CISPA
HELMHOLTZ CENTER FOR
INFORMATION SECURITY

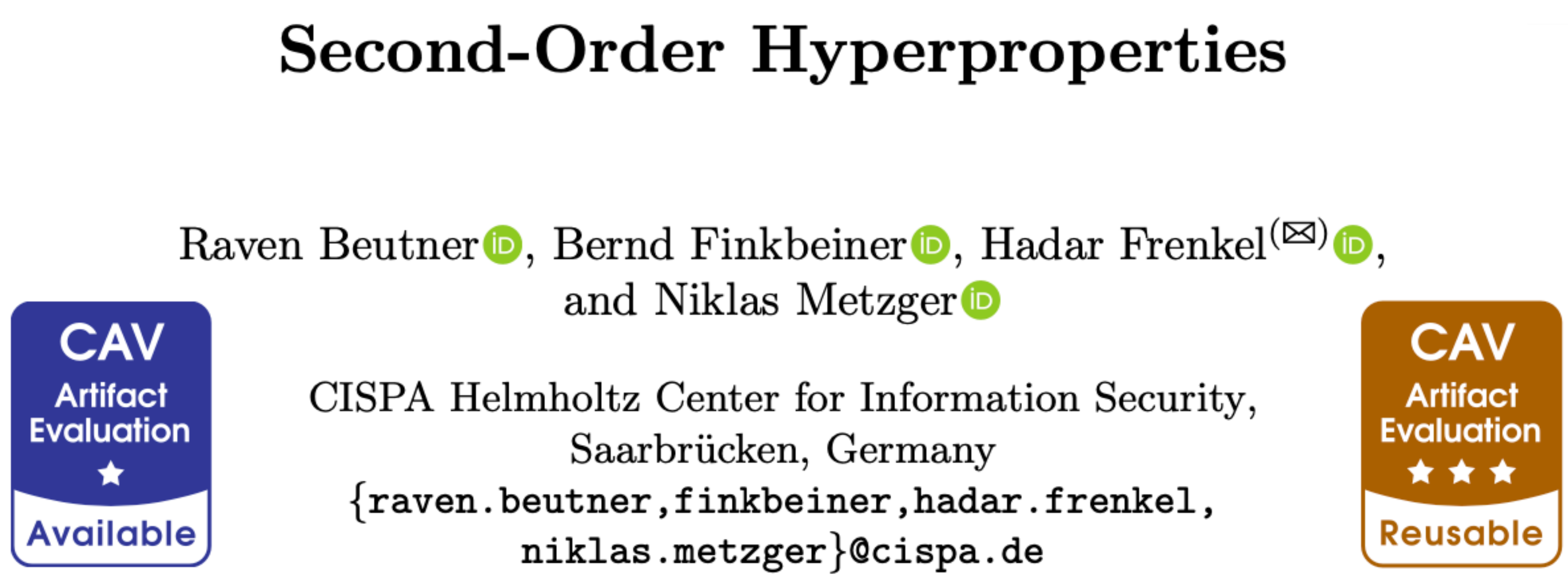
Why do we need Second-Order Hyperlogics?

Hadar Frenkel

CISPA Helmholtz Center for Information Security
Saarbrücken, Germany

HYPER workshop
18 July @ CAV 2023

Joint work with



Raven Beutner **Bernd Finkbeiner** **Niklas Metzger**

CISPA Helmholtz Center for Information Security
Saarbrücken, Germany

Friday afternoon!

Second-Oder Hyperproperties. Beutner, Finkbeiner, **F.**, Metzger (CAV 2023)

Hyper^{2}LTL

Why?

$$\psi := \ a_\pi \mid \neg\psi \mid \psi \wedge \psi \mid \bigcirc\psi \mid \psi \, \mathsf{U} \, \psi$$

$$\varphi := \exists \pi \in X \,.\, \varphi \mid \forall \pi \in X \,.\, \varphi \mid \exists X \,.\, \varphi \mid \forall X \,.\, \varphi$$

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Common Knowledge

Asynchronous Hyperproperties

Trace Theory

Causality

Generic reasoning and algorithms

Hyper^{2}LTL

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Temporal Logics for Hyperproperties. Clarkson, Finkbeiner, Koleini, Micinski, Rabe, and Sánchez. (POST 2014).

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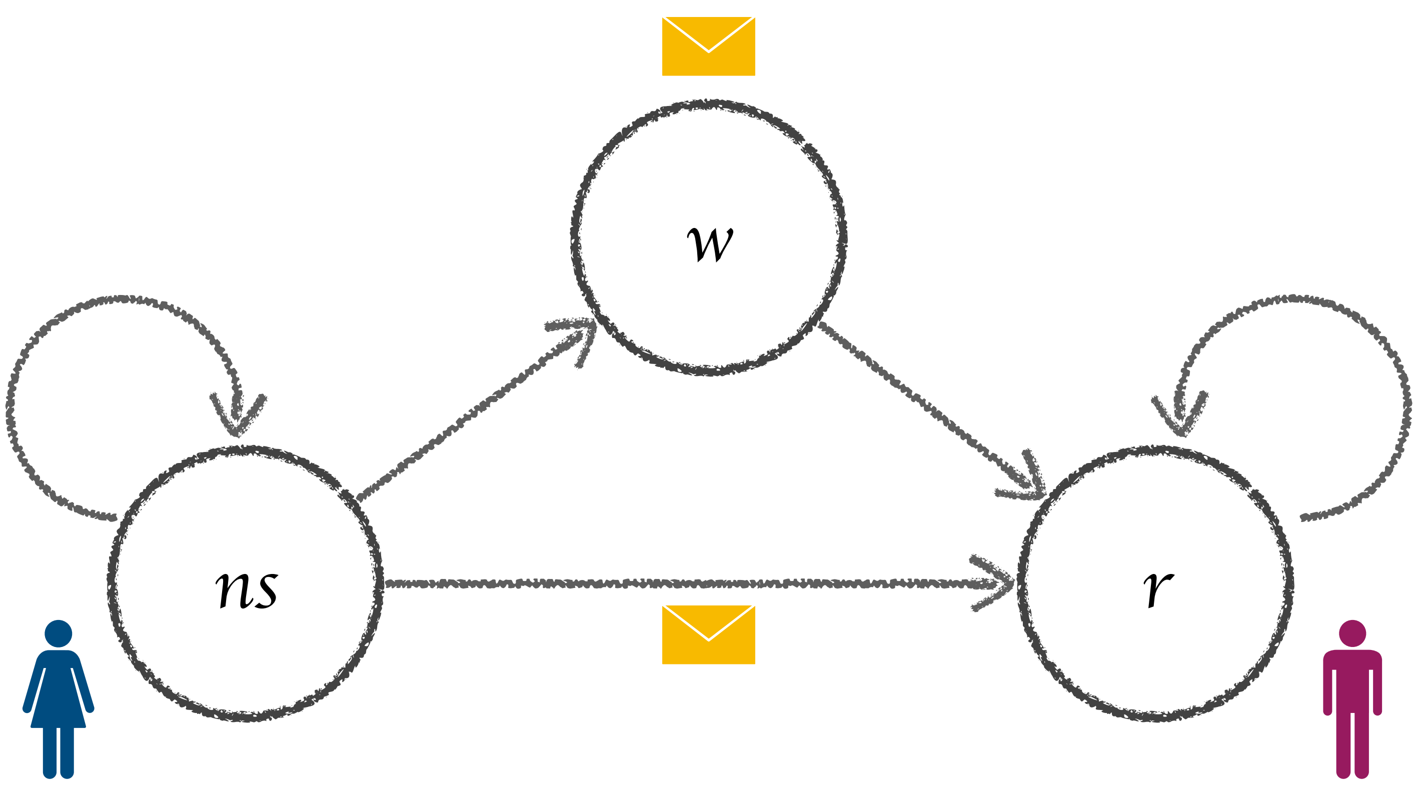
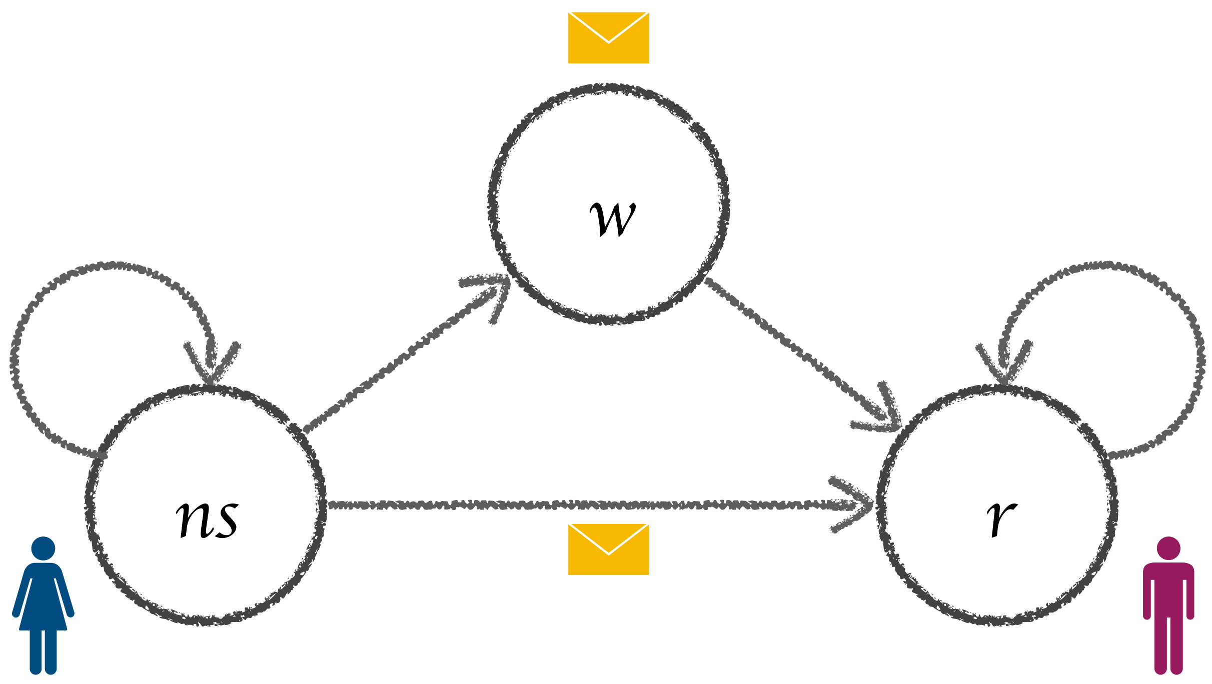
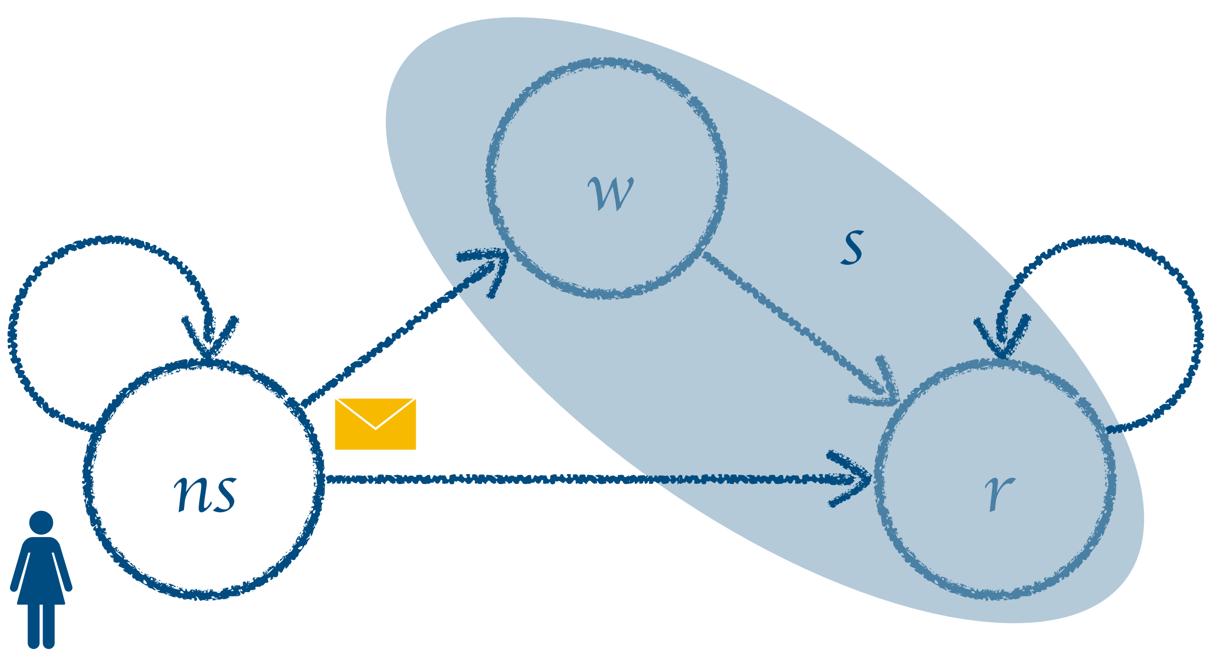
Communication in Multi-Agent Systems



Communication in Multi-Agent Systems





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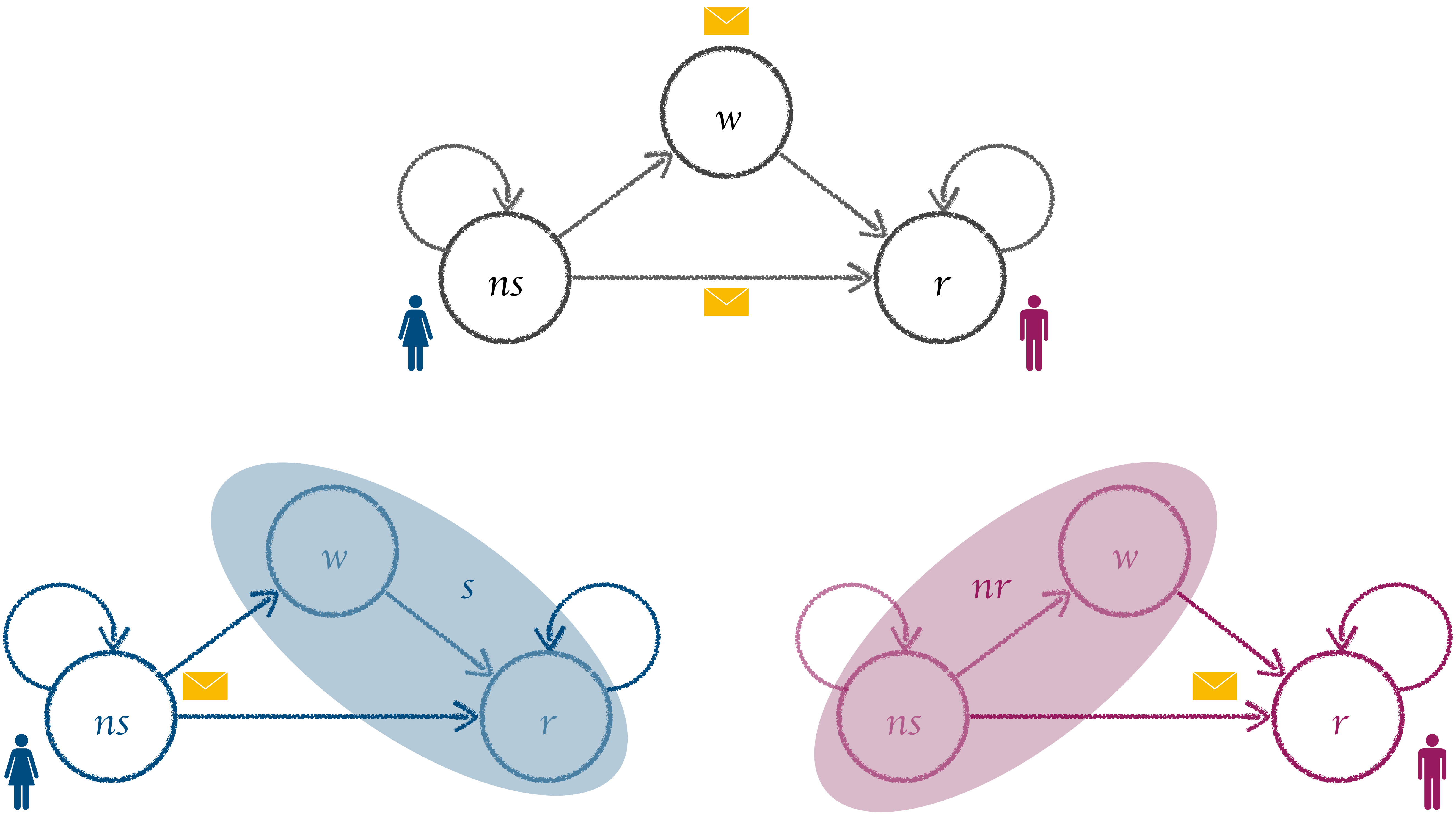

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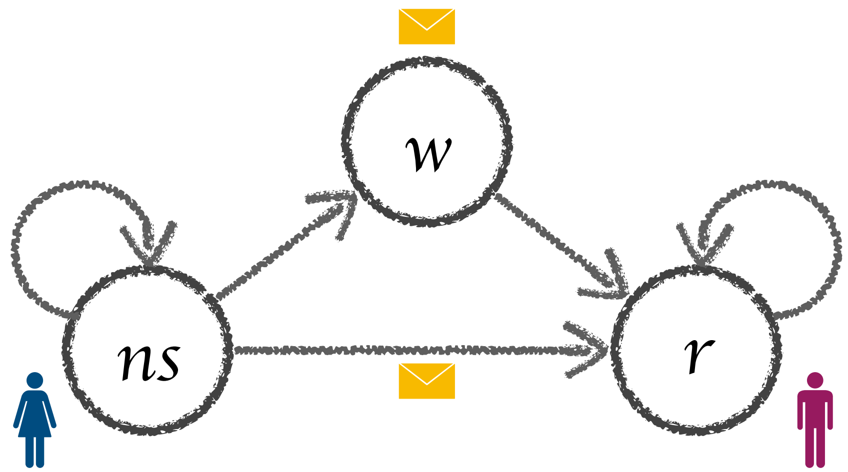
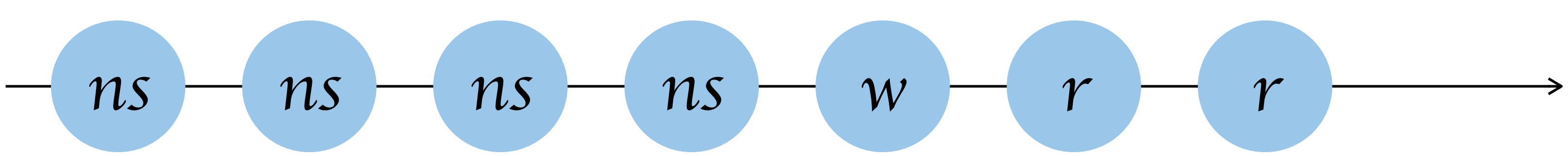
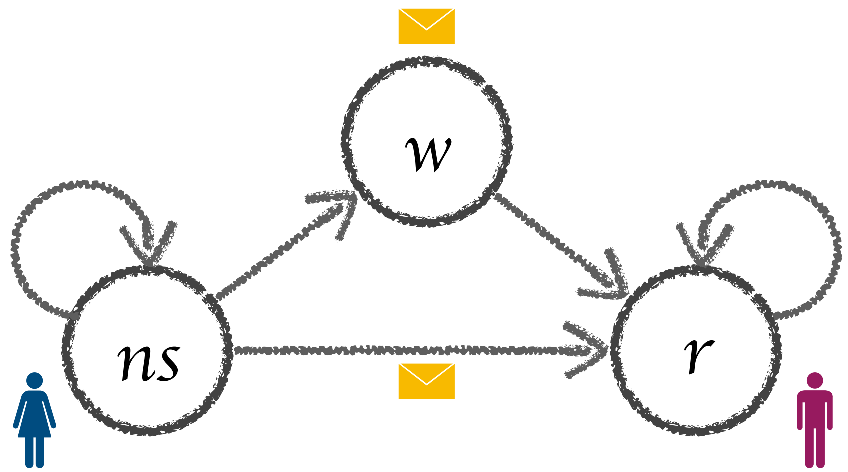
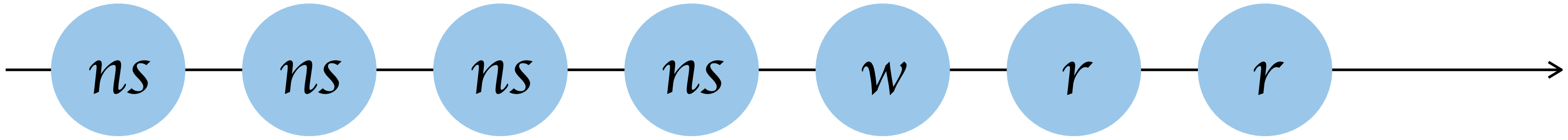




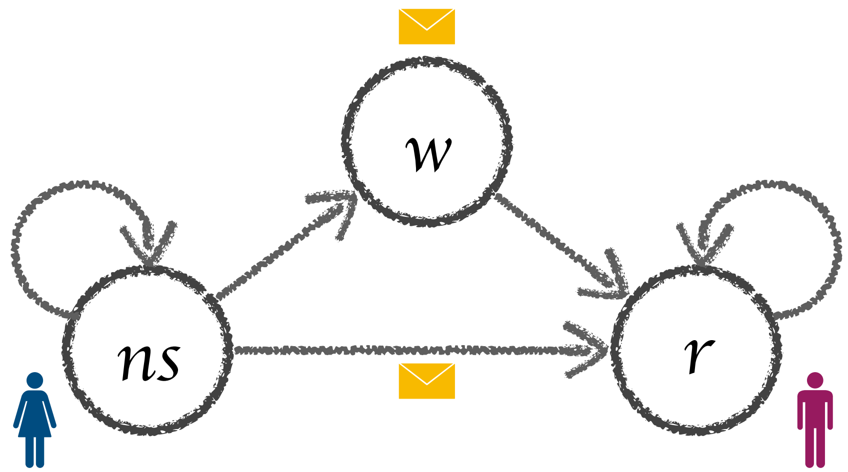
Communication in Multi-Agent Systems

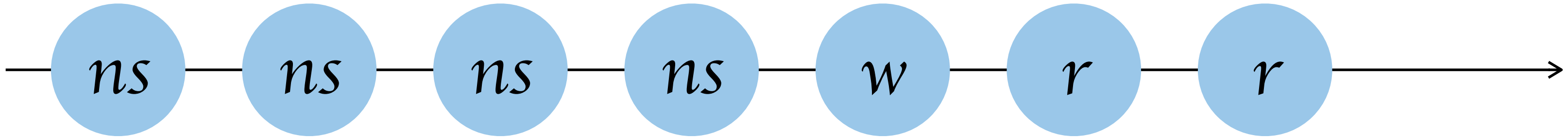




eventually r ?

Communication in Multi-Agent Systems

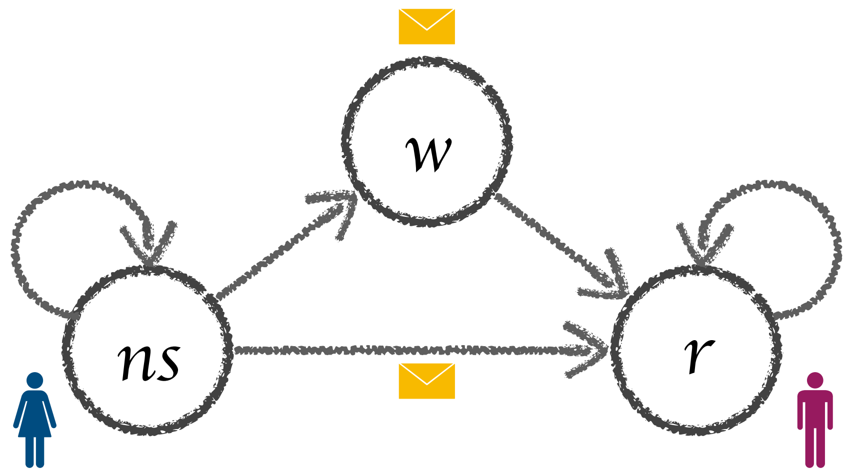


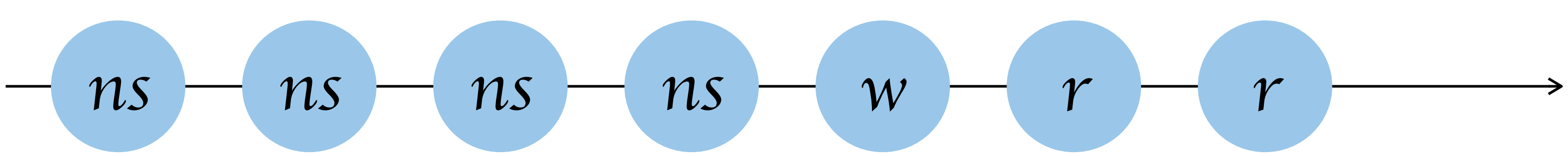


eventually knows r ?

eventually r ?

Communication in Multi-Agent Systems





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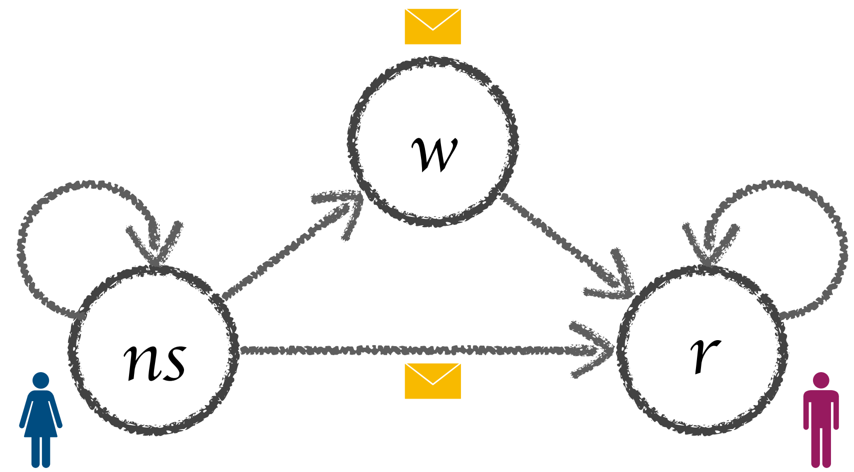
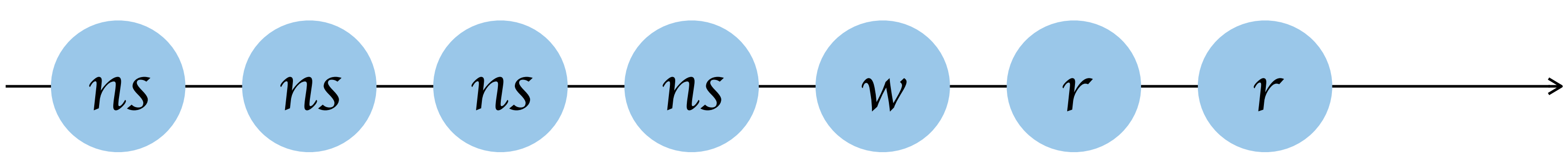
Communication in Multi-Agent Systems





eventually common knowledge r ?

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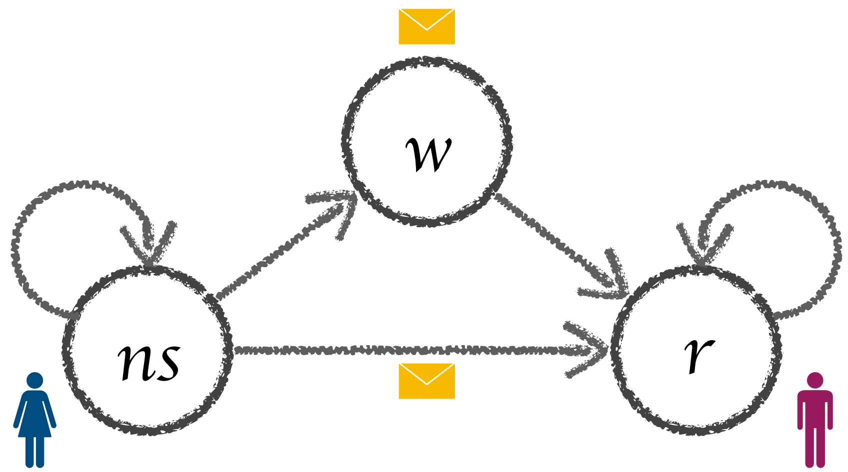
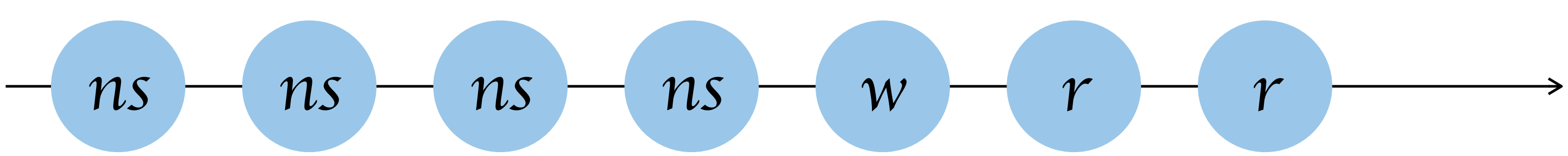
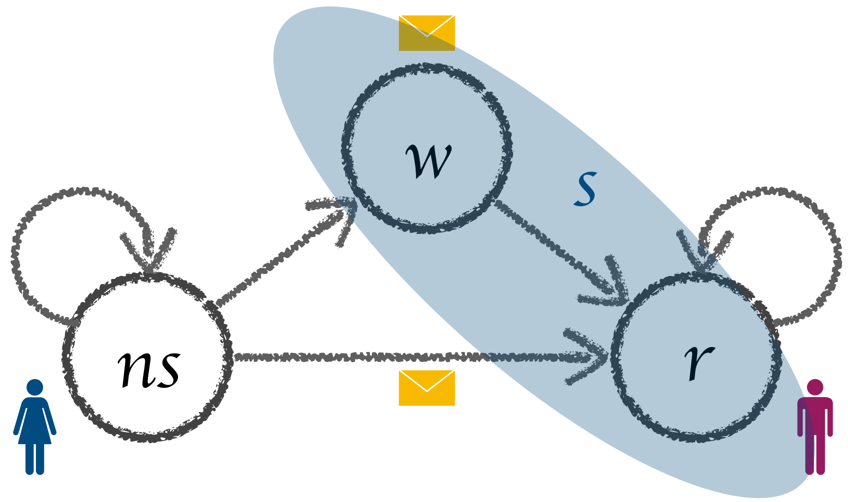




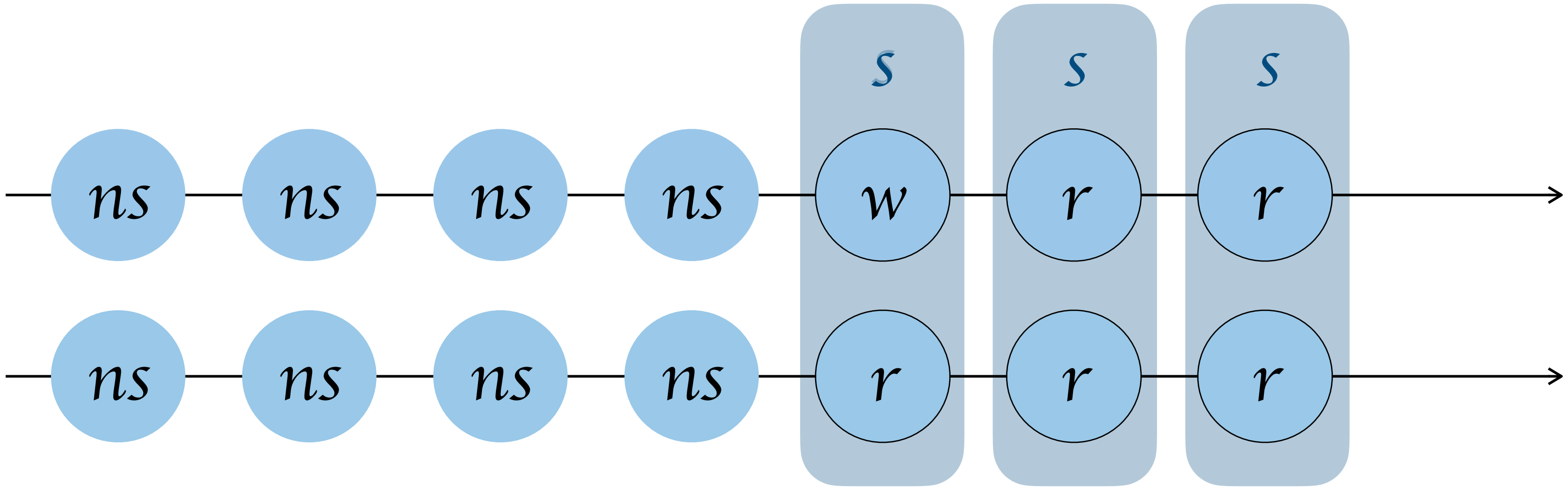
A trace property

eventually r ?

Communication in Multi-Agent Systems



A hyperproperty



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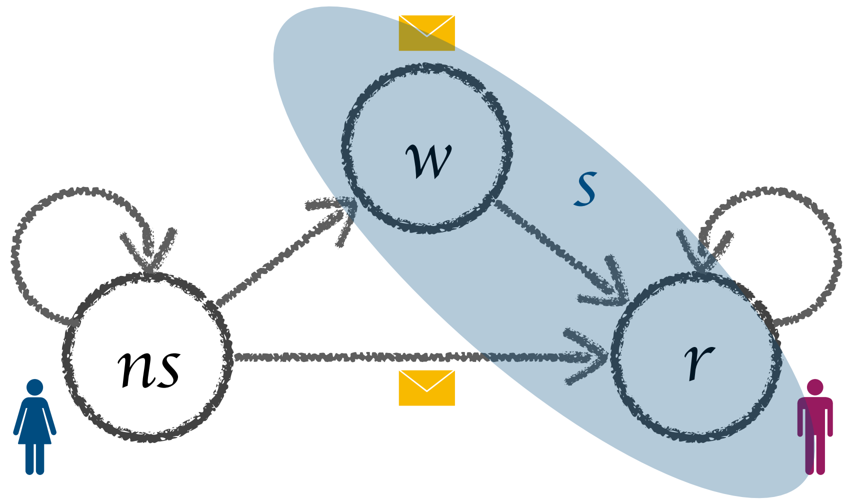

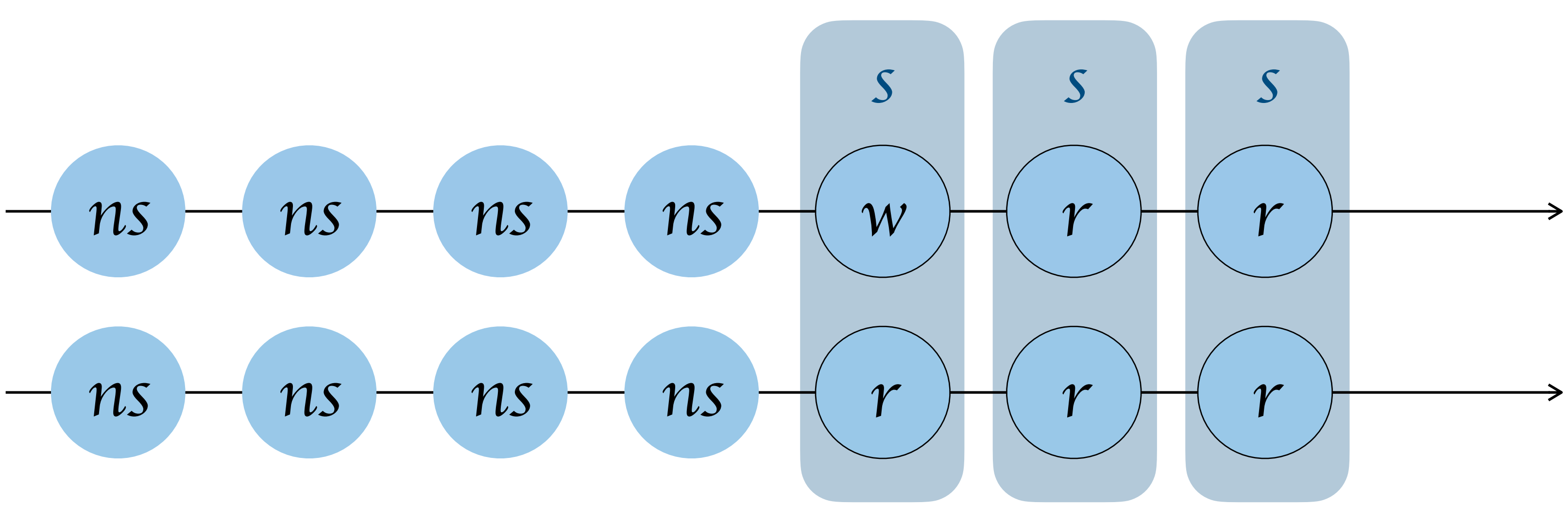

A hyperproperty

eventually know r ?

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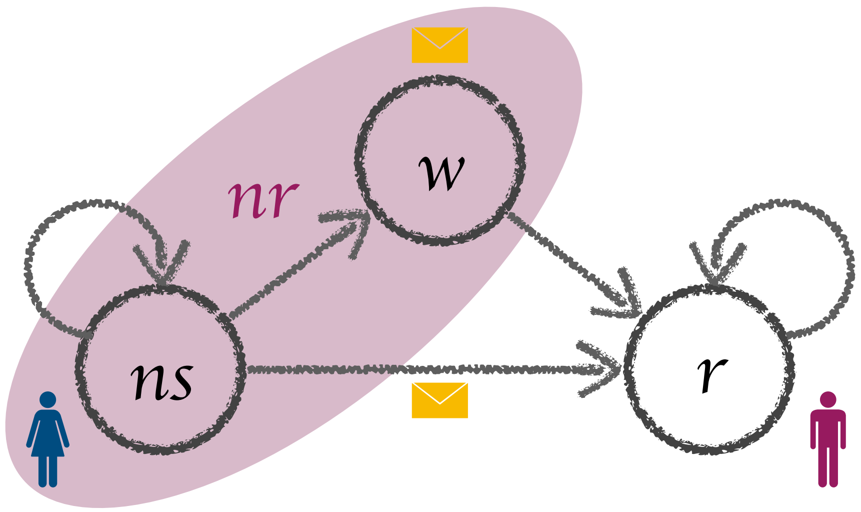
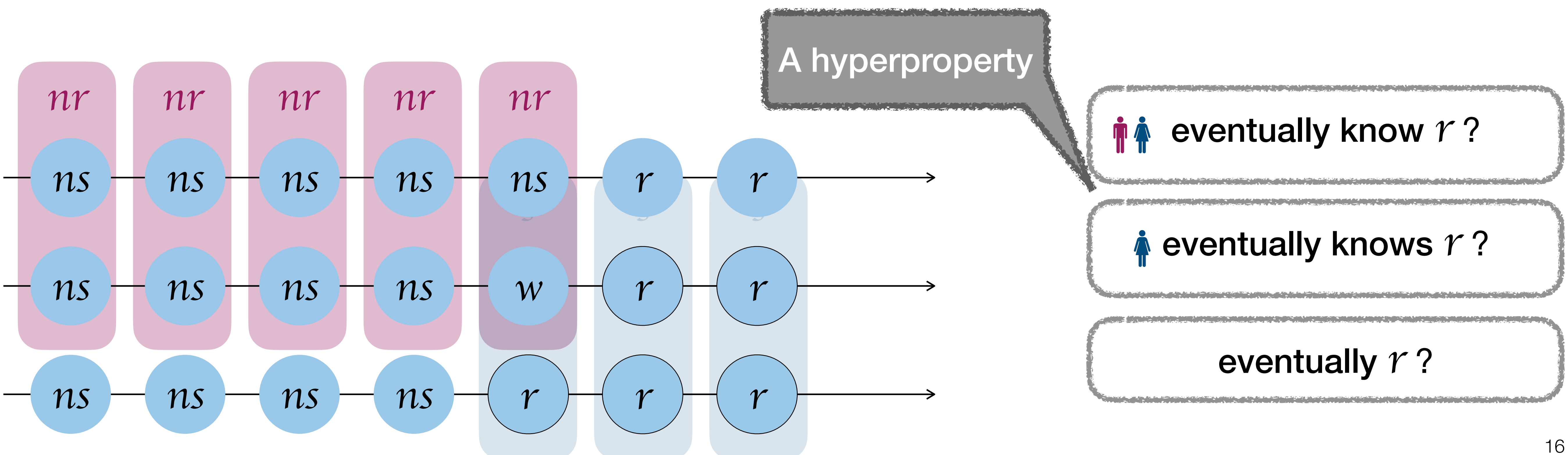
Communication in Multi-Agent Systems





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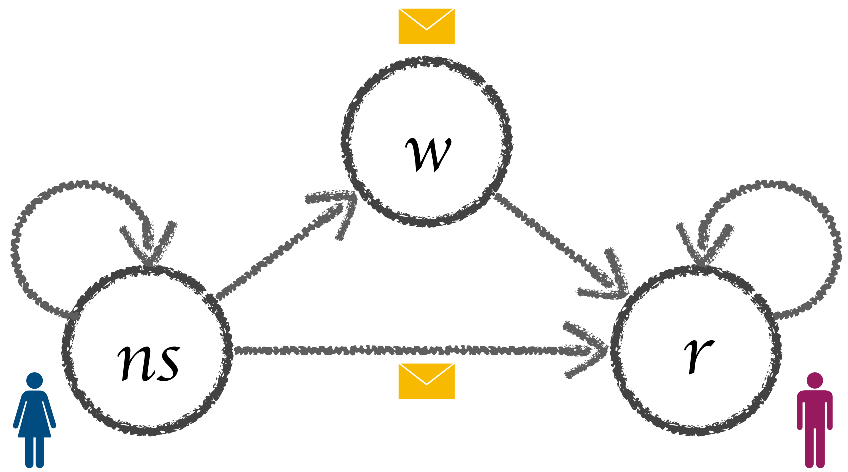
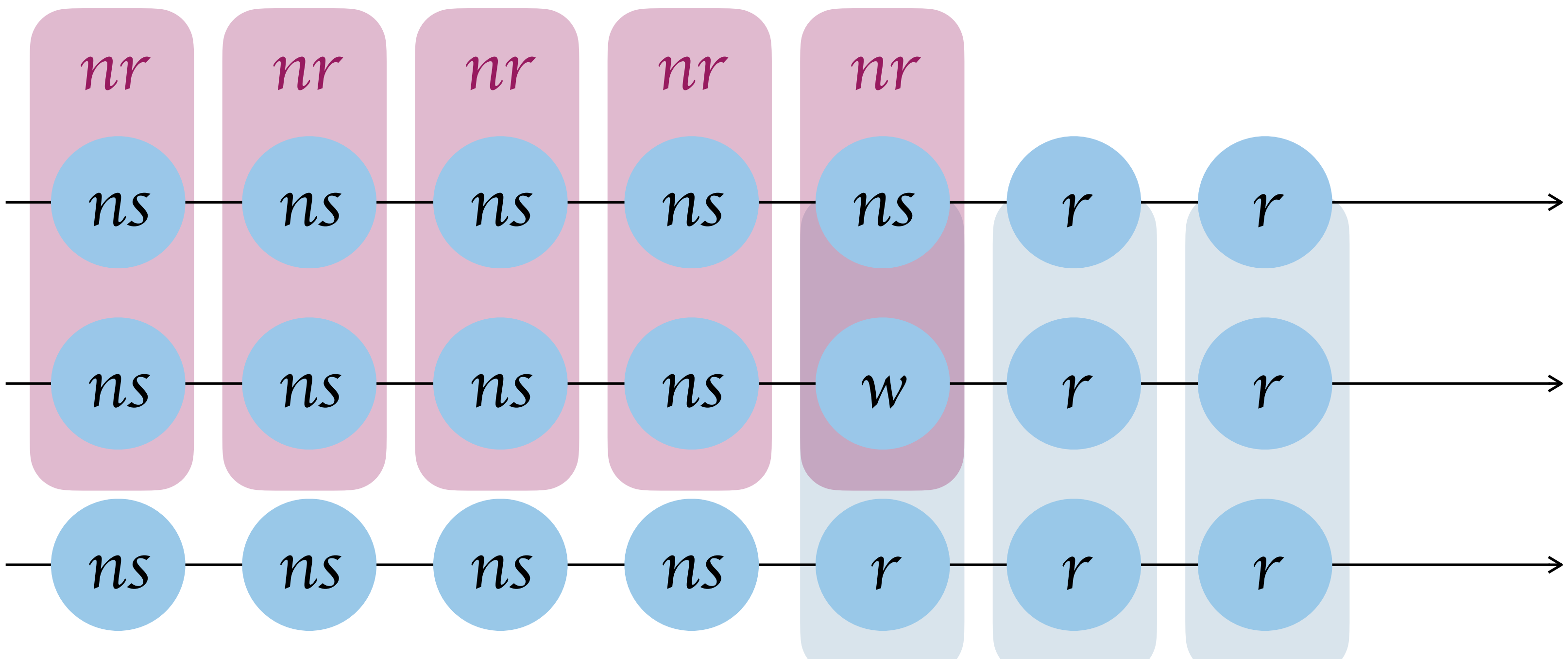


eventually common knowledge r ?



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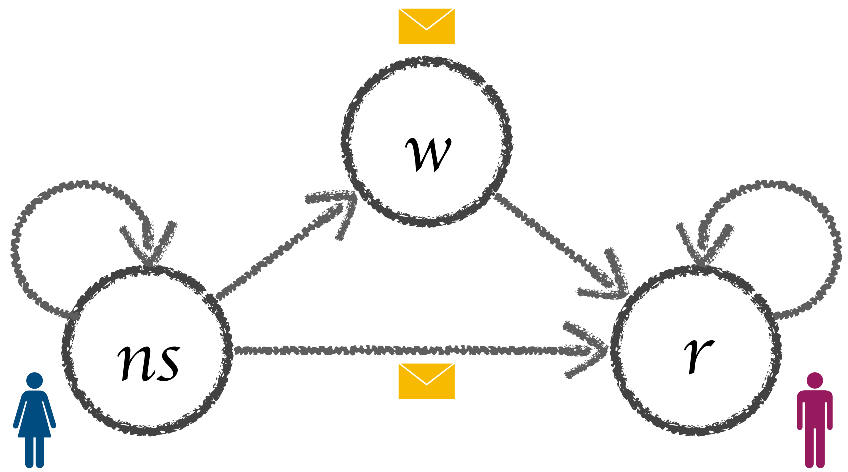
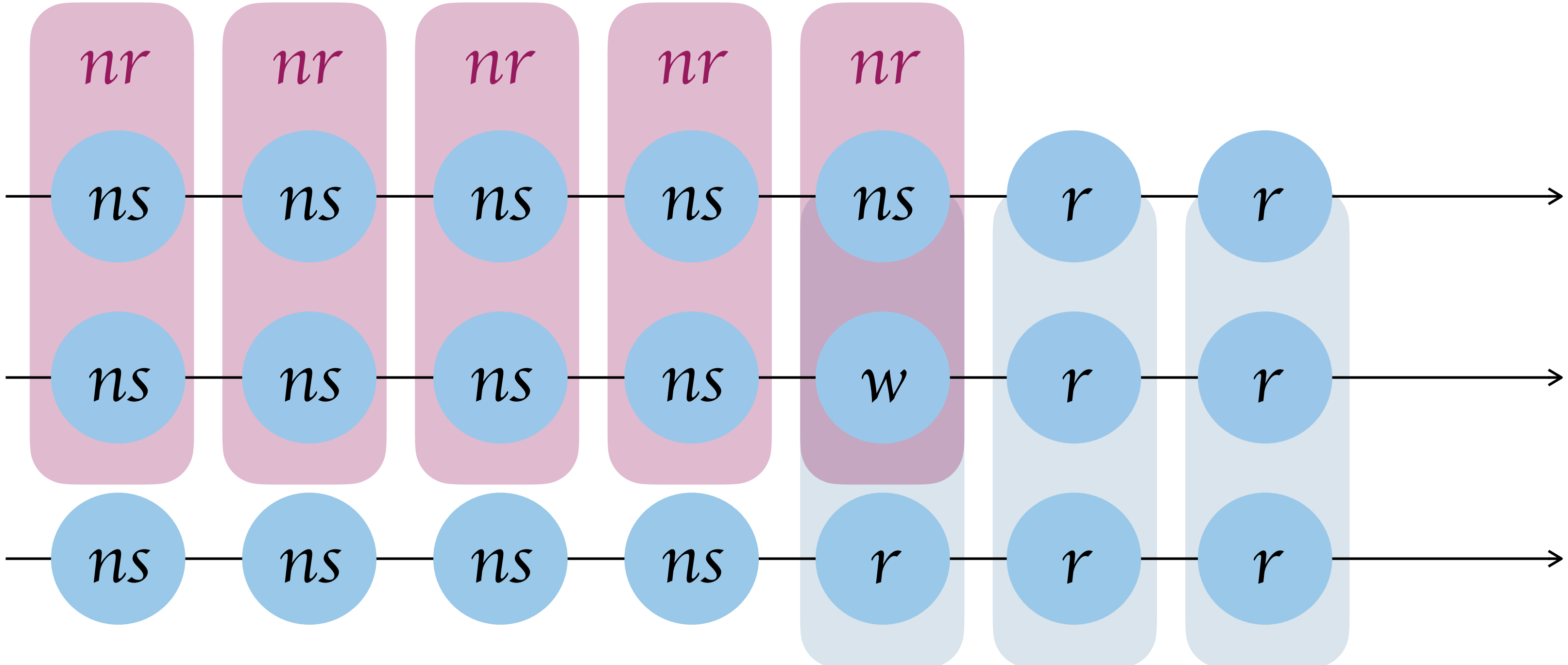


Simultaneous action

Consensus

eventually common knowledge r ?



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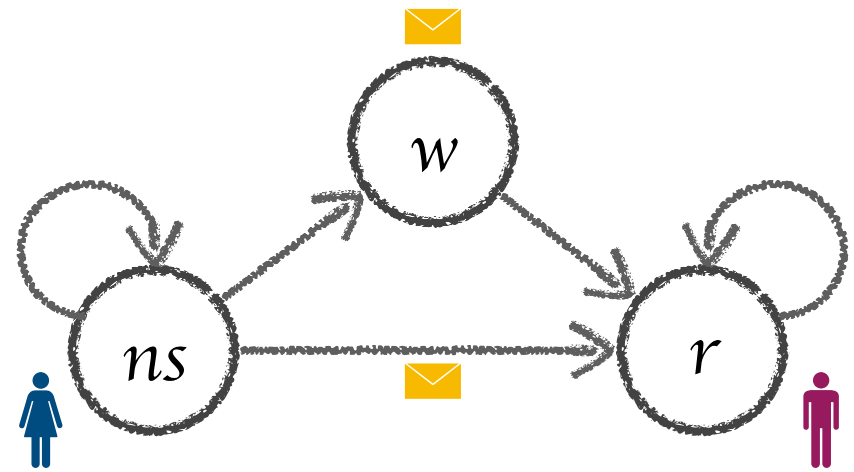

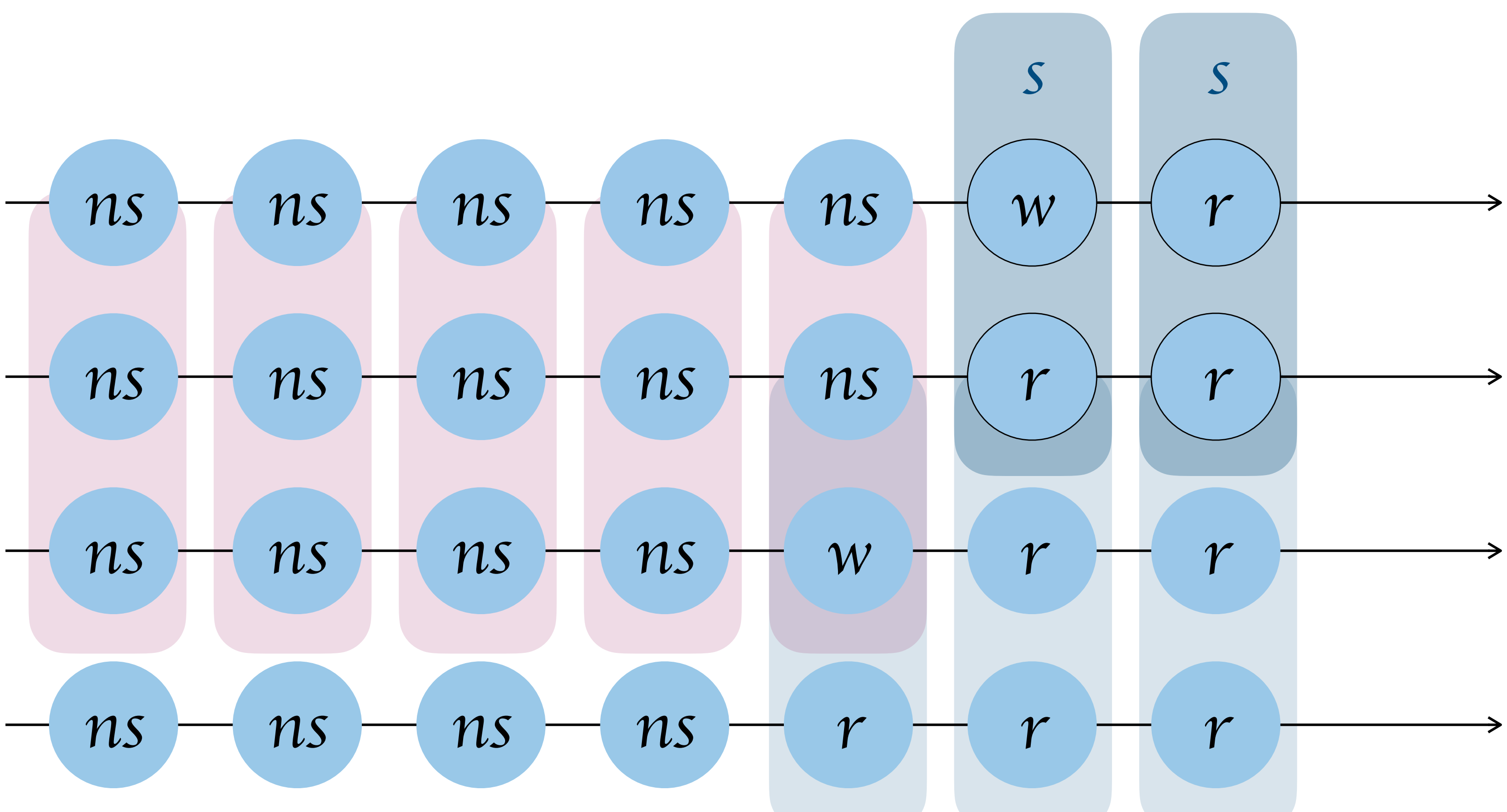

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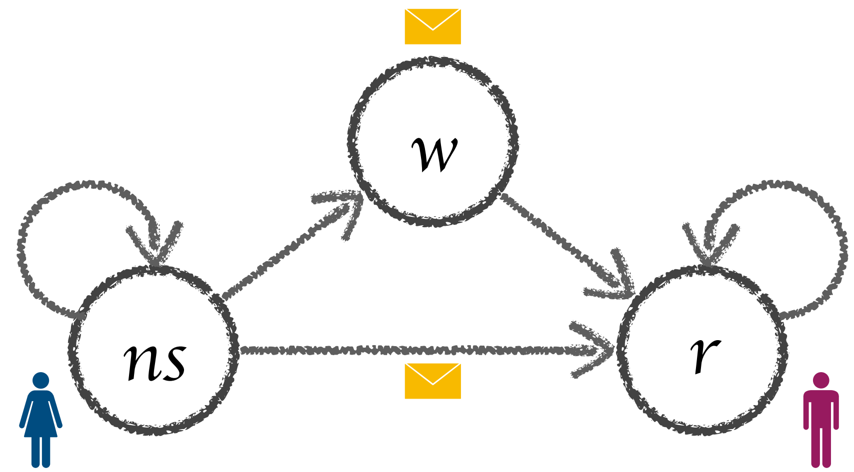

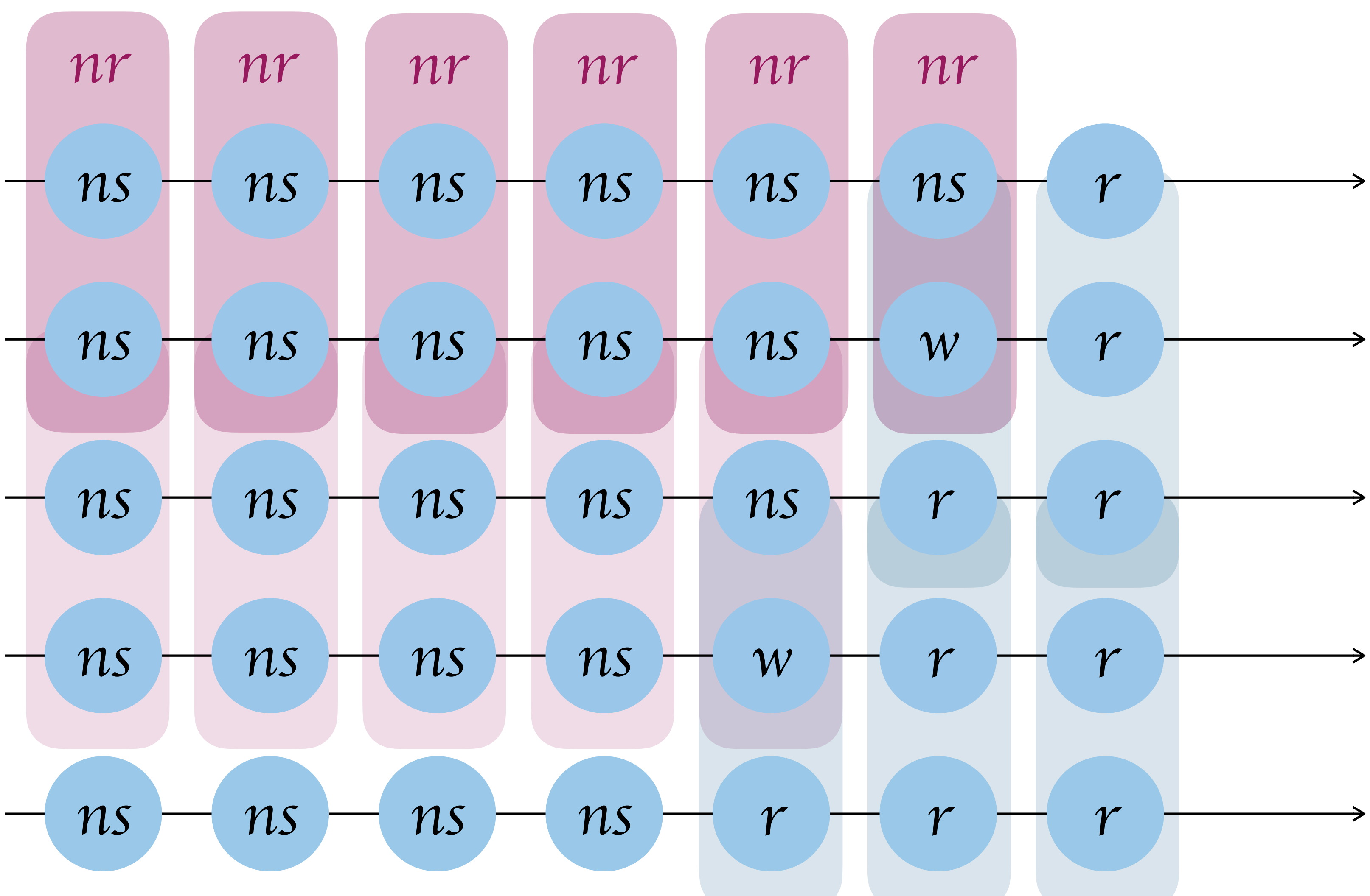

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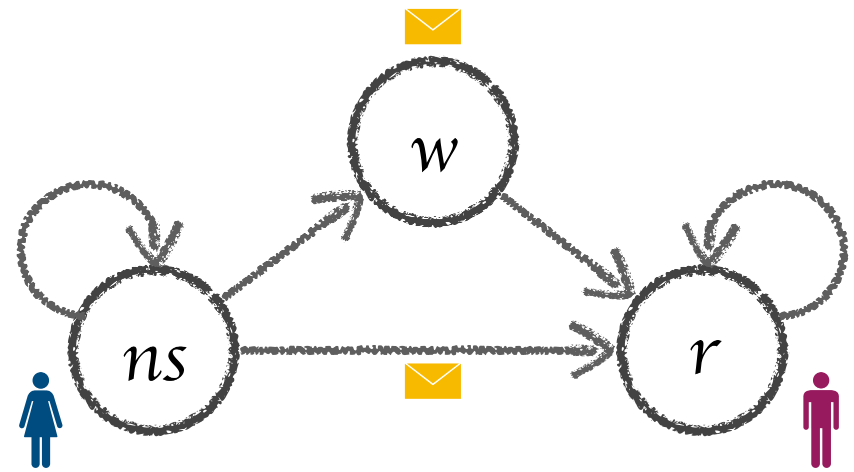

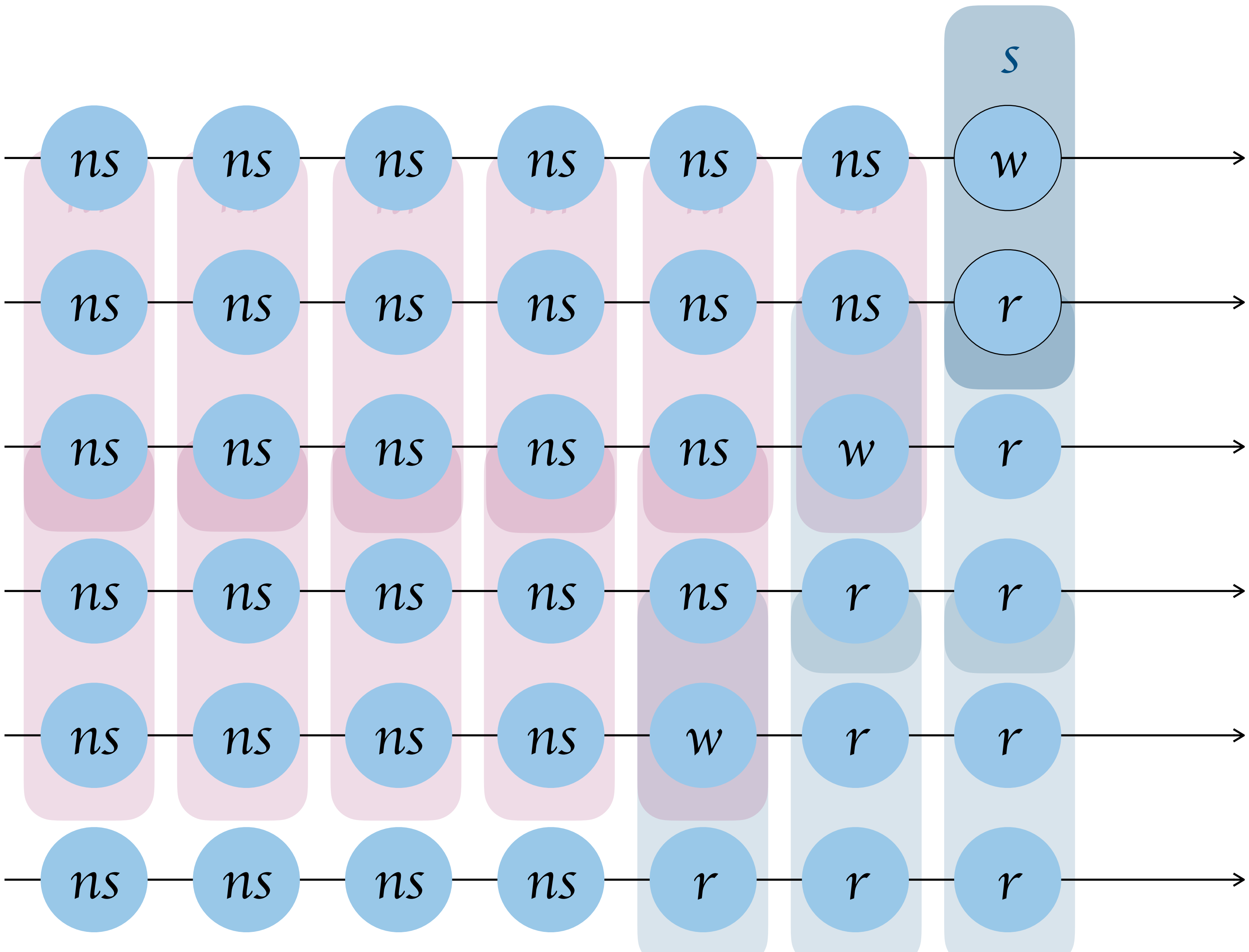

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Temporal Epistemic Logics

- $\text{LTL}_{K,C}$
  LTL + (common) knowledge

- HyperCTL^*_{lp}
  HyperCTL^* + knowledge

- MCK, MCMAS
  Model checking knowledge and time

The complexity of reasoning about knowledge and time. Halpern and Vardi (STOC 1986)

Unifying hyper and epistemic temporal logics. Bozzelli, Maubert, Pinchinat (FoSSaCS 2015)

MCK: model checking the logic of knowledge. Gammie, van der Meyden (CAV 2004)

MCMAS: an open-source model checker for the verification of multi-agent systems. Lomuscio, Qu, Raimondi (Int. J. Softw. Tools Technol. 2015)

Hyper²LTL

$$\psi := \ a_\pi \mid \neg\psi \mid \psi \wedge \psi \mid \bigcirc\psi \mid \psi \,\mathcal{U}\, \psi$$

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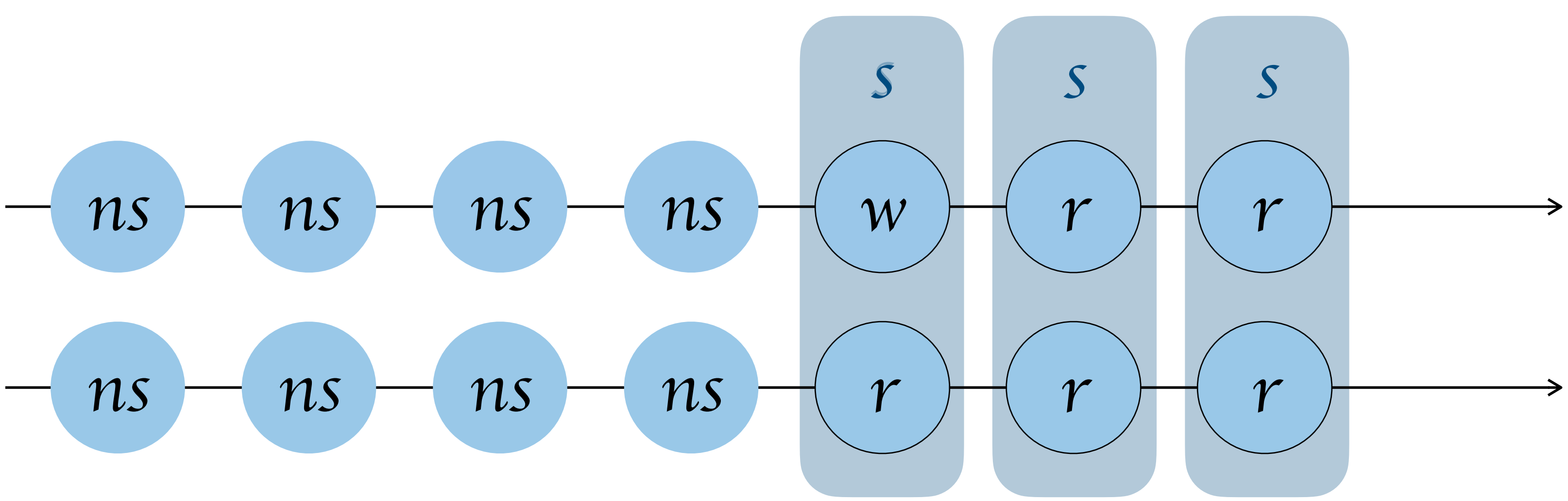
$$\exists\pi\forall\pi' \, . \, (\pi \equiv \pi') \rightarrow \Diamond(r_\pi \wedge r_{\pi'})$$



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$$\Box\Big((ns_\pi \leftrightarrow ns_{\pi'}) \wedge (s_\pi \leftrightarrow s_{\pi'})\Big)$$

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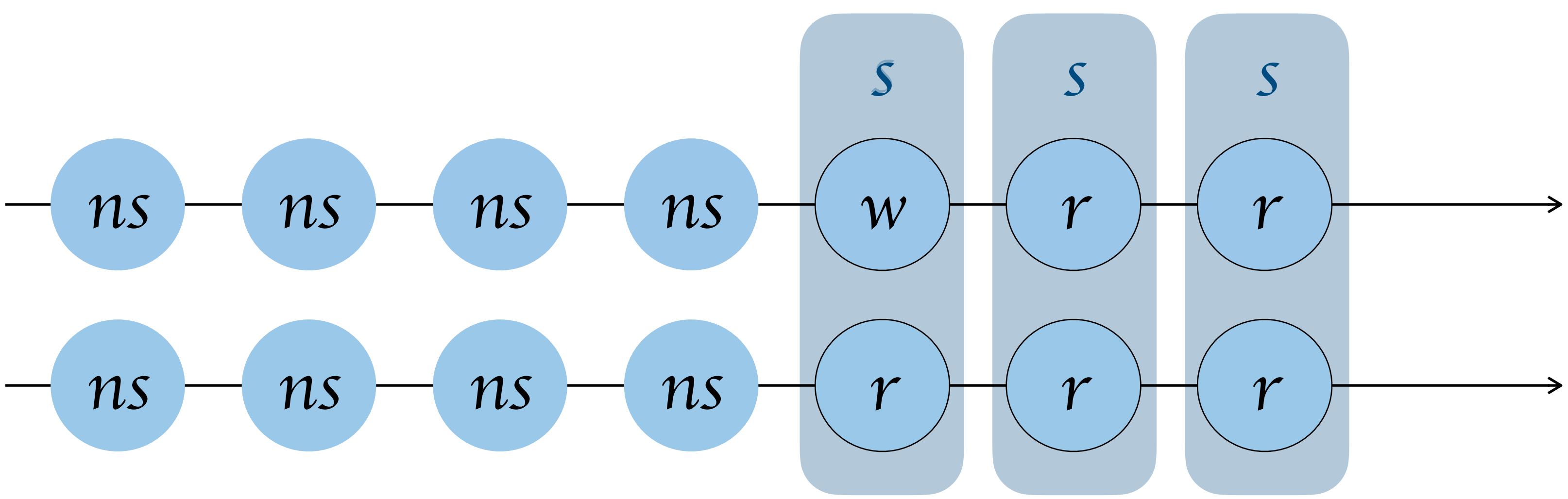

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trace-set variable

$\mathcal{G}$ – system traces

$\mathcal{U}$ – Σ^ω

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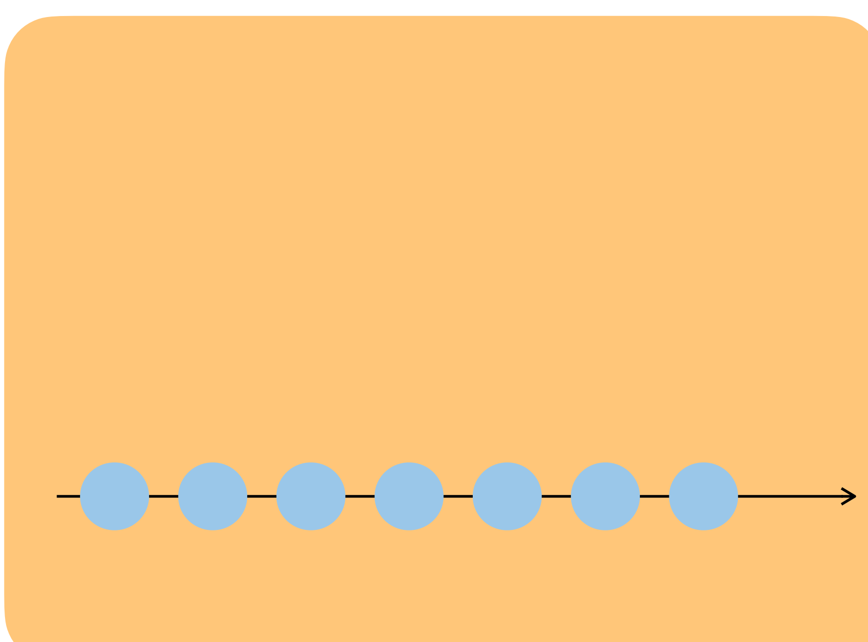
$$\exists \pi \,.\, \exists X \,.\, \pi \in X$$



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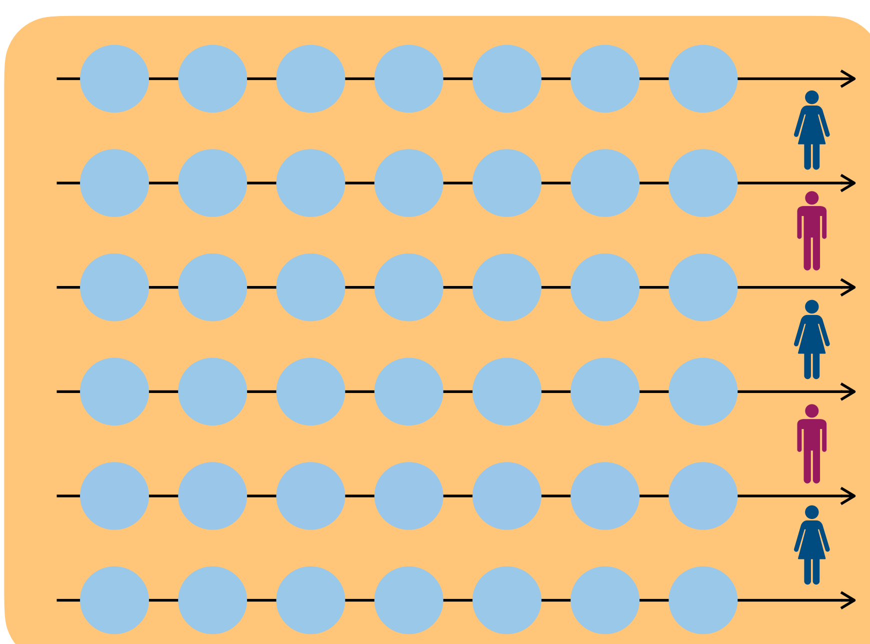

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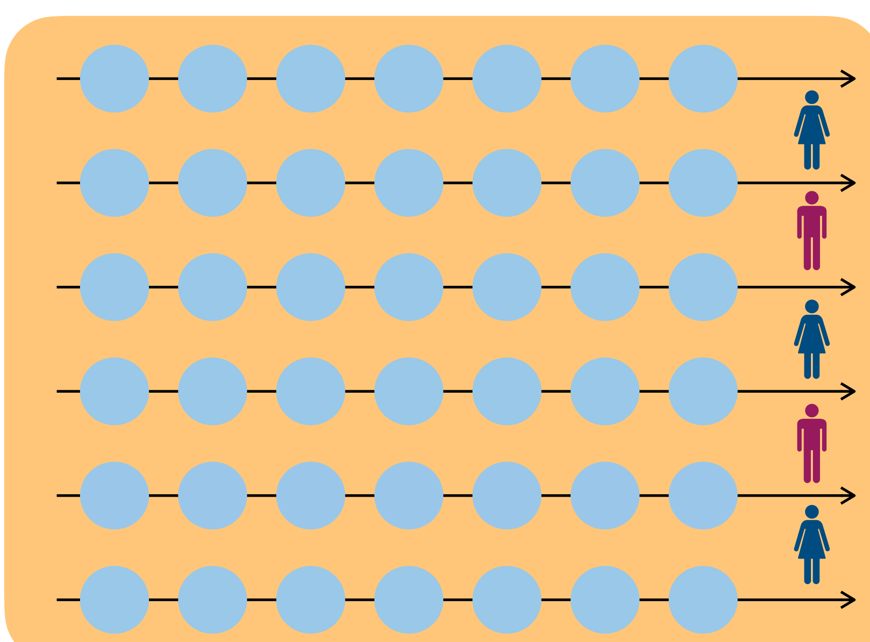
$$\forall \pi' \in X \,.\, \Diamond r_{\pi'}$$



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Observational determinism

$$\forall \pi_1 . \forall \pi_2 . (l_{\pi_1} \leftrightarrow l_{\pi_2}) \rightarrow \square (l_{\pi_1} \leftrightarrow l_{\pi_2})$$

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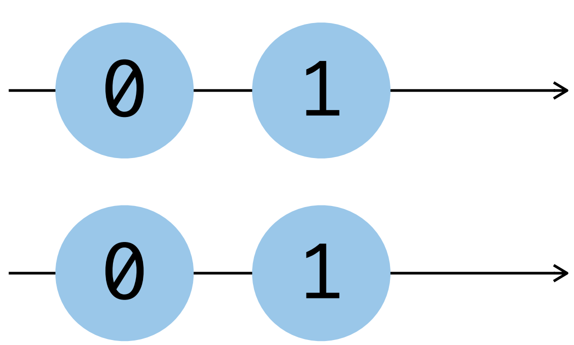
```
l := 0
if h then
  l := 1
else
  l := l + 1
```



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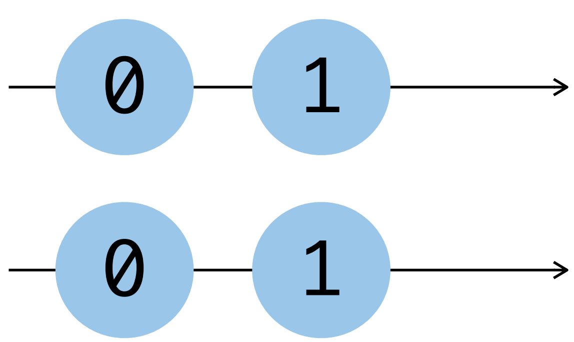
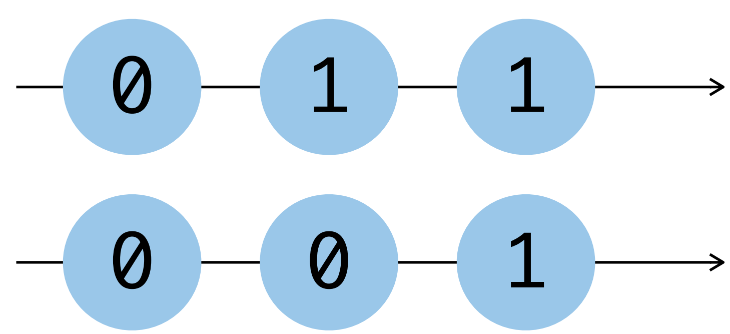
```





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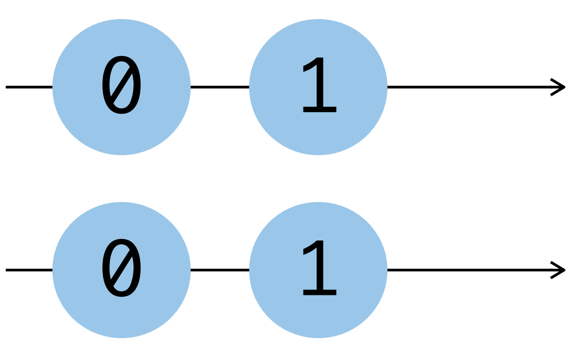


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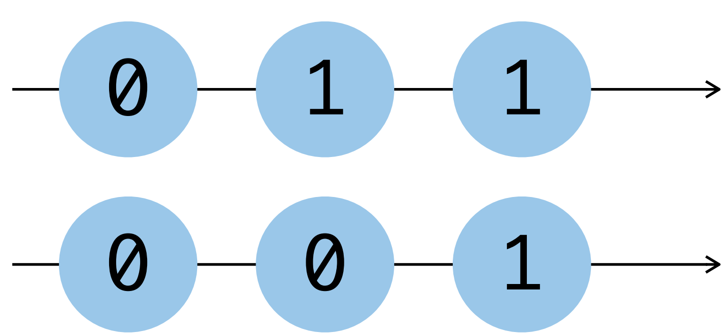

Access to shared resources

Scheduler decisions

Compiler optimizations

Asynchronous Hyperproperties

- HyperLTL$_S$
  HyperLTL + stuttering w.r.t a set of LTL formulas

- AHLTL
  HyperLTL + trajectories

- H_μ
  fixpoint-based logic

Asynchronous extensions of HyperLTL. Bozzelli, Peron, Sánchez (LICS 2021)
A temporal logic for asynchronous hyperproperties. Baumeister, Coenen, Bonakdarpour, Finkbeiner, Sánchez (CAV 2021)
Automata and fixpoints for asynchronous hyperproperties. Gutsfeld, Müller-Olm, Ohrem (POPL 2021)

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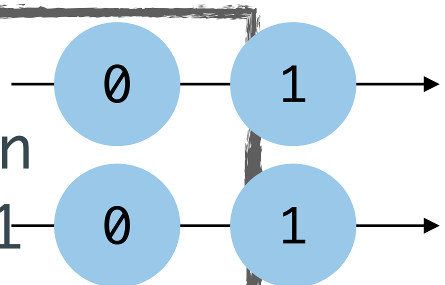


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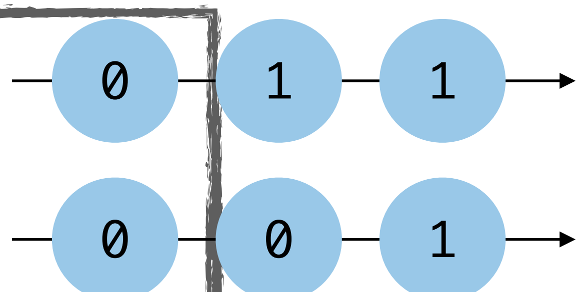
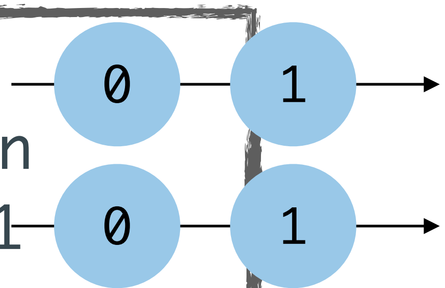


Asynchronous Hyperproperties

Observational determinism

$$\forall \pi_1 . \forall \pi_2 . (l_{\pi_1} \leftrightarrow l_{\pi_2}) \rightarrow \Box (l_{\pi_1} \leftrightarrow l_{\pi_2})$$

```
l := 0
if h then
   l := 1
else
   l := l + 1
```

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l := 0
if h then
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```

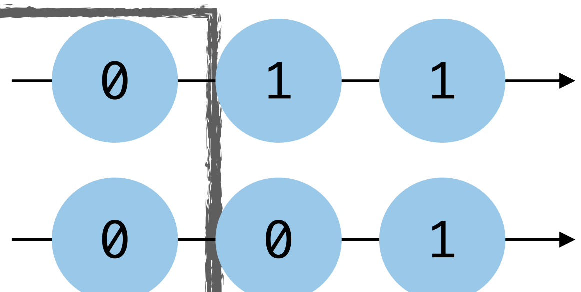

$\forall \pi_i . \exists X_i . \pi_i \in X_i$

X_i — all stutter-equivalent traces to π_i

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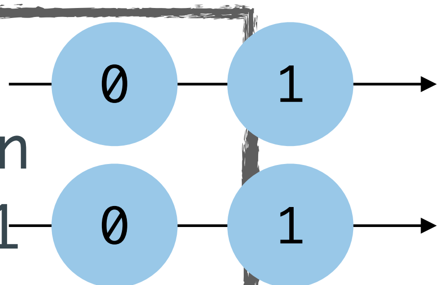


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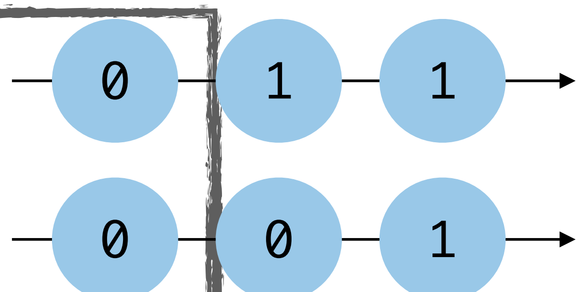

$$\forall \pi_i . \exists X_i . \pi_i \in X_i$$

$$\forall \pi \in X_i . \forall \pi' \in \mathcal{U} . (\pi =_{AP} \pi') \, \mathcal{U}$$
$$(\pi =_{AP} \pi' \wedge \Box \bigwedge_{a \in AP} a_\pi \leftrightarrow \bigcirc a_{\pi'}) \rightarrow \pi' \in X$$

Asynchronous Hyperproperties

Observational determinism

$$\forall \pi_1 . \forall \pi_2 . (l_{\pi_1} \leftrightarrow l_{\pi_2}) \rightarrow \Box (l_{\pi_1} \leftrightarrow l_{\pi_2})$$

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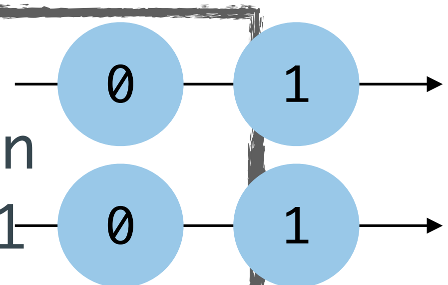


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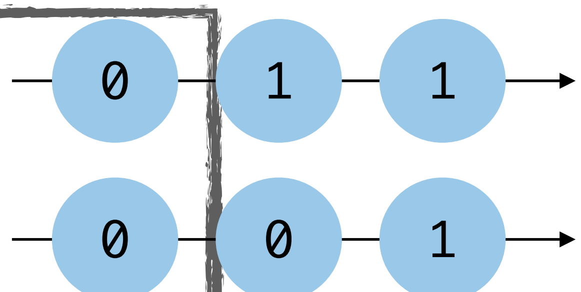

$\forall \pi_i . \exists X_i . \pi_i \in X_i$

X_i — all stutter-equivalent traces to π_i

$$\exists \pi_1 \in X_1 . \exists \pi_2 \in X_2 . (l_{\pi_1} \leftrightarrow l_{\pi_2}) \rightarrow \Box (l_{\pi_1} \leftrightarrow l_{\pi_2})$$

Hyper^{2}LTL

Why?

$$\psi := a_\pi \mid \neg\psi \mid \psi \wedge \psi \mid \bigcirc\psi \mid \psi \, \mathsf{U} \, \psi$$

$$\varphi := \exists \pi \in X \,.\, \varphi \mid \forall \pi \in X \,.\, \varphi \mid \exists X \,.\, \varphi \mid \forall X \,.\, \varphi$$

Common Knowledge

Asynchronous Hyperproperties

Trace Theory

Causality

Generic reasoning

Hyper^{2}LTL

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$$\psi := a_\pi \mid \neg\psi \mid \psi \wedge \psi \mid \bigcirc\psi \mid \psi \,\mathsf{U}\, \psi$$

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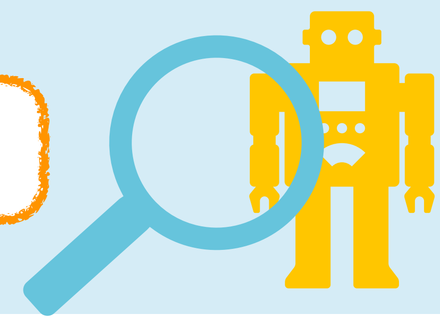
Common Knowledge

Asynchronous Hyperproperties

Trace Theory

Causality



trace property = set of traces

Generic reasoning

Hyper²LTL

Why?

$$\psi := a_\pi \mid \neg\psi \mid \psi \wedge \psi \mid \bigcirc\psi \mid \psi \, \mathcal{U} \, \psi$$

$$\varphi := \exists \pi \in X \,.\, \varphi \mid \forall \pi \in X \,.\, \varphi \mid \exists X \,.\, \varphi \mid \forall X \,.\, \varphi$$

Common Knowledge

Asynchronous Hyperproperties

Trace Theory

Causality

Generic reasoning and algorithms

Model Checking Hyper^{2}LTL$_{fp}$

Friday afternoon @CAV

$$\varphi = \exists \pi_1 . \forall X_1 . \forall \pi_2 \in X_1 . \ldots . \exists X_k . \exists \pi_k \in X_k . \psi$$

Interpreted over general set assignments

Model Checking Hyper^{2}LTL$_{fp}$

Friday afternoon @CAV

$$\varphi = \exists \pi_1 . \forall X_1 . \forall \pi_2 \in X_1 \ldots . \exists X_k . \exists \pi_k \in X_k . \psi$$

under some conditions

under some conditions

Model Checking Hyper²LTL$_{\mathrm{fp}}$

Friday afternoon @CAV

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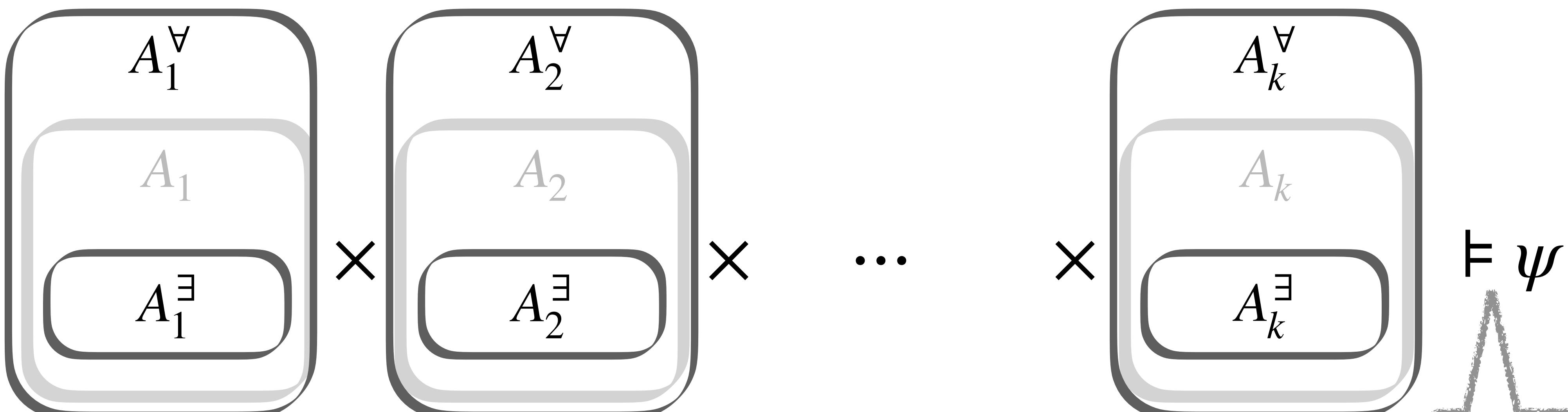
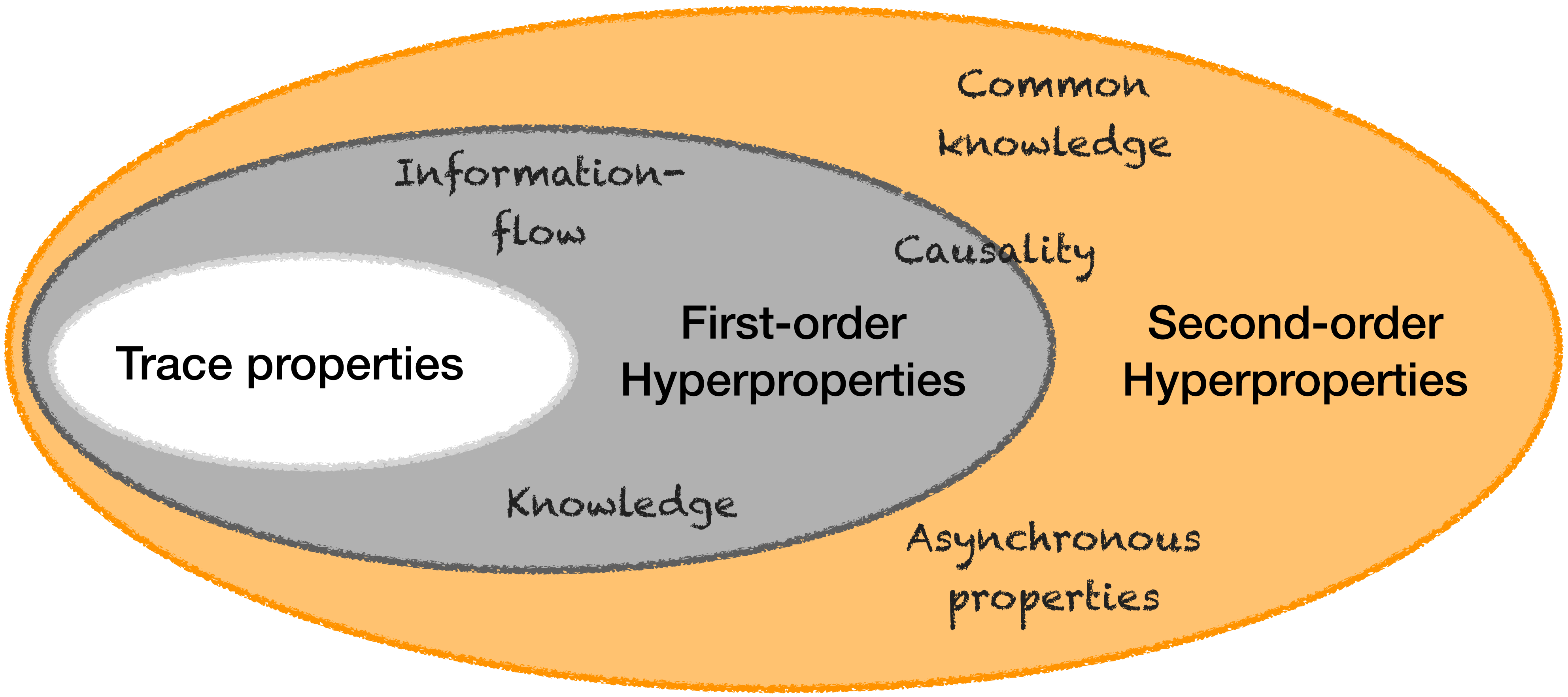


HyperLTL model checking

Why do we need **Second-Order Hyperlogics?**

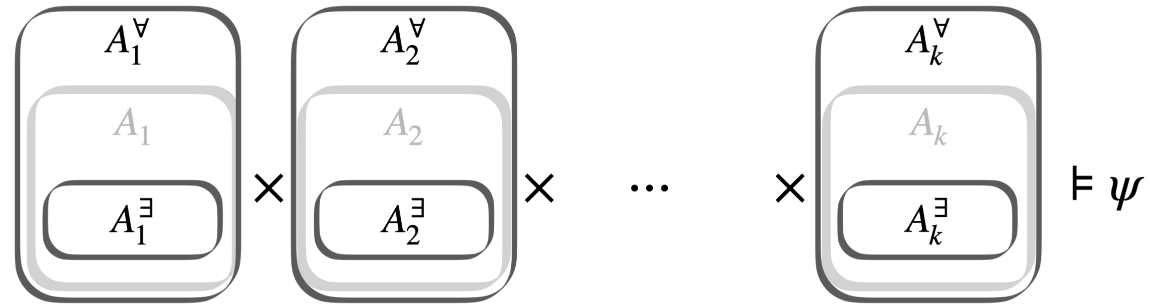


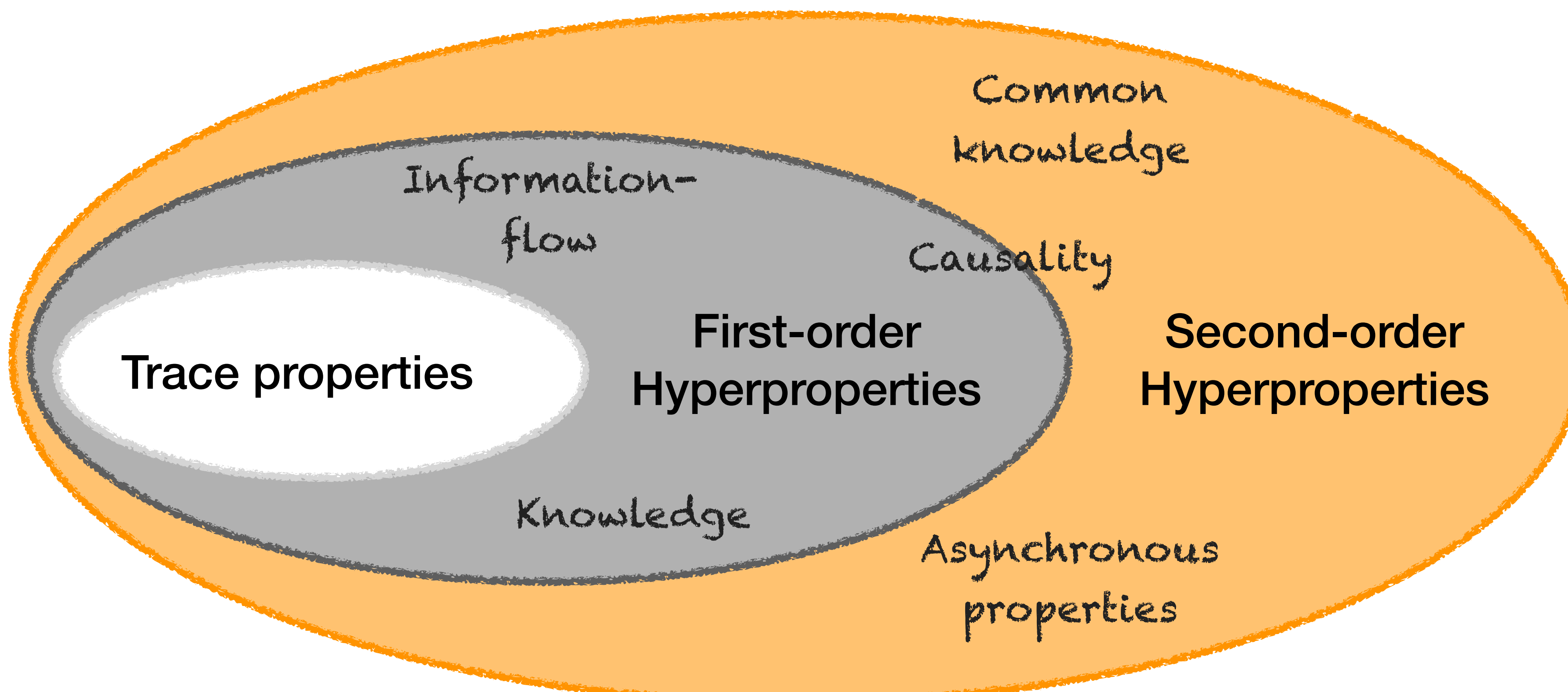
Why do we need Second-Order Hyperlogics?

Hyper²LTL

$$\psi := a_\pi \mid \neg\psi \mid \psi \wedge \psi \mid \bigcirc\psi \mid \psi \,\mathcal{U}\, \psi$$

$$\varphi := \exists\pi.\varphi \mid \forall\pi.\varphi \mid \exists X.\varphi \mid \forall X.\varphi$$



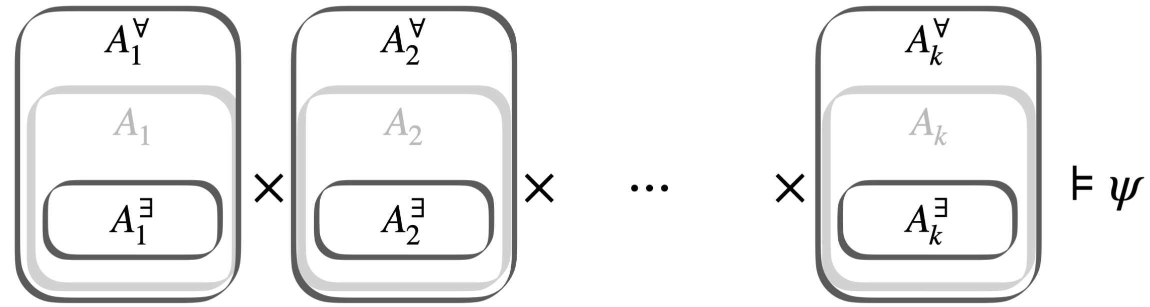


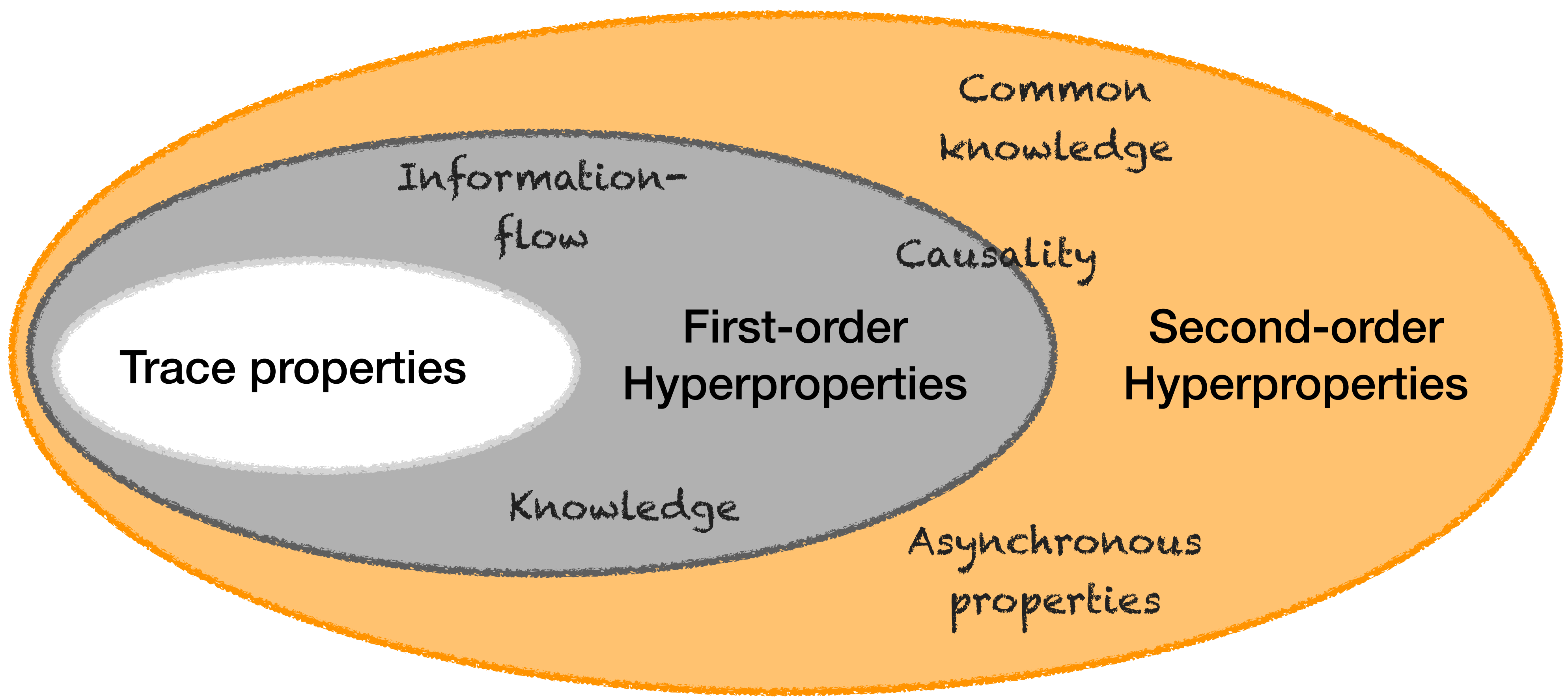
Why do we need **Second-Order Hyperlogics?**

Hyper²LTL

$$\psi := a_\pi \mid \neg\psi \mid \psi \wedge \psi \mid \bigcirc\psi \mid \psi \, \mathcal{U} \, \psi$$

$$\varphi := \exists \pi . \varphi \mid \forall \pi . \varphi \mid \exists X . \varphi \mid \forall X . \varphi$$





Thank you!