Efficient Parallel Path Checking

Lars Kuhtz

Universität des Saarlandes

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Path checking

Path checking problem [Markey,Schnoebelen 2003]

Given a formula φ and a finite path σ. Decide $\sigma \models \varphi$.

Related problems

Regular expressions NL

Semi-extended regular expressions logCFL

Star-free regular expressions P

Extended regular expressions P

Theorem ([Kuhtz,Finkbeiner 2009])

The path checking problem for LTL is efficiently parallelizable.

Path checking in NC

Proof idea

1. Construct equivalent Boolean circuit.
2. Decompose circuit into planar monotone sub-circuits.
3. Evaluate circuit in NC using [Yang 1991] as oracle.

Theorem (Yang 1991. Delcher,Kosaraju 1995)

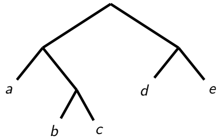
The planar monotone circuit value problem is in NC*.*

LTL – construction of circit

Construction of circuit

1. Obtain monotone formula (de Morgan's law).
2. Obtain monotone Boolean circuit $\mathcal{C}$ (expansion laws).

$$(a\,\mathrm{U}(b\,\mathrm{U}c))\,\mathrm{U}(d\,\mathrm{U}e)$$



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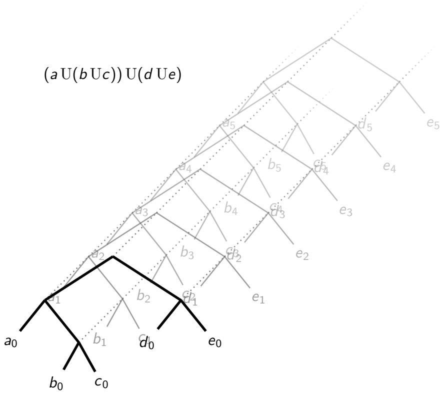

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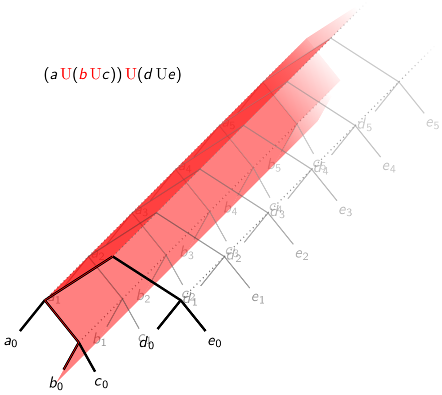

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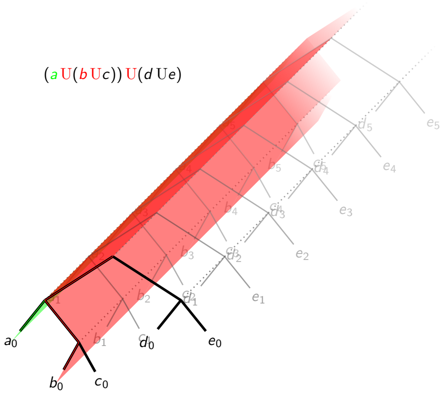

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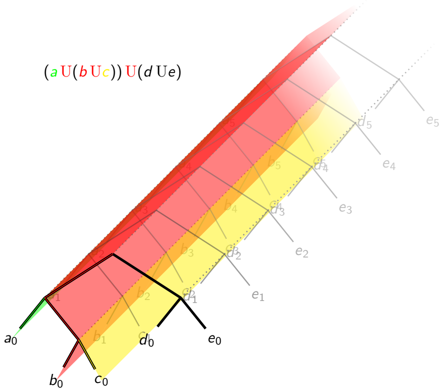

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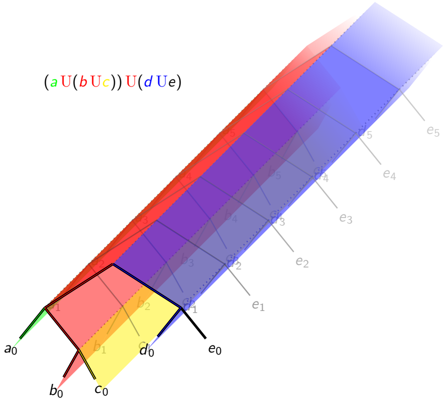

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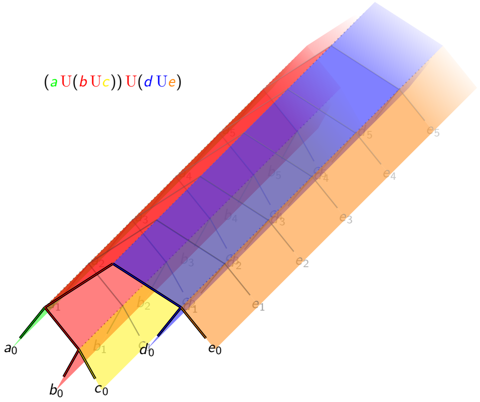
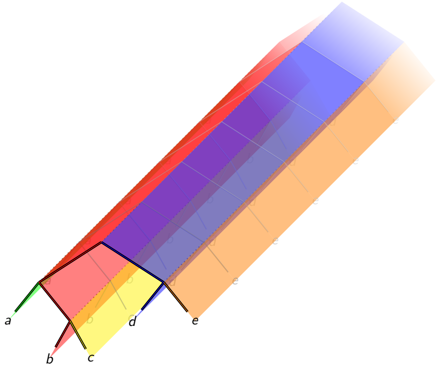


LTL – decomposition of circuit

Topology of circuit

Tree of planar monotone circuits induced by structure of formula.



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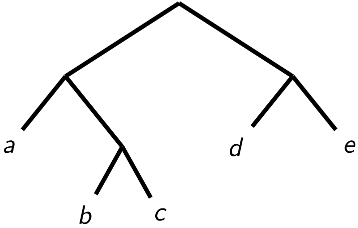

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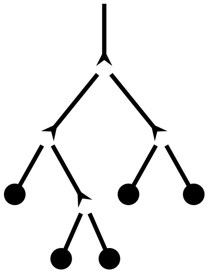
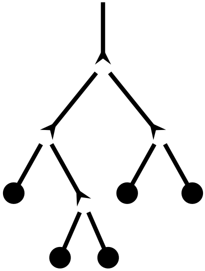


LTL – evaluation of the circuit

Evaluation of circuit

- Tree contraction (AC^1 reduction).
- Use [Yang 1991] as oracle for evaluation of planar monotone circuits.



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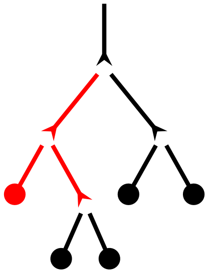

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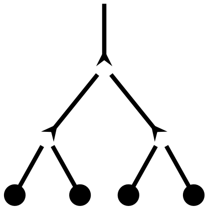

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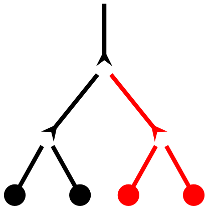

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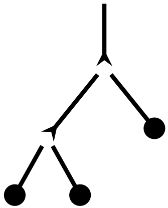

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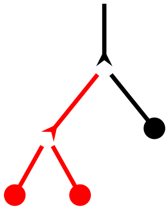

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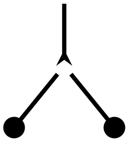

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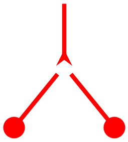

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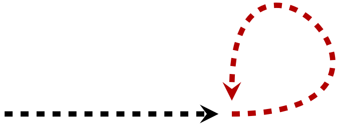
Ultimately periodic paths



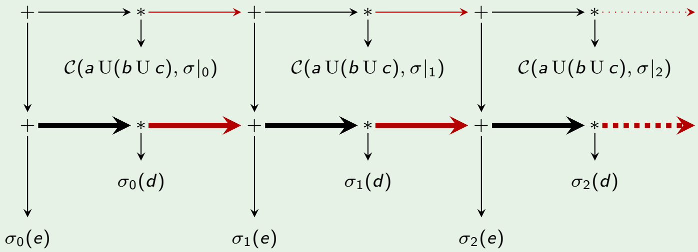
Theorem ([Markey,Schnoebelen 2003])

For any LTL+Past formula φ and ultimately periodic path π, one can build in logspace a formula φ' and a finite path π' s.t.

$$\pi \models \varphi \ \textit{iff}\ \pi' \models \varphi'.$$

LTL+Past

Example $((a \,\mathrm{U}\, (b \,\mathrm{U}\, c)) \,\mathrm{U}\, (d \,\mathrm{U}\, e))$



LTL+Past

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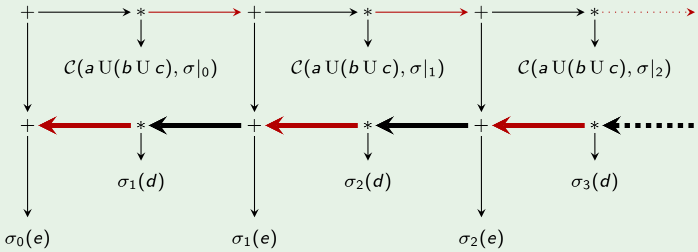

CTL over finite trees

Construction of circuit

1. Obtain monotone formula (de Morgan's law).
2. Obtain monotone Boolean circuit (expansion laws).

Topology of circuit

Tree of trees:

- Inner tree: induced by structure of formula.
- Outer tree: induced by structure of model.

Evaluation of circuit

- Tree contraction (AC^1 reduction) on outer tree.
- Use [Kuhtz,Finkbeiner 2009] for evaluation of inner tree.