Causality-based LTL Model Checking without Automata

joint work with Bernd Finkbeiner

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- $\pm$ $\forall\pi.\Box\Phi$ / $\exists\pi.\Diamond\Phi$
- ✓ Infinite-state
- ✓ Multi-threading

$$T_1 \parallel \ldots \parallel T_n \models \Box\neg(at_{l_3} \wedge at_{m_3})$$

[CONCUR 2013]

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[CAV 2014]

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Termination: the first termination prover that scales to a large number of non-trivial concurrent threads.

	Terminator		T2		AProVE		**Arctor** [1]	
Threads	Time(s)	Mem.(MB)	Time(s)	Mem.(MB)	Time(s)	Mem.(MB)	Time(s)	Mem.(MB)
1	3.37	26	2.42	38	3.17	237	**0.002**	**2.3**
2	1397	1394	3.25	44	6.79	523	**0.002**	**2.6**
3	×	MO	U(29.2)	253	U(26.6)	1439	**0.002**	**2.6**
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Example: individual accessibility for semaphores

Fair scheduling: $\Box\Diamond(at_2 \wedge r_{free}) \implies \Box\Diamond at_3$

Termination of critical sections: $\Box(at_3 \implies \Diamond at_1)$

Individual accessibility: $\Box(at_2 \implies \Diamond at_3)$

$$\varphi \equiv \bigwedge_{i \in 1..n}(Scheduling_i \wedge Termination_i) \implies Accessibility_1$$

Translation of $\neg\varphi$ into a Büchi automaton, **ltl3ba**:

Threads	Time (sec)	|Automaton| (MB)
2	0.005	0.002
3	0.09	0.38
4	9.6	8.6
5	1295	185
6	TO	X

Our approach

Causality

A relationship between two events, when the occurrence of first event is recognized as a necessary prerequisite for the occurrence of the second

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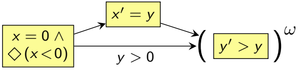
- **Proof objects: concurrent traces**
  compactly represent sets of program runs, by specifying events that should *necessarily* occur in the run, and the partial order between them



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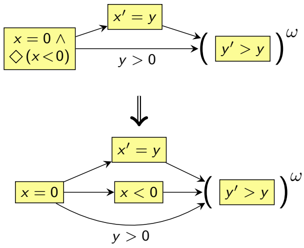

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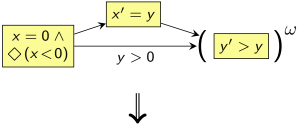

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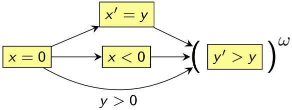
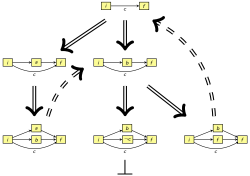
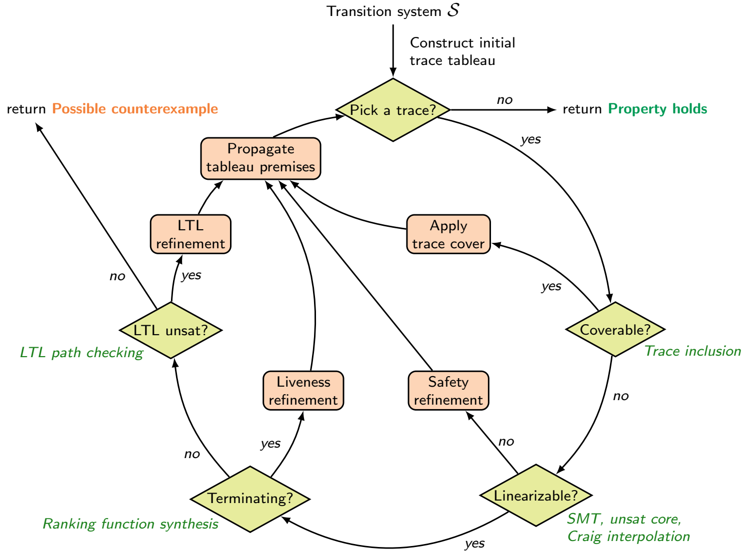


- **Proof construction: tableau search based on causal loops**
  causal loops $\equiv$ infinitely-looping trace transformations
  - root trace captures all possible counterexamples
  - tableau branches according to applications of proof rules
  - termination when all leaves are contradictory, or covered by causal loops



LTL Model Checking Algorithm



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Want to learn more? See the poster, and talk to me!