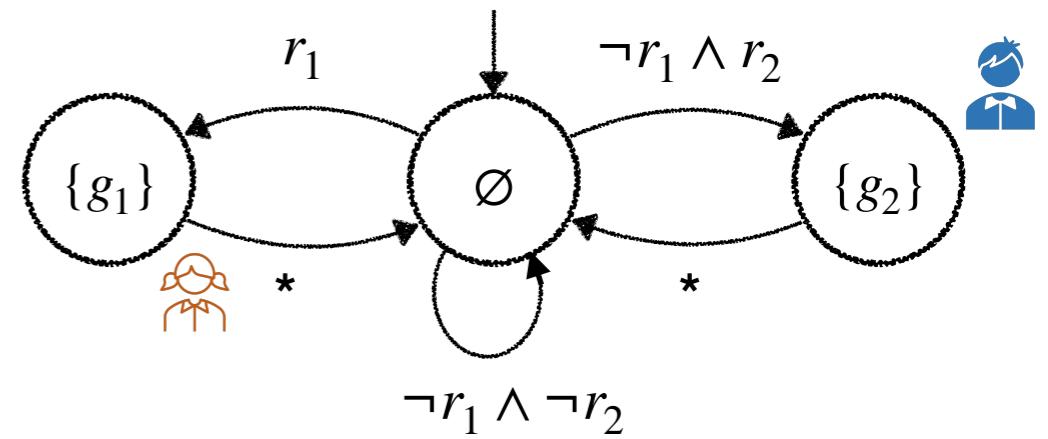
- Cause: an ω -regular property

Quantified Propositional Temporal Logic – QPTL

LTL + quantification over propositions

$\exists q . q \wedge \square (q \leftrightarrow \bigcirc \neg q) \wedge \square (q \rightarrow a)$ – “ a holds at every odd position”

Causes as Trace Properties



$$\pi \left[\begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline \hline \hline \end{array} \left(\begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline g_1 & \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline \\ \hline \end{array} \right)^\omega \not\models \diamond g_2$$

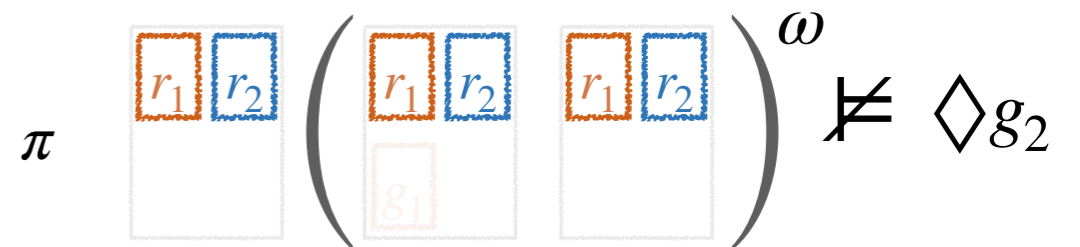
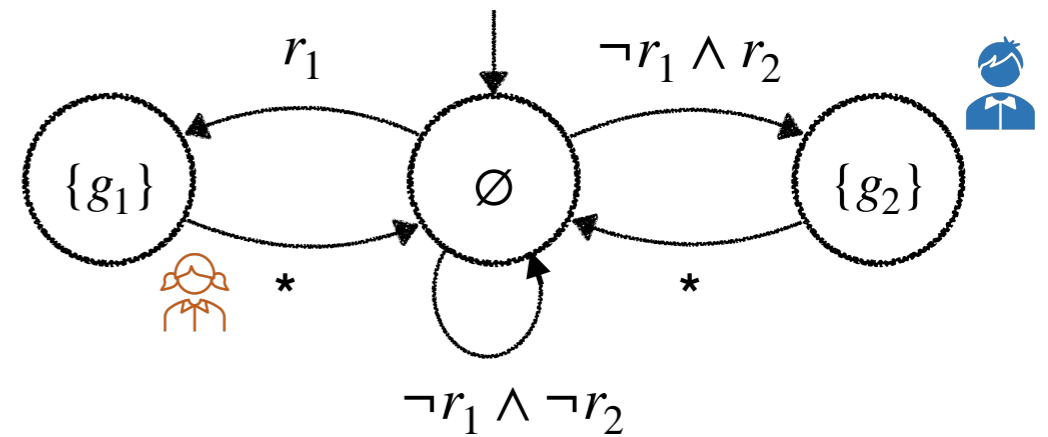
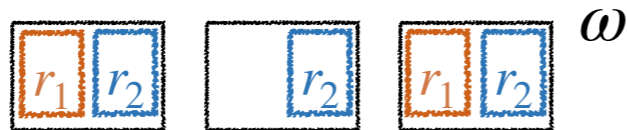
Causes as Trace Properties

CF: \forall counterfactual \exists contingency s.t. $\diamond g_2$ holds

Closest input sequences
s.t. C doesn't hold

$$C = r_1 \wedge \bigcirc r_1$$

$$\neg C = \neg r_1 \vee \bigcirc \neg r_1$$



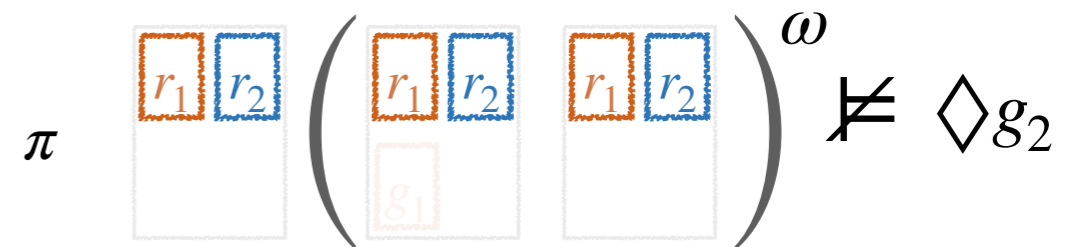
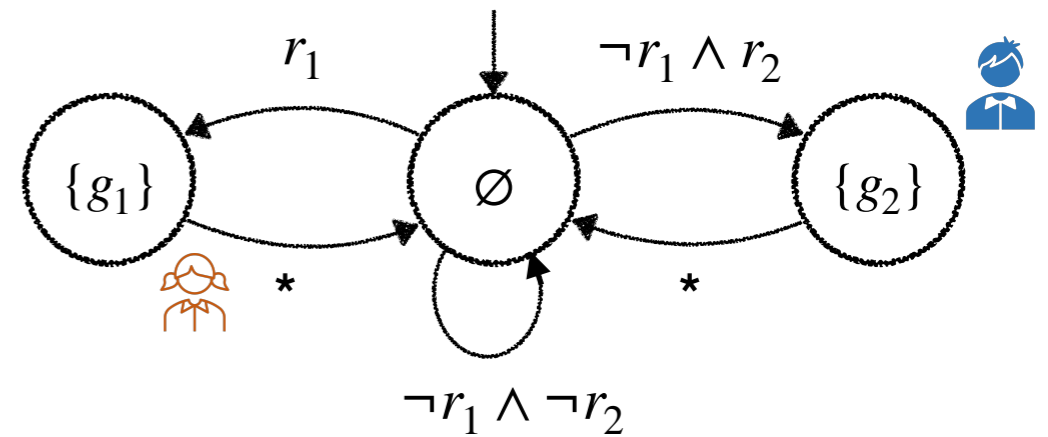
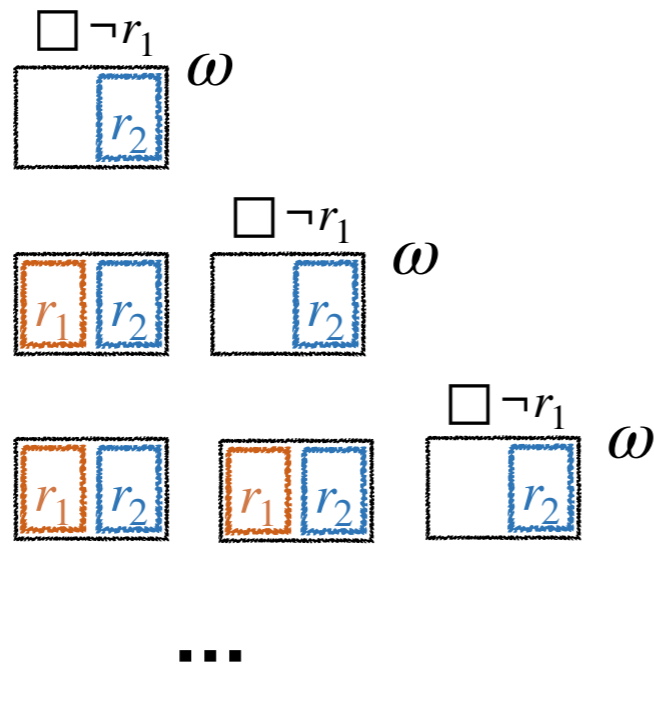
Causes as Trace Properties

CF: \forall counterfactual \exists contingency s.t. $\diamond g_2$ holds

Closest input sequences
s.t. C doesn't hold

$$C = \square \diamond r_1$$

$$\neg C = \diamond \square \neg r_1$$

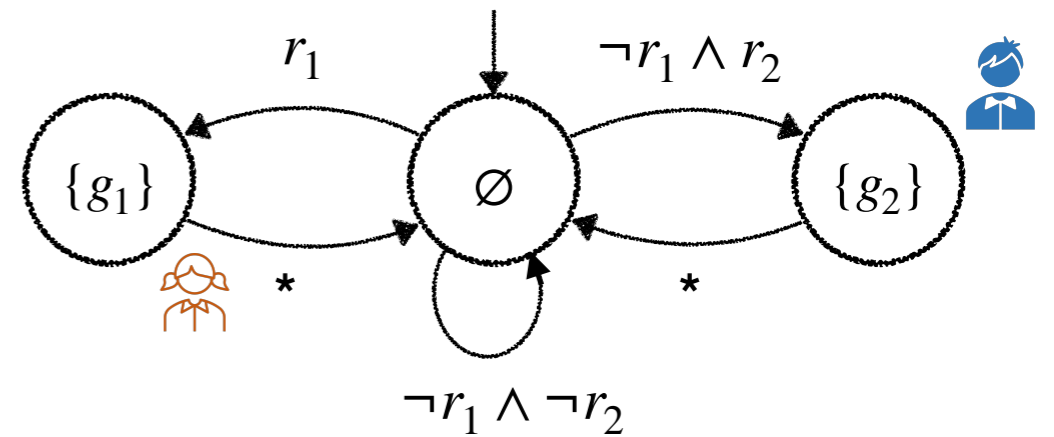


Compare traces that have the same **rejection structure**

Causes as Trace Properties

CF: \forall counterfactual \exists contingency s.t. $\diamond g_2$ holds

Closest input sequences
s.t. C doesn't hold



HyperQPTL formula

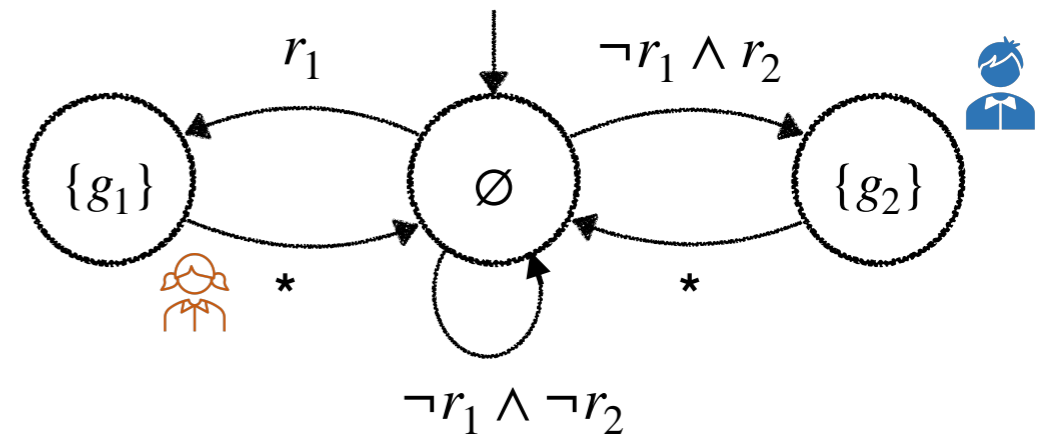
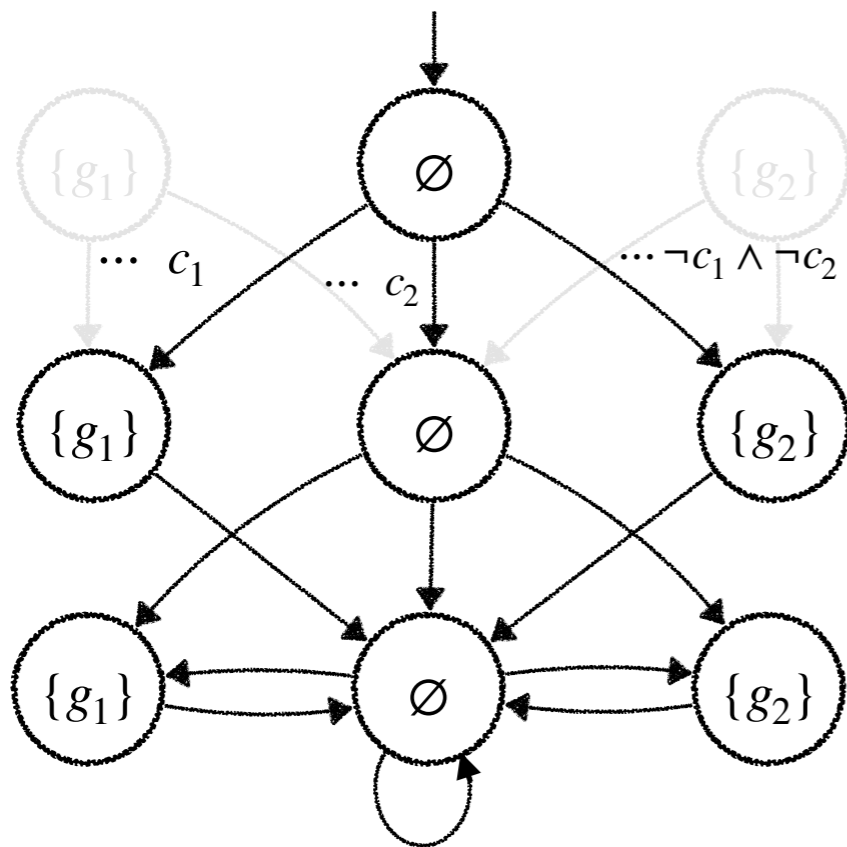
$\Psi_{struct}(\pi_1, \pi_2) : \pi_1, \pi_2$ satisfy all sub-formulas of C at the same positions

$$\pi \begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline \end{array} \left(\begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline g_1 & \end{array} \begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline \end{array} \right)^\omega \not\models \diamond g_2$$

Compare traces that have the same **rejection structure**

Causes as Trace Properties

CF: \forall counterfactual \exists contingency s.t. $\diamond g_2$ holds



$$\pi \begin{matrix} r_1 & r_2 \\ \hline & \end{matrix} \left(\begin{matrix} r_1 & r_2 \\ \hline g_1 & \end{matrix} \quad \begin{matrix} r_1 & r_2 \\ \hline & \end{matrix} \right)^\omega \not\models \diamond g_2$$

Counterfactual automaton additional inputs $[c_1, c_2]$ set a contingency

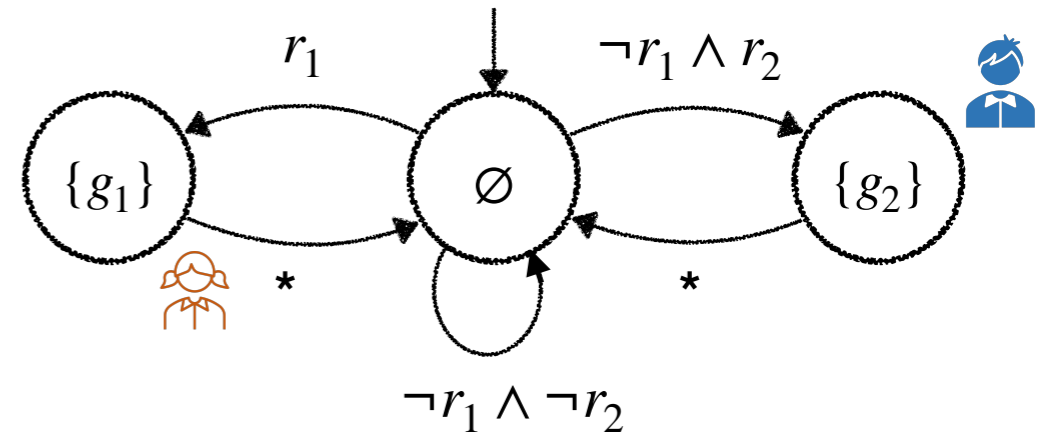
Causes as Trace Properties

MIN: There is no C' such that $C' \rightarrow C$
and $C' \models \text{SAT} \ \& \ \text{CF}$

HyperQPTL formula

No lasso-shaped trace can be removed from C

a trace that does not satisfy the effect, or does not contribute for counterfactual traces



$$\pi \left[\begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline \end{array} \right] \left(\left(\begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline g_1 & \end{array} \right) \begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline \end{array} \right)^\omega \not\models \Diamond g_2$$

Causes as Trace Properties

Given a candidate cause C , verify:

SAT: $\pi \models C$

CF: \forall counterfactual \exists contingency s.t. φ holds

MIN: There is no C' such that $C' \rightarrow C$ and $C' \models \text{SAT} \ \& \ \text{CF}$

$\Rightarrow C$ is a cause of φ on π

HyperQPTL

SAT

Verify C on π

CF

Counterfactuals: traces with the same rejection structure
Contingencies: using the counterfactual automaton

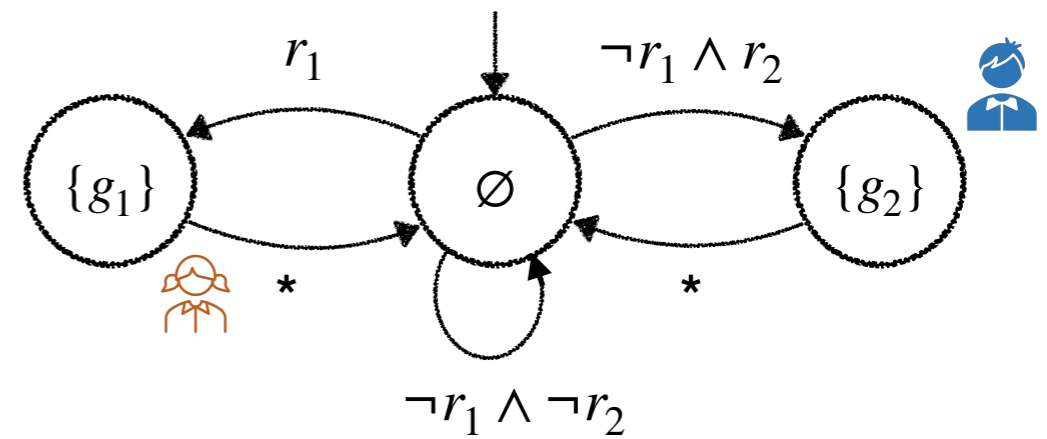
*Decidable via
HyperQPTL model-
checking!*

MIN

No lasso-shaped trace can be removed from C

Unfair Arbitrator

Is $\Box r_1$ the cause for $\Box \neg g_2$?



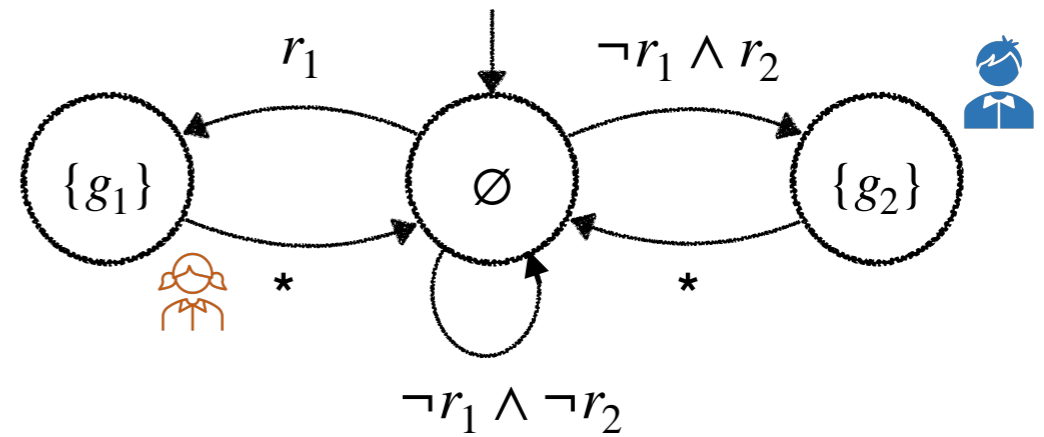
$$\pi \left(\begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline \end{array} \left(\begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline g_1 & \end{array} \quad \begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline \end{array} \right)^\omega \right) \not\models \Diamond g_2$$

Unfair Arbiter

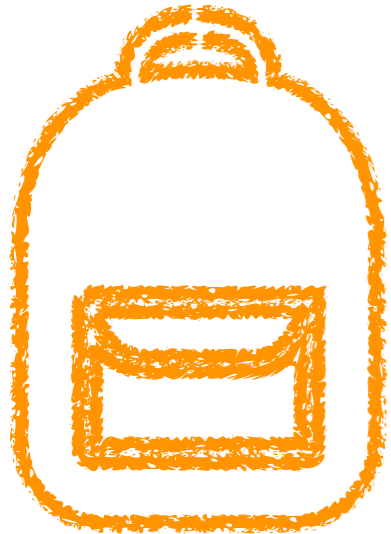
$$\exists q . q \wedge \square (q \leftrightarrow \bigcirc \neg q) \wedge \square (q \rightarrow r_1)$$

r_1 holds at every odd position

is a cause for $\square \neg g_2$ on π



$$\pi \quad \begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline \end{array} \left(\begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline g_1 & \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline r_1 & r_2 \\ \hline & \\ \hline \end{array} \right)^\omega \not\models \diamond g_2$$



Hyperproperties

Halpern & Pearl
Causality

Formalism for
expressing
HP causality
for reactive
systems

Causality in
reactive
systems

**Causes as
sets of events**

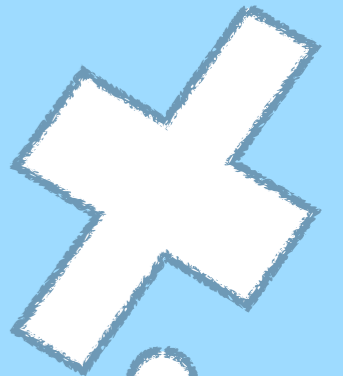
Finding a
cause via
HyperLTL
model
checking

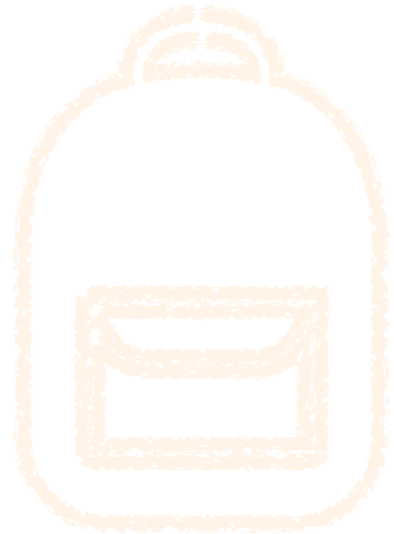
Algorithm for
finding all
causes

Verifying a
cause via
HyperQPTL
model
checking

**Causes as
temporal
properties**

**Causality as a
Hyperproperty**





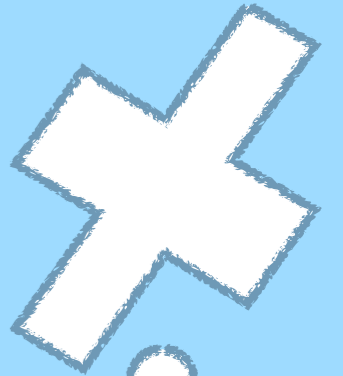
Hyperproperties

Halpern & Pearl
Causality

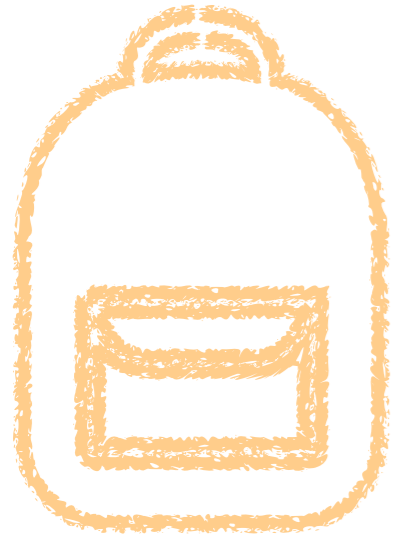
**Find
temporal
causes**

**Effects for
temporal causes as
Hyperproperties**

**Causality as a
Hyperproperty**



**Applications
– Repair**



Hyperproperties

Halpern & Pearl
Causality

Causality in
reactive
systems

Causes as
sets of events

Thank you!
Questions?

Causes as
temporal
properties

Causality as a
Hyperproperty

