Deciding Hyperproperties

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The 27th International Conference on Concurrency Theory

Canada, 23.-26. August 2016

Information Leakage

- Heartbleed - 4.5m patient information leaked
- Goto Fail - encryption of >300m devices broken
- Shellshock - web servers attackable for 22 years

HyperLTL - A Logic for Information-flow Control

[Clarkson, Finkbeiner, Koleini, Micinski, Rabe, Sánchez, ’14]

- Observational Determinism: “Program appears deterministic to low security users.”

  $\forall \pi . \forall \pi' . \square (I_\pi = I_{\pi'}) \rightarrow \square (O_\pi = O_{\pi'})$

- Generalized Noninterference: “. . . additionally low-security outputs may not be altered by injection of high-security inputs.”

  $\forall \pi . \forall \pi' . \exists \pi'' . \ \square (HighI_\pi = HighI_{\pi''}) \wedge \square (O_{\pi'} = O_{\pi''})$

HyperLTL - An Extension of LTL

LTL

- logical connectives: $\vee$, $\neg$
  temporal connectives:
  $\square$ - *globally*
  $\bigcirc$ - *next*
- $\square a$ is satisfied by $\{a\}^\omega$ as well as $\{a, b\}^\omega$
- $\square a \wedge \square \neg a$ is unsatisfiable.
- defines a set of computation traces
  (trace property)

HyperLTL

- LTL + explicit trace quantifiers:
  Observational Determinism:
  $$\forall \pi . \forall \pi' . \quad \square(I_\pi = I_{\pi'}) \rightarrow \square(O_\pi = O_{\pi'})$$
- $\exists \pi . \exists \pi' . \ \square a_\pi \wedge \square \neg a_{\pi'}$
  is satisfiable by $\{\{a\}^\omega, \{b\}^\omega\}$.
- defines a set of sets of computation traces
  (hyperproperty)

Trace Properties and Hyperproperties

Definition

Let $TR := (2^{AP})^\omega$ be the set of all traces. A trace property $T \subseteq TR$ is a set of execution traces.

Example (a on every position, $\Box a$)

$\{ \{a\}^\omega, \{a, b\}^\omega \dots \}$

Definition

A hyperproperty [Clarkson, Schneider, '10] $H \subseteq 2^{TR}$ is a set of sets of execution traces.

Example (one trace with a on every position, one without a)

$\{ \{ \{a\}^\omega, \{\}^\omega \}, \{ \{a\}^\omega, \{b\}^\omega \},$
$\{ \{a, b\}^\omega, \{\}^\omega \}, \{ \{a, b\}^\omega, \{b\}^\omega \} \dots \}$

Satisfiability of HyperLTL

Definition (HyperLTL-SAT)

Let φ be an HyperLTL formula. HyperLTL-SAT is the problem to decide whether there exists a non-empty trace set T satisfying φ.

Example (Application)

Two versions of Observational Determinism:

- $\forall \pi. \forall \pi'. \square (I_\pi = I_{\pi'}) \rightarrow \square (O_\pi = O_{\pi'})$
- $\forall \pi. \forall \pi'. (I_\pi = I_{\pi'}) \rightarrow \square (O_\pi = O_{\pi'})$

Which variation is stronger?

Challenge

LTL Satisfiability Solving

- Translate LTL formula into Büchi automaton
- Check the automaton for emptiness
- PSPACE-complete

HyperLTL Satisfiability Solving

- A Hyperproperty is not necessarily ω-regular
- Standard automata approach cannot be applied

Key Results

- HyperLTL-SAT is PSPACE-complete for alternation-free formulas
- HyperLTL-SAT is EXPSPACE-complete for $\exists^*\forall^*$ formulas
- HyperLTL-SAT is undecidable for $\forall\exists$ formulas

HyperLTL Syntax

Syntax

$\psi ::= \exists \pi.\ \psi \mid \forall \pi.\ \psi \mid \varphi$

$\varphi ::= a_\pi \mid \neg \varphi \mid \varphi \vee \varphi \mid \bigcirc \varphi \mid \varphi\, \mathcal{U}\, \varphi$

- Quantifier Prefix with arbitrary alternation
- Then quantifier-free LTL formula with trace variables
- $\wedge, \rightarrow, \leftrightarrow, \square, \diamond$ derived in the usual way
- $X_\pi = X_{\pi'}$ syntactic sugar for $\bigwedge_{x \in X}(x_\pi \leftrightarrow x_{\pi'})$

Example

"All executions have the light on at the same time."

$\forall \pi.\ \forall \pi'.\ \square(on_\pi \leftrightarrow on_{\pi'})$

HyperLTL Semantics

Semantics w.r.t. Trace Environment $\Pi : Var \mapsto TR$

$\Pi \models_T \forall \pi.\varphi$	*iff*	*for all* $t \in T$, *s.t.* $\Pi[\pi \mapsto t] \models_T \varphi$
$\Pi \models_T a_\pi$	*iff*	$a \in \Pi(\pi)[0]$
$\Pi \models_T \Box\varphi$	*iff*	$\forall i \geq 0 : \Pi[i, \infty] \models_T \varphi$

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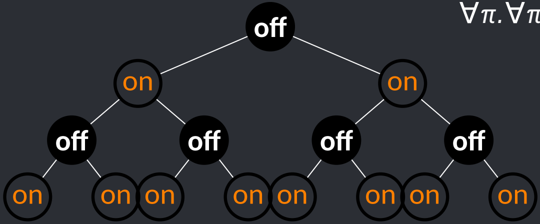

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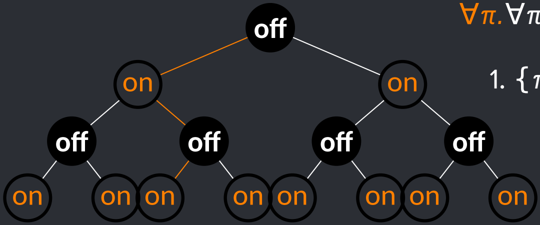

1. $\{\pi \mapsto t\} \models_M \forall \pi' . \; \Box(\ldots)$

HyperLTL Semantics

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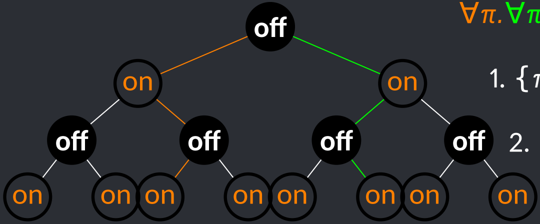

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HyperLTL Semantics

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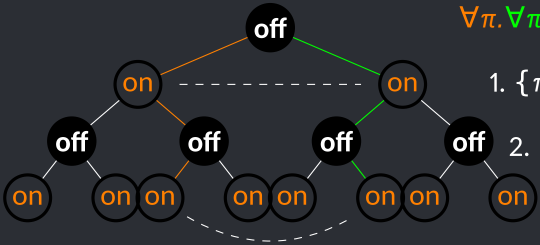
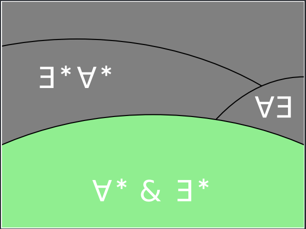


1. $\{\pi \mapsto t\} \models_M \forall \pi' . \ \Box(\ldots)$
2. $\{\pi \mapsto t, \pi' \mapsto t'\} \models_T \ \Box(\ldots)$
3. $\forall i \geq 0 : \ \{\pi \mapsto t[i, \infty], \pi' \mapsto t'[i, \infty]\} \models_T on_\pi \leftrightarrow on_{\pi'}$

Outline - Solving HyperLTL-SAT



1. Alternation-free fragments (∀* & ∃*)
2. Alternation starting with existential quantifier (∃*∀*)
3. Alternation starting with universal quantifier (∀*∃*)

Existential Fragment

Theorem

$\exists^*$ *HyperLTL-SAT is* ***PSPACE-complete***.

Example

$\exists \pi_0 \exists \pi_1.\ \square a_{\pi_0} \wedge \square b_{\pi_0} \wedge \square c_{\pi_0} \wedge \square a_{\pi_1} \wedge \square \neg c_{\pi_1}$

Idea:

Replace indexed atomic propositions with fresh atomic propositions.

$$\square a_0 \wedge \square b_0 \wedge \square c_0 \wedge \square a_1 \wedge \square \neg c_1$$

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$$\Box a_0 \land \Box b_0 \land \Box c_0 \land \Box a_1 \land \Box \neg c_1$$
$$t : \{a_0, b_0, c_0, a_1\}^\omega$$

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$$\square a_0 \wedge \square b_0 \wedge \square c_0 \wedge \square a_1 \wedge \square \neg c_1$$
$$t : \{a_0, b_0, c_0, a_1\}^\omega$$

$$T = \{\{a, b, c\}^\omega, \{a\}^\omega\}$$

Universal Fragment

Theorem

$\forall^*$ *HyperLTL-SAT is* ***PSPACE-complete***.

Example

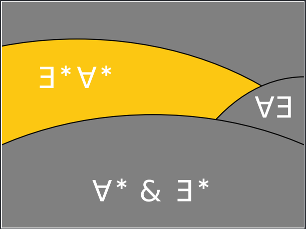
$$\forall\pi\forall\pi'.\,\Box b_\pi \land \Box\neg b_{\pi'} \quad\equiv\quad \Box b \land \Box \neg b$$

$\Downarrow$ $\Downarrow$ ... $\Downarrow$

t t ... unsatisfiable

Idea: Discard indexes from indexed propositions

Outline - Solving HyperLTL-SAT



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$\exists^*\forall^*$ HyperLTL-SAT

Lemma

For every $\exists\pi_1 \ldots \exists\pi_n \forall\pi'_1 \ldots \forall\pi'_m . \varphi$ HyperLTL formula, there exists an equisatisfiable $\exists^*$ HyperLTL formula.

Example

$$\exists\pi_0\exists\pi_1\forall\pi'_0\forall\pi'_1.\ (\Box a_{\pi'_0} \land \Box b_{\pi'_1}) \land (\Box c_{\pi_0} \land \Box d_{\pi_1})$$

Idea: Unroll universal quantifiers

$$\begin{aligned}\exists\pi_0\exists\pi_1.\ (\Box a_{\pi_0} \land \Box b_{\pi_0}) &\land (\Box c_{\pi_0} \land \Box d_{\pi_1})\\ \land(\Box a_{\pi_1} \land \Box b_{\pi_0}) &\land (\Box c_{\pi_0} \land \Box d_{\pi_1})\\ \land(\Box a_{\pi_0} \land \Box b_{\pi_1}) &\land (\Box c_{\pi_0} \land \Box d_{\pi_1})\\ \land(\Box a_{\pi_1} \land \Box b_{\pi_1}) &\land (\Box c_{\pi_0} \land \Box d_{\pi_1})\end{aligned}$$

Complexity of $\exists^*\forall^*$ HyperLTL-SAT

Theorem

Let n be the number of existential quantifier and m be the number of universal quantifier. $\exists^*\forall^*$ HyperLTL-SAT is EXPSPACE-complete.

- Unrolling results in formula of size $O(n^m)$.
- Hardness follows from an encoding of an EXPSPACE-bounded Turing machine in this fragment.

$\exists^*\forall^b$ HyperLTL-SAT

Theorem

Bounded $\exists^*\forall^b$ HyperLTL-SAT is PSPACE-complete.

Observation: In practice, many properties of interest quantify universally over pairs of traces

$\forall\pi.\forall\pi'.\Box(I_\pi = I_{\pi'}) \rightarrow \Box(O_\pi = O_{\pi'})$

$\forall\pi.\forall\pi'.(I_\pi = I_{\pi'}) \rightarrow \Box(O_\pi = O_{\pi'})$

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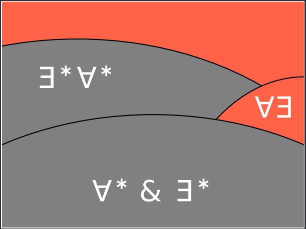

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The Power of ∀∃

$$\forall\pi\exists\pi'.\ a_{\pi'} \qquad (1)$$

$$\wedge\ \square(a_\pi \rightarrow \bigcirc\square\neg a_\pi) \qquad (2)$$

$$\wedge\ \square(a_\pi \rightarrow \bigcirc a_{\pi'}) \qquad (3)$$

$t_1 :\ \{a\}(\{\,\})^\omega$

$t_2 :\ \{\,\}\{a\}(\{\,\})^\omega$

$t_3 :\ \{\,\}\{\,\}\{a\}(\{\,\})^\omega$

$\ldots$

$\rightarrow$ Model has infinitely many traces.

Encoding of Posts Correspondence Problem

Example PCP instance:

I	II	III
a	ab	bba
baa	aa	bb

Encoding of Posts Correspondence Problem

Example PCP solution:

III	II	III	I
bba	ab	bba	a
bb	aa	bb	baa

HyperLTL encoding:

1. exists a “solution”-trace π_s,
   where top matches bottom

Encoding of Posts Correspondence Problem

Example PCP solution:

III	II	III	I
bba	ab	bba	a
bb	aa	bb	baa

HyperLTL encoding:

1. exists a “solution”-trace π_s, where top matches bottom

	b	b	a	a	b	b	b	a	a	#	#	$\cdots$
	b	b	a	a	b	b	b	a	a	#	#	$\cdots$
π_s												

Encoding of Posts Correspondence Problem

Example PCP solution:

III	II	III	I
bba	ab	bba	a
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b b a | a b b b a a # # · · ·
b b | a a b b b a a # # · · ·
π_S III

HyperLTL encoding:

1. exists a "solution"-trace π_S, where top matches bottom
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Encoding of Posts Correspondence Problem

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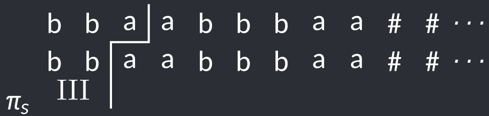

HyperLTL encoding:

1. exists a “solution”-trace π_s, where top matches bottom
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π_s

b b a | a b b b a a # # · · ·
b b | a a b b b a a # # · · ·
III

π'_s

a b b b a a # # · · ·
a a b b b a a # · · ·

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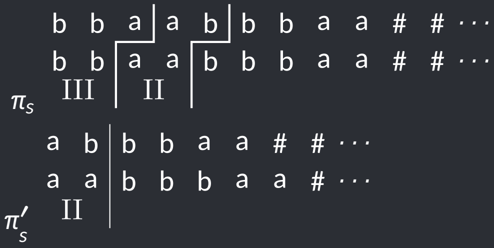

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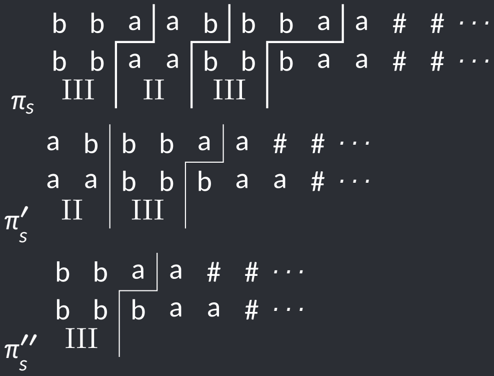

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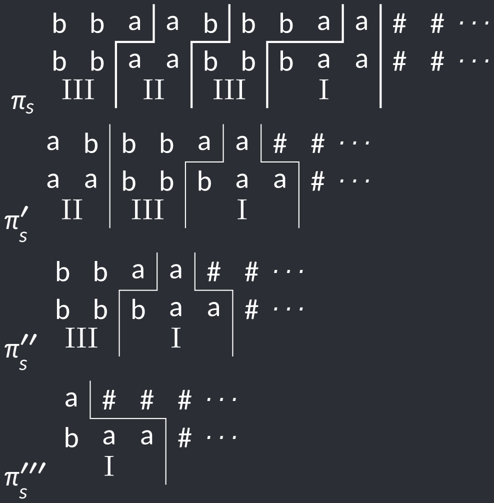

Summary

$\exists^*$	$\forall^*$	$\exists^*\forall^*$	*b*-Bounded $\exists^*\forall^*$	$\forall\exists$
PSpace-complete	**PSpace-complete**	**EXPSpace-complete**	**PSpace-complete**	undecidable

Application: Implication Checking of Quantifier-alternation-free Hyperproperties

ψ implies φ?

Check the negation $\neg(\psi \rightarrow \varphi)$ for unsatisfiability.

Example

To determine whether the HyperLTL formula $\forall \pi_0 \ldots \forall \pi_n.\ \psi$ implies the HyperLTL formula $\forall \pi'_0 \ldots \forall \pi'_m.\ \varphi$, we check the $\exists^n \forall^m$ formula $\exists \pi_1 \ldots \pi_n \forall \pi'_0 \ldots \pi'_m.\ \psi \wedge \neg \varphi$ for unsatisfiability.

Theorem

Implication Checking of quantifier-alternation-free HyperLTL formulas is EXPSPACE-complete.

Summary & Conclusion

$\exists^*$	$\forall^*$	$\exists^*\forall^*$	*b*-Bounded $\exists^*\forall^*$	$\forall\exists$
PSpace-complete	**PSpace-complete**	**EXPSpace-complete**	**PSpace-complete**	undecidable

- Satisfiability of alternation-free formulas is decidable
- Implication and equivalence of alternation-free formulas are decidable
- Full logic is undecidable:
  HyperLTL is much more powerful than LTL

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Picture: http://russia-insider.com/sites/insider/files/20110226_bbd001_0.jpg