Causality-based LTL Model Checking without Automata

joint work with Bernd Finkbeiner

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Reactive
Systems

Motivation

LTL

A well-established basis for specification, verification, and synthesis of reactive programs. We consider two decision problems:

- Satisfiability/validity: $\models \varphi$
- Model checking against a program: $P \models \varphi$

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Automata-based LTL Model Checking

The standard way to model check a program P against an LTL property φ:

1. translate $\neg\varphi$ into a Büchi automaton A
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Main problem: LTL formulas are often not small!

They describe necessary assumptions of, e.g.:

- fairness
- termination
- allowed request/response pairs

Example: individual accessibility for semaphores

Thread 1

```
while (true) {
  l1:  noncritical;
  l2:  request r;
  l3:  critical;
  l4:  release r;
}
```

Thread 2

```
while (true) {
  m1:  noncritical;
  m2:  request r;
  m3:  critical;
  m4:  release r;
}
```

Thread 3

```
while (true) {
  n1:  noncritical;
  n2:  request r;
  n3:  critical;
  n4:  release r;
}
```

LTL Properties

Fair scheduling: $\varphi_F \equiv \Box\Diamond(at_2 \wedge r_{free}) \implies \Box\Diamond at_3$

Termination of critical sections: $\varphi_T \equiv \Box(at_3 \implies \Diamond at_1)$

Individual **A**ccessibility: $\varphi_A \equiv \Box(at_2 \implies \Diamond at_3)$

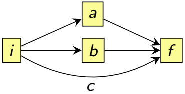
$$\varphi \equiv \bigwedge_{i \in 1..n}(\varphi_{F_i} \wedge \varphi_{T_i}) \implies \varphi_{A_1}$$

Translation of $\neg\varphi$ into a Büchi automaton: **ltl3ba**

Threads	Time (sec)	Memory (MB)	|Automaton| (MB)
2	0.005	4.2	0.002
3	0.09	5.0	0.38
4	9.6	14.7	8.6
5	1295	139	185
6	TO	X	X

Our approach

- **Proof objects: concurrent traces**
  allow to capture temporal order, constraints, independence



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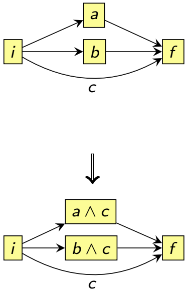
- **Proof rules based on causality**
  causality $\equiv$ language-preserving trace transformations



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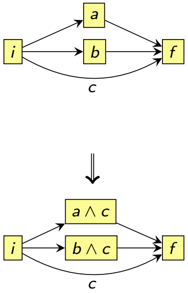
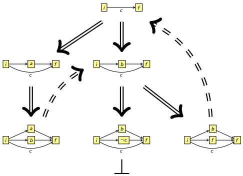
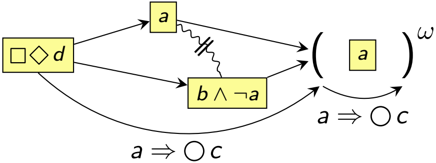


- **Proof rules based on causality**
  causality ≡ language-preserving trace transformations

- **Proof construction: tableau search based on causal loops**
  causal loops ≡ infinitely-looping trace transformations



Concurrent traces



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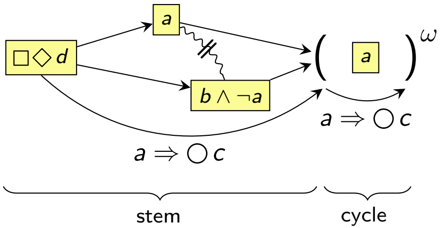

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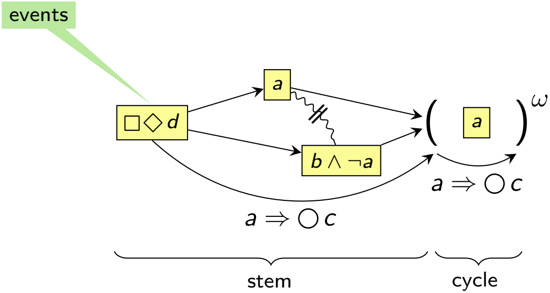

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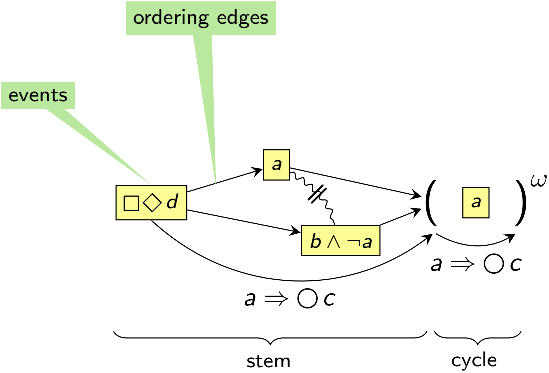
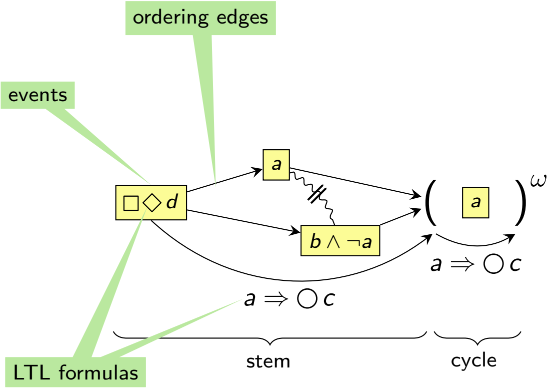
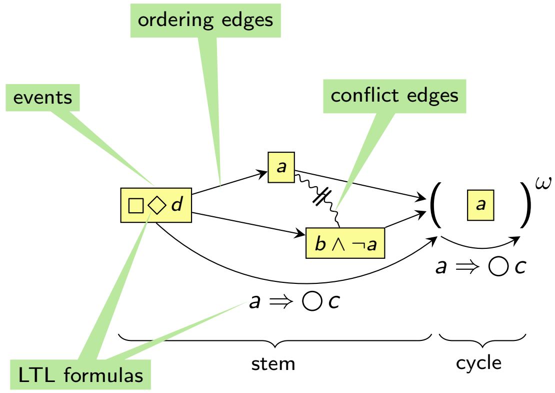


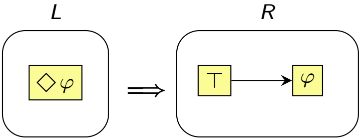
Concurrent traces



Concurrent traces



LTL proof rules



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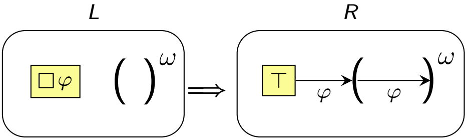
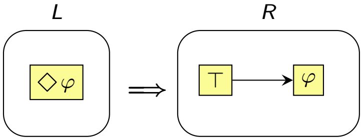


Globally

LTL proof rules



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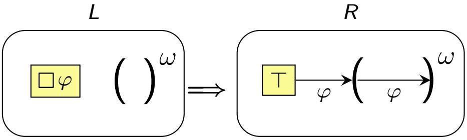

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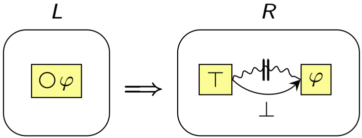

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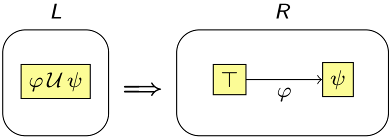
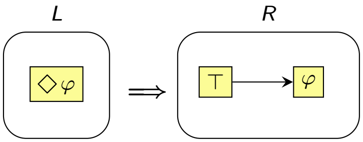


Until

LTL proof rules



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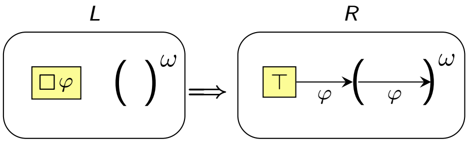

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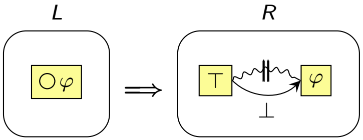

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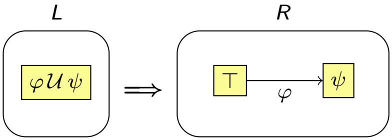

Until

Other proof rules

for safety [K., Finkbeiner, Concur 2013], and termination [K., Finkbeiner, CAV 2014]

LTL satisfiability: $\Box\Diamond p \wedge \Box\Diamond\neg p$

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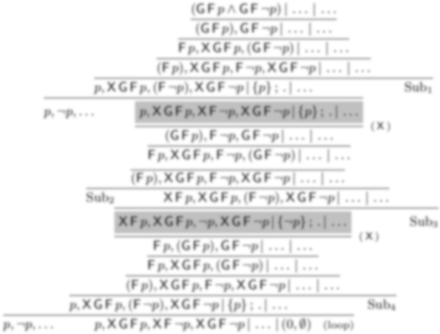
(G F p ∧ G F ¬p) | … | …
(G F p), G F ¬p | … | …
F p, X G F p, (G F ¬p) | … | …
(F p), X G F p, F ¬p, X G F ¬p | … | …
p, X G F p, (F ¬p), X G F ¬p | {p} : . | …
Sub₁
p, ¬p, …
p, X G F p, X F ¬p, X G F ¬p | {p} : . | …
(X)
(G F p), F ¬p, G F ¬p | … | …
F p, X G F p, F ¬p, (G F ¬p) | … | …
(F p), X G F p, F ¬p, X G F ¬p | … | …
Sub₂
X F p, X G F p, (F ¬p), X G F ¬p | … | …
X F p, X G F p, ¬p, X G F ¬p | {¬p} : . | …
Sub₃
(X)
F p, (G F p), G F ¬p | … | …
F p, X G F p, (G F ¬p) | … | …
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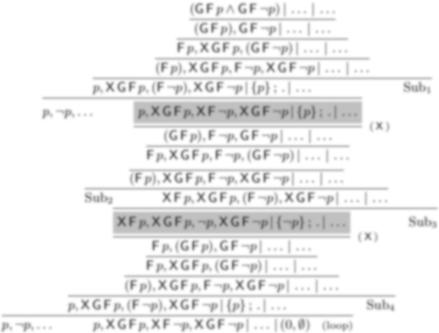
p, X G F p, (F ¬p), X G F ¬p | {p} : . | …
Sub₄
p, ¬p, …
p, X G F p, X F ¬p, X G F ¬p | … | (0, ∅)
(loop)

[Schwendimann, 1998,
A New One-Pass Tableau Calculus for PLTL]

Tools: **LWB**, **pltl**, **LTL Tableau**,. . .

LTL satisfiability: $\Box\Diamond p \wedge \Box\Diamond\neg p$

$$\boxed{\Box\Diamond p \ \wedge\ \Box\Diamond\neg p} \longrightarrow \left(\ \boxed{\top}\ \right)^{\omega}$$


(GF $p \wedge$ GF $\neg p$) | … | …
(GF p), GF $\neg p$ | … | …
F p, XGF p, (GF $\neg p$) | … | …
(F p), XGF p, F $\neg p$, XGF $\neg p$ | … | …
p, XGF p, (F $\neg p$), XGF $\neg p$ | $\{p\}$: . | …
Sub$_1$
$p, \neg p, \ldots$
p, XGF p, XF $\neg p$, XGF $\neg p$ | $\{p\}$: . | …
(X)
(GF p), F $\neg p$, GF $\neg p$ | … | …
F p, XGF p, F $\neg p$, (GF $\neg p$) | … | …
(F p), XGF p, F $\neg p$, XGF $\neg p$ | … | …
Sub$_2$
XF p, XGF p, (F $\neg p$), XGF $\neg p$ | … | …
XF p, XGF p, $\neg p$, XGF $\neg p$ | $\{\neg p\}$: . | …
Sub$_3$
(X)
F p, (GF p), GF $\neg p$ | … | …
F p, XGF p, (GF $\neg p$) | … | …
(F p), XGF p, F $\neg p$, XGF $\neg p$ | … | …
p, XGF p, (F $\neg p$), XGF $\neg p$ | $\{p\}$: . | …
Sub$_4$
$p, \neg p, \ldots$
p, XGF p, XF $\neg p$, XGF $\neg p$ | … | $(0, \emptyset)$
(loop)

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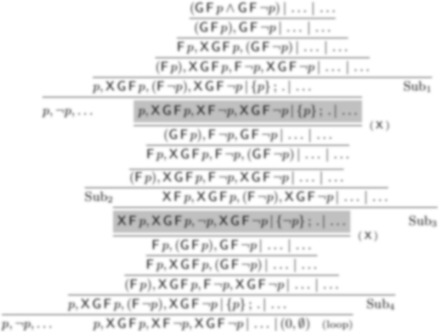

(GF p ∧ GF ¬p) | … | …
(GF p), GF ¬p | … | …
F p, XGF p, (GF ¬p) | … | …
(F p), XGF p, F ¬p, XGF ¬p | … | …
p, XGF p, (F ¬p), XGF ¬p | {p} ; . | …
Sub₁
p, ¬p, …
p, XGF p, XF ¬p, XGF ¬p | {p} ; . | …
(X)
(GF p), F ¬p, GF ¬p | … | …
F p, XGF p, F ¬p, (GF ¬p) | … | …
(F p), XGF p, F ¬p, XGF ¬p | … | …
Sub₂
XF p, XGF p, (F ¬p), XGF ¬p | … | …
XF p, XGF p, ¬p, XGF ¬p | {¬p} ; . | …
Sub₃
(X)
F p, (GF p), GF ¬p | … | …
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(F p), XGF p, F ¬p, XGF ¬p | … | …
p, XGF p, (F ¬p), XGF ¬p | {p} ; . | …
Sub₄
p, ¬p, …
p, XGF p, XF ¬p, XGF ¬p | … | (0, ∅)
(loop)

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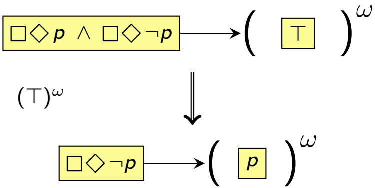

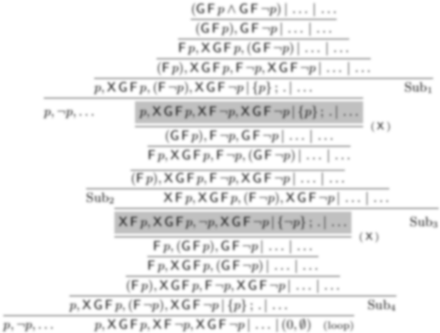
(GF p ∧ GF ¬p) | … | …
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p, XGF p, (F ¬p), XGF ¬p | {p} : . | …
Sub₁
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(X)
(GF p), F ¬p, GF ¬p | … | …
F p, XGF p, F ¬p, (GF ¬p) | … | …
(F p), XGF p, F ¬p, XGF ¬p | … | …
Sub₂
XF p, XGF p, (F ¬p), XGF ¬p | … | …
XF p, XGF p, ¬p, XGF ¬p | {¬p} : . | …
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(X)
F p, (GF p), GF ¬p | … | …
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p, XGF p, (F ¬p), XGF ¬p | {p} : . | …
Sub₄
p, ¬p, …
p, XGF p, XF ¬p, XGF ¬p | … | (0, ∅)
(loop)

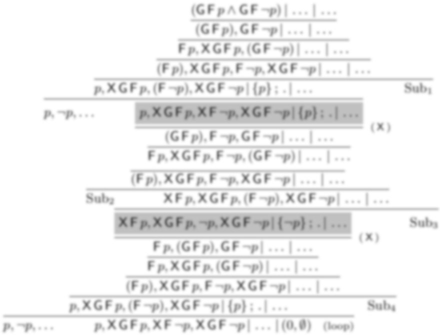
[Schwendimann, 1998,
A New One-Pass Tableau Calculus for PLTL]
Tools: **LWB**, **pltl**, **LTL Tableau**,. . .

LTL satisfiability: $\Box\Diamond p \land \Box\Diamond\neg p$

$$\begin{tikzcd}
\boxed{\Box\Diamond p \land \Box\Diamond\neg p} \arrow[r] \arrow[d, Rightarrow] & \left(\boxed{\top} \right)^{\omega} \\
\boxed{\Box\Diamond\neg p} \arrow[r] & \left(\boxed{p} \right)^{\omega}
\end{tikzcd}$$

$(\top)^{\omega}$

$(p)^{\omega}$


(GF p ∧ GF ¬p) | … | …
(GF p), GF ¬p | … | …
F p, XGF p, (GF ¬p) | … | …
(F p), XGF p, F ¬p, XGF ¬p | … | …
p, XGF p, (F ¬p), XGF ¬p | {p} : . | …
Sub₁
p, ¬p, …
p, XGF p, XF ¬p, XGF ¬p | {p} : . | …
(X)
(GF p), F ¬p, GF ¬p | … | …
F p, XGF p, F ¬p, (GF ¬p) | … | …
(F p), XGF p, F ¬p, XGF ¬p | … | …
Sub₂
XF p, XGF p, (F ¬p), XGF ¬p | … | …
XF p, XGF p, ¬p, XGF ¬p | {¬p} : . | …
Sub₃
(X)
F p, (GF p), GF ¬p | … | …
F p, XGF p, (GF ¬p) | … | …
(F p), XGF p, F ¬p, XGF ¬p | … | …
p, XGF p, (F ¬p), XGF ¬p | {p} : . | …
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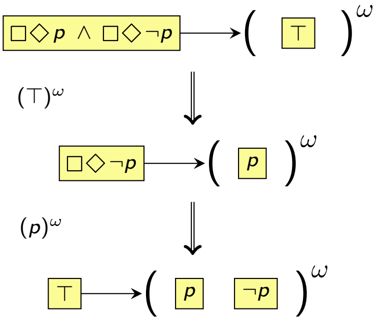

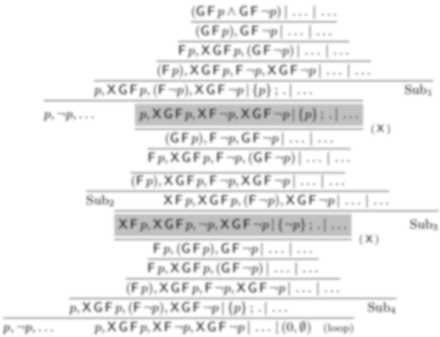
(G F p ∧ G F ¬p) | … | …
(G F p), G F ¬p | … | …
F p, X G F p, (G F ¬p) | … | …
(F p), X G F p, F ¬p, X G F ¬p | … | …
p, X G F p, (F ¬p), X G F ¬p | {p} : . | …
Sub₁
p, ¬p, …
p, X G F p, X F ¬p, X G F ¬p | {p} : . | …
(X)
(G F p), F ¬p, G F ¬p | … | …
F p, X G F p, F ¬p, (G F ¬p) | … | …
(F p), X G F p, F ¬p, X G F ¬p | … | …
Sub₂
X F p, X G F p, (F ¬p), X G F ¬p | … | …
X F p, X G F p, ¬p, X G F ¬p | {¬p} : . | …
Sub₃
(X)
F p, (G F p), G F ¬p | … | …
F p, X G F p, (G F ¬p) | … | …
(F p), X G F p, F ¬p, X G F ¬p | … | …
p, X G F p, (F ¬p), X G F ¬p | {p} : . | …
Sub₄
p, ¬p, …
p, X G F p, X F ¬p, X G F ¬p | … | (0, ∅)
(loop)

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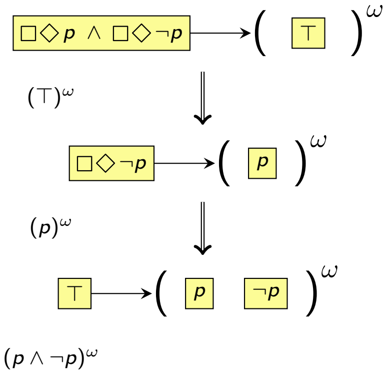


$\boxed{\Box\Diamond p \ \wedge\ \Box\Diamond\neg p} \longrightarrow \left(\ \boxed{\top}\ \right)^{\omega}$

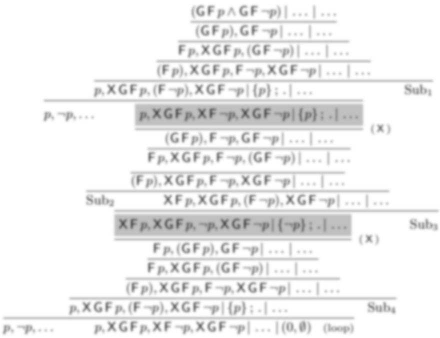
$(\top)^{\omega}$

$\boxed{\Box\Diamond\neg p} \longrightarrow \left(\ \boxed{p}\ \right)^{\omega}$

$(p)^{\omega}$

$\boxed{\top} \longrightarrow \left(\ \boxed{p}\quad \boxed{\neg p}\ \right)^{\omega}$

$(p \wedge \neg p)^{\omega}$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F p, XGF p, (GF ¬p) | … | …
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p, XGF p, (F ¬p), XGF ¬p | {p} : . | …
Sub₁
p, ¬p, …
p, XGF p, XF ¬p, XGF ¬p | {p} : . | …
(✗)
(GF p), F ¬p, GF ¬p | … | …
F p, XGF p, F ¬p, (GF ¬p) | … | …
(F p), XGF p, F ¬p, XGF ¬p | … | …
Sub₂
XF p, XGF p, (F ¬p), XGF ¬p | … | …
XF p, XGF p, ¬p, XGF ¬p | {¬p} : . | …
Sub₃
(✗)
F p, (GF p), GF ¬p | … | …
F p, XGF p, (GF ¬p) | … | …
(F p), XGF p, F ¬p, XGF ¬p | … | …
p, XGF p, (F ¬p), XGF ¬p | {p} : . | …
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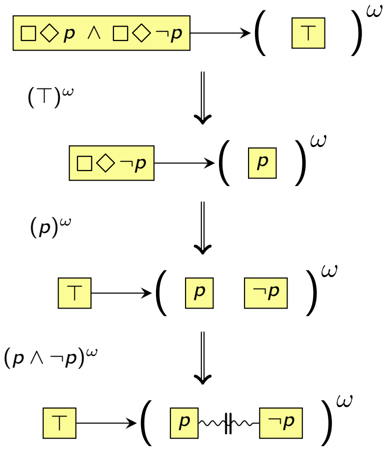
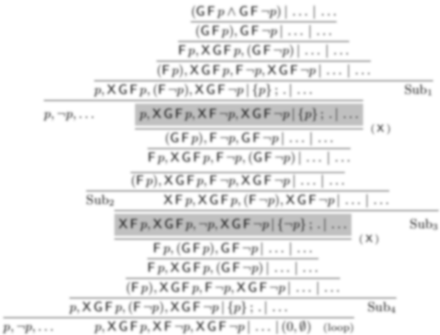
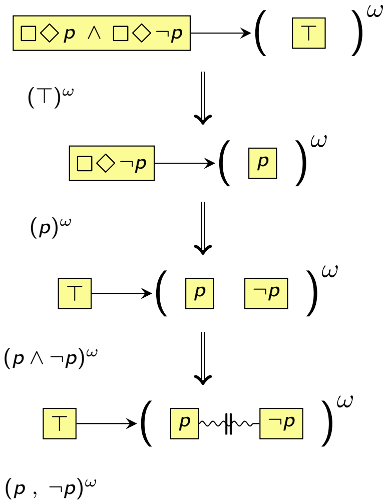


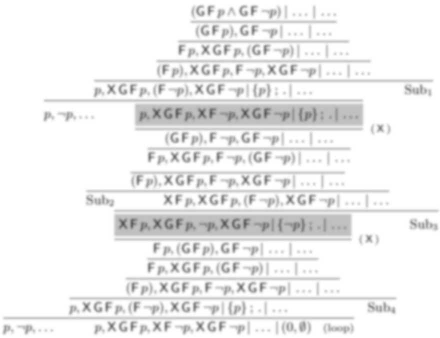

(GF p ∧ GF ¬p) | … | …
(GF p), GF ¬p | … | …
F p, XGF p, (GF ¬p) | … | …
(F p), XGF p, F ¬p, XGF ¬p | … | …
p, XGF p, (F ¬p), XGF ¬p | {p} : . | …
Sub₁
p, ¬p, …
p, XGF p, XF ¬p, XGF ¬p | {p} : . | …
(X)
(GF p), F ¬p, GF ¬p | … | …
F p, XGF p, F ¬p, (GF ¬p) | … | …
(F p), XGF p, F ¬p, XGF ¬p | … | …
Sub₂
XF p, XGF p, (F ¬p), XGF ¬p | … | …
XF p, XGF p, ¬p, XGF ¬p | {¬p} : . | …
Sub₃
(X)
F p, (GF p), GF ¬p | … | …
F p, XGF p, (GF ¬p) | … | …
(F p), XGF p, F ¬p, XGF ¬p | … | …
p, XGF p, (F ¬p), XGF ¬p | {p} : . | …
Sub₄
p, ¬p, …
p, XGF p, XF ¬p, XGF ¬p | … | (0, ∅)
(loop)

[Schwendimann, 1998,
A New One-Pass Tableau Calculus for PLTL]

Tools: **LWB**, **pltl**, **LTL Tableau**,. . .

LTL satisfiability: $\Box\Diamond p \wedge \Box\Diamond\neg p$


$\Box\Diamond p \wedge \Box\Diamond\neg p$
$(\top)^\omega$
$(\top)^\omega$
$\Box\Diamond\neg p$
$(p)^\omega$
$(p)^\omega$
$\top$
$(p \quad \neg p)^\omega$
$(p \wedge \neg p)^\omega$
$\top$
$(p \quad \neg p)^\omega$
$(p\ ,\ \neg p)^\omega$



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Tools: **LWB**, **pltl**, **LTL Tableau**, . . .

LTL satisfiability: $\Diamond p \wedge \Box(p \implies \bigcirc p) \implies \Diamond\Box p$

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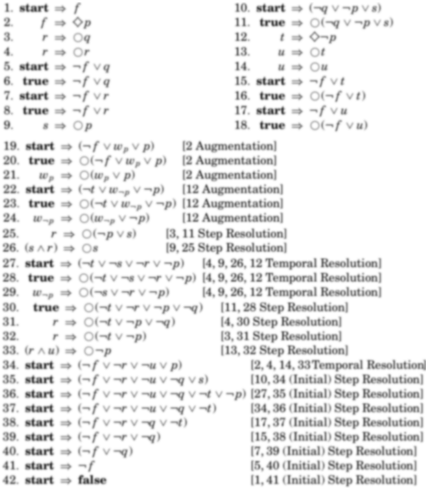

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[Fischer, Dixon, Peim, 2001,
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Tools: **TSPASS**, **TRP++**, **TeMP**,. . .

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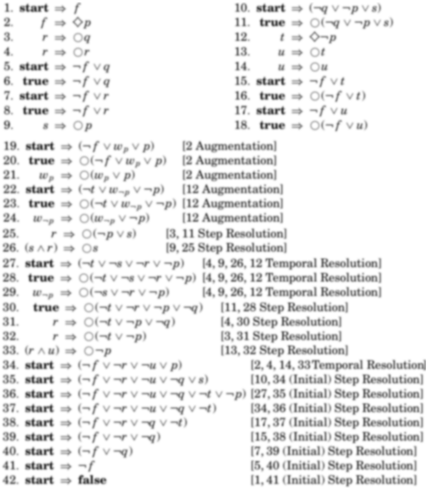

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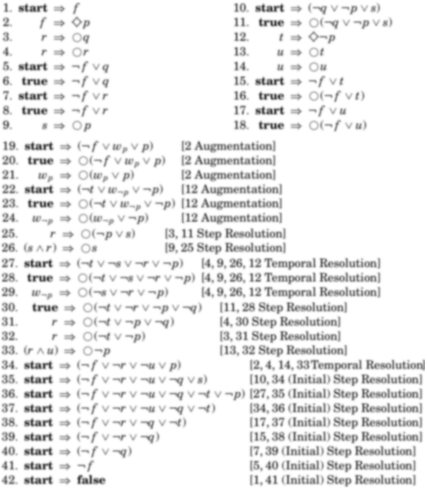

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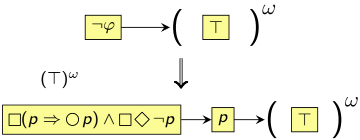

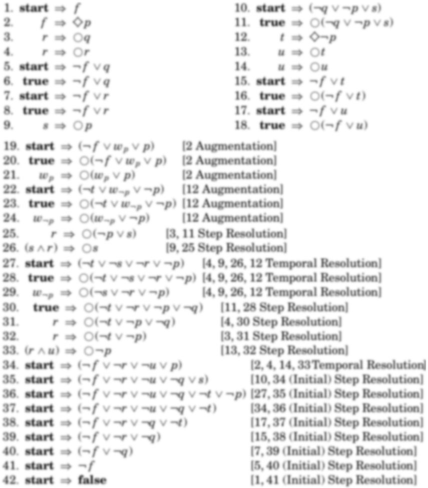

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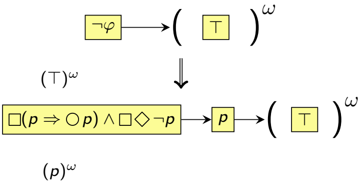

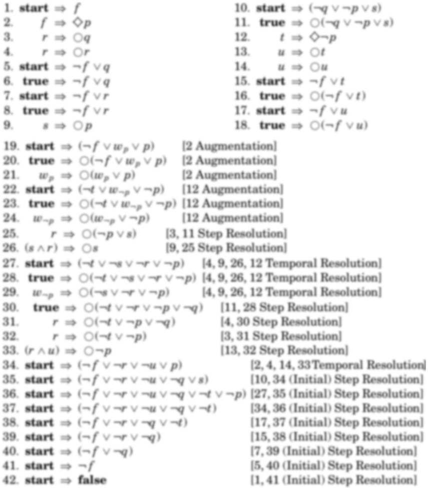

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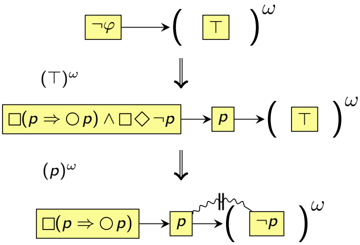

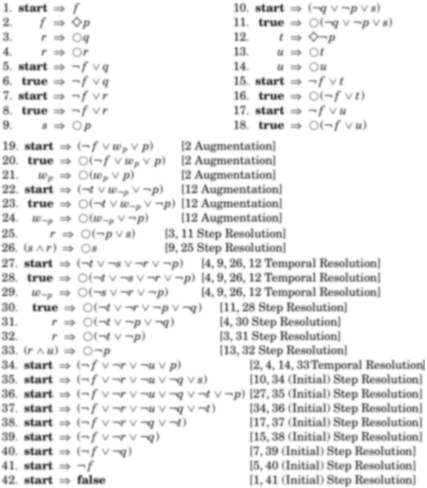

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[Fischer, Dixon, Peim, 2001,
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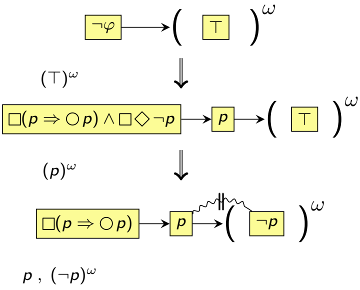

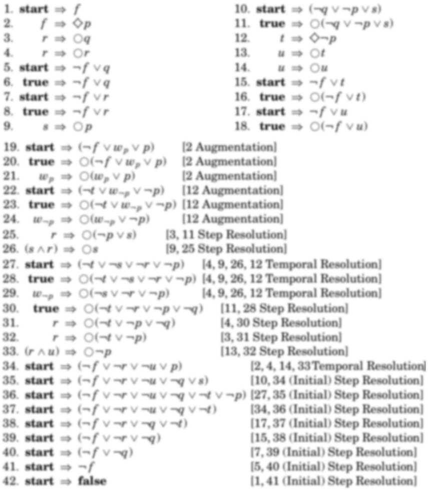

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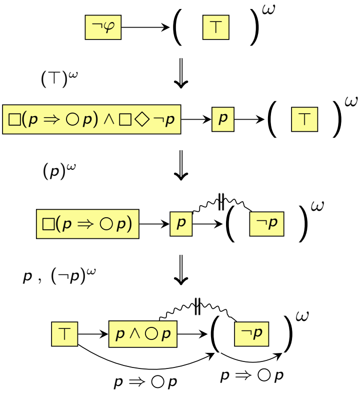

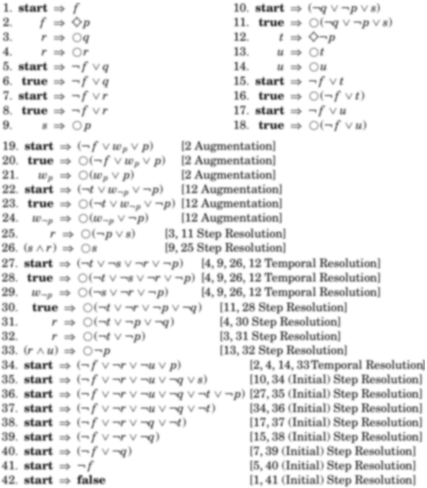

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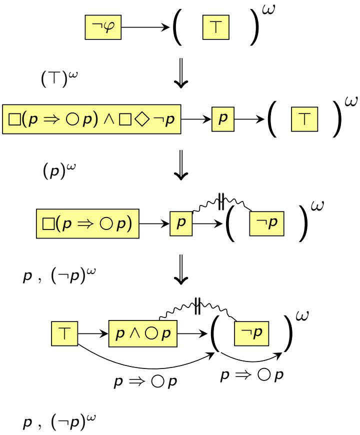

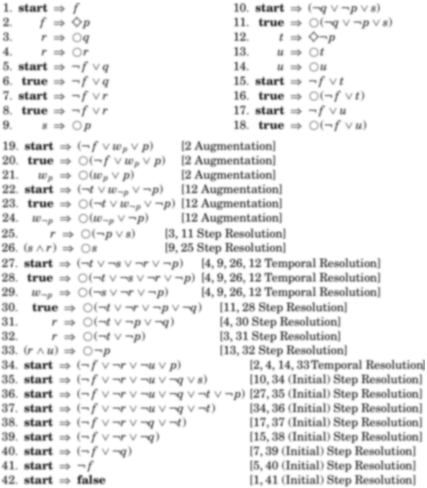

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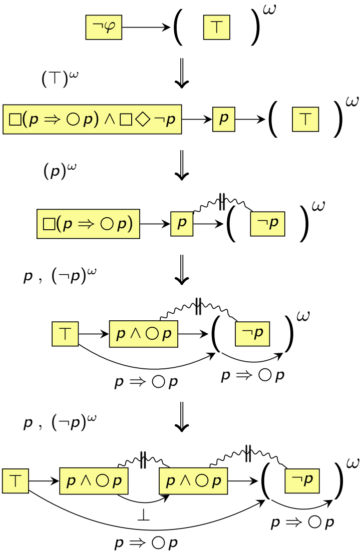

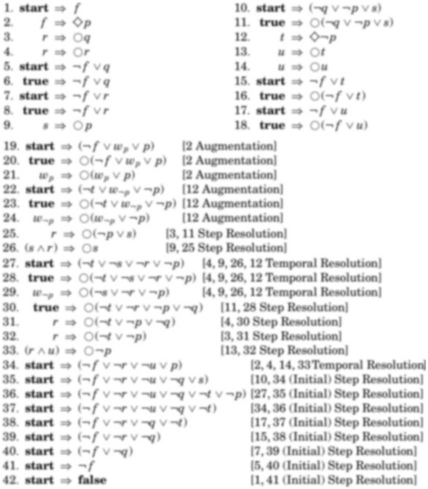

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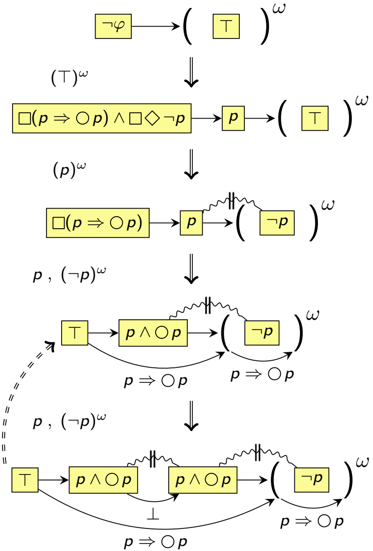

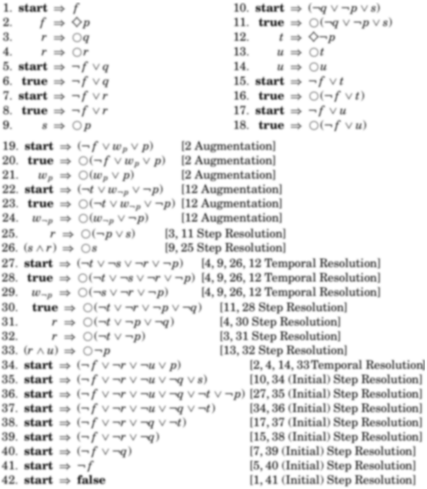

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33. $(r \wedge u) \Rightarrow \bigcirc\neg p$ [13, 32 Step Resolution]
34. $\textbf{start} \Rightarrow (\neg f \vee \neg r \vee \neg u \vee p)$ [2, 4, 14, 33Temporal Resolution]
35. $\textbf{start} \Rightarrow (\neg f \vee \neg r \vee \neg u \vee \neg q \vee s)$ [10, 34 (Initial) Step Resolution]
36. $\textbf{start} \Rightarrow (\neg f \vee \neg r \vee \neg u \vee \neg q \vee \neg t \vee \neg p)$ [27, 35 (Initial) Step Resolution]
37. $\textbf{start} \Rightarrow (\neg f \vee \neg r \vee \neg u \vee \neg q \vee \neg t)$ [34, 36 (Initial) Step Resolution]
38. $\textbf{start} \Rightarrow (\neg f \vee \neg r \vee \neg q \vee \neg t)$ [17, 37 (Initial) Step Resolution]
39. $\textbf{start} \Rightarrow (\neg f \vee \neg r \vee \neg q)$ [15, 38 (Initial) Step Resolution]
40. $\textbf{start} \Rightarrow (\neg f \vee \neg q)$ [7, 39 (Initial) Step Resolution]
41. $\textbf{start} \Rightarrow \neg f$ [5, 40 (Initial) Step Resolution]
42. $\textbf{start} \Rightarrow \textbf{false}$ [1, 41 (Initial) Step Resolution]

[Fischer, Dixon, Peim, 2001,
Clausal Temporal Resolution]
Tools: **TSPASS**, **TRP++**, **TeMP**,. . .

LTL model checking

Thread 1

```
while (true) {
  $l_1$:  noncritical;
  $l_2$:  request r;
  $l_3$:  critical;
  $l_4$:  release r;
}
```

Thread 2

```
while (true) {
  $m_1$:  noncritical;
  $m_2$:  request r;
  $m_3$:  critical;
  $m_4$:  release r;
}
```

Thread 3

```
while (true) {
  $n_1$:  noncritical;
  $n_2$:  request r;
  $n_3$:  critical;
  $n_4$:  release r;
}
```

LTL Properties

Fair scheduling: $\varphi_F \equiv \Box\Diamond(at_2 \wedge r_{free}) \implies \Box\Diamond at_3$

Termination of critical sections: $\varphi_T \equiv \Box(at_3 \implies \Diamond at_1)$

Individual **A**ccessibility: $\varphi_A \equiv \Box(at_2 \implies \Diamond at_3)$

$$\varphi \equiv \bigwedge_{i\in 1..n}(\varphi_{F_i} \wedge \varphi_{T_i}) \implies \varphi_{A_1}$$

Translation of $\neg\varphi$ into a Büchi automaton: **ltl3ba**

Threads	Time (sec)	Memory (MB)	|Automaton| (MB)
2	0.005	4.2	0.002
3	0.09	5.0	0.38
4	9.6	14.7	8.6
5	1295	139	185
6	TO	X	X

LTL model checking

Thread 1

```
while (true) {
  l1:  noncritical;
  l2:  request r;
  l3:  critical;
  l4:  release r;
}
```

Thread 2

```
while (true) {
  m1:  noncritical;
  m2:  request r;
  m3:  critical;
  m4:  release r;
}
```

Thread 3

```
while (true) {
  n1:  noncritical;
  n2:  request r;
  n3:  critical;
  n4:  release r;
}
```

$$
\begin{array}{lllll}
\Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond at_{l_1}) & & \\
\Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond at_{n_1}) & &
\end{array}
$$

LTL model checking

Thread 1

```
while (true) {
  l1:  noncritical;
  l2:  request r;
  l3:  critical;
  l4:  release r;
}
```

Thread 2

```
while (true) {
  m1:  noncritical;
  m2:  request r;
  m3:  critical;
  m4:  release r;
}
```

Thread 3

```
while (true) {
  n1:  noncritical;
  n2:  request r;
  n3:  critical;
  n4:  release r;
}
```

$$\begin{array}{llllll}
\Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond\, at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond\, at_{l_1}) & & \\
\Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond\, at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond\, at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond\, at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond\, at_{n_1}) & &
\end{array}$$

1. Thread 1 stays at l_1

LTL model checking

Thread 1

```
while (true) {
  $l_1$:  noncritical;
  $l_2$:  request r;
  $l_3$:  critical;
  $l_4$:  release r;
}
```

Thread 2

```
while (true) {
  $m_1$:  noncritical;
  $m_2$:  request r;
  $m_3$:  critical;
  $m_4$:  release r;
}
```

Thread 3

```
while (true) {
  $n_1$:  noncritical;
  $n_2$:  request r;
  $n_3$:  critical;
  $n_4$:  release r;
}
```

$$\begin{array}{lllll}
\Box\Diamond(at_{l_2} \land r_{free}) \implies \Box\Diamond at_{l_3} & \land & \Box(at_{l_3} \implies \Diamond at_{l_1}) & & \\
\Box\Diamond(at_{m_2} \land r_{free}) \implies \Box\Diamond at_{m_3} & \land & \Box(at_{m_3} \implies \Diamond at_{m_1}) & \land & \Diamond(at_{l_2} \land \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \land r_{free}) \implies \Box\Diamond at_{n_3} & \land & \Box(at_{n_3} \implies \Diamond at_{n_1}) & &
\end{array}$$

1. Thread 1 stays at l_1

LTL model checking

Thread 1

```
while (true) {
  l1:  noncritical;
  l2:  request r;
  l3:  critical;
  l4:  release r;
}
```

Thread 2

```
while (true) {
  m1:  noncritical;
  m2:  request r;
  m3:  critical;
  m4:  release r;
}
```

Thread 3

```
while (true) {
  n1:  noncritical;
  n2:  request r;
  n3:  critical;
  n4:  release r;
}
```

$$\begin{array}{lllll}
\Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond\, at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond\, at_{l_1}) & & \\
\Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond\, at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond\, at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond\, at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond\, at_{n_1}) & &
\end{array}$$

1. Thread 1 stays at l_1
2. Thread 1 moves to l_2 and stays there

LTL model checking

Thread 1

```
while (true) {
  $l_1$:  noncritical;
  $l_2$:  request r;
  $l_3$:  critical;
  $l_4$:  release r;
}
```

Thread 2

```
while (true) {
  $m_1$:  noncritical;
  $m_2$:  request r;
  $m_3$:  critical;
  $m_4$:  release r;
}
```

Thread 3

```
while (true) {
  $n_1$:  noncritical;
  $n_2$:  request r;
  $n_3$:  critical;
  $n_4$:  release r;
}
```

$$\begin{array}{lclclcl}
\Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond at_{l_1}) & & & & \\
\Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond at_{n_1}) & & & &
\end{array}$$

1. Thread 1 stays at l_1
2. Thread 1 moves to l_2 and stays there

LTL model checking

Thread 1

```
while (true) {
  $l_1$:  noncritical;
  $l_2$:  request r;
  $l_3$:  critical;
  $l_4$:  release r;
}
```

Thread 2

```
while (true) {
  $m_1$:  noncritical;
  $m_2$:  request r;
  $m_3$:  critical;
  $m_4$:  release r;
}
```

Thread 3

```
while (true) {
  $n_1$:  noncritical;
  $n_2$:  request r;
  $n_3$:  critical;
  $n_4$:  release r;
}
```

$$
\begin{array}{lllll}
\Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond at_{l_1}) & & \\
\Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond at_{n_1}) & &
\end{array}
$$

1. Thread 1 stays at l_1
2. Thread 1 moves to l_2 and stays there
3. Someone should request and hold the resource. Who?

LTL model checking

Thread 1

```
while (true) {
  $l_1$:  noncritical;
  $l_2$:  request r;
  $l_3$:  critical;
  $l_4$:  release r;
}
```

Thread 2

```
while (true) {
  $m_1$:  noncritical;
  $m_2$:  request r;
  $m_3$:  critical;
  $m_4$:  release r;
}
```

Thread 3

```
while (true) {
  $n_1$:  noncritical;
  $n_2$:  request r;
  $n_3$:  critical;
  $n_4$:  release r;
}
```

$$
\begin{array}{lclcl}
\Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond\, at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond\, at_{l_1}) & & \\
\Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond\, at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond\, at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond\, at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond\, at_{n_1}) & &
\end{array}
$$

1. Thread 1 stays at l_1
2. Thread 1 moves to l_2 and stays there
3. Someone should request and hold the resource. Who?

LTL model checking

Thread 1

```
while (true) {
  $l_1$:  noncritical;
  $l_2$:  request r;
  $l_3$:  critical;
  $l_4$:  release r;
}
```

Thread 2

```
while (true) {
  $m_1$:  noncritical;
  $m_2$:  request r;
  $m_3$:  critical;
  $m_4$:  release r;
}
```

Thread 3

```
while (true) {
  $n_1$:  noncritical;
  $n_2$:  request r;
  $n_3$:  critical;
  $n_4$:  release r;
}
```

$$\begin{array}{lllll}
\Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond\, at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond\, at_{l_1}) & & \\
\Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond\, at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond\, at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond\, at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond\, at_{n_1}) & &
\end{array}$$

1. Thread 1 stays at l_1
2. Thread 1 moves to l_2 and stays there
3. Someone should request and hold the resource. Who?
    - Suppose, it's thread 2

LTL model checking

Thread 1

```
while (true) {
  $l_1$:  noncritical;
  $l_2$:  request r;
  $l_3$:  critical;
  $l_4$:  release r;
}
```

Thread 2

```
while (true) {
  $m_1$:  noncritical;
  $m_2$:  request r;
  $m_3$:  critical;
  $m_4$:  release r;
}
```

Thread 3

```
while (true) {
  $n_1$:  noncritical;
  $n_2$:  request r;
  $n_3$:  critical;
  $n_4$:  release r;
}
```

$$\begin{array}{lclclcl}
\Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond at_{l_1}) & & & & \\
\Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond at_{n_1}) & & & &
\end{array}$$

1. Thread 1 stays at l_1
2. Thread 1 moves to l_2 and stays there
3. Someone should request and hold the resource. Who?
   - Suppose, it's thread 2

LTL model checking

Thread 1

```
while (true) {
  l1:  noncritical;
  l2:  request r;
  l3:  critical;
  l4:  release r;
}
```

Thread 2

```
while (true) {
  m1:  noncritical;
  m2:  request r;
  m3:  critical;
  m4:  release r;
}
```

Thread 3

```
while (true) {
  n1:  noncritical;
  n2:  request r;
  n3:  critical;
  n4:  release r;
}
```

$$\begin{array}{lllll}
\Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond\, at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond\, at_{l_1}) & & \\
\Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond\, at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond\, at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond\, at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond\, at_{n_1}) & &
\end{array}$$

❶ Thread 1 stays at l_1

❷ Thread 1 moves to l_2 and stays there

❸ Someone should request and hold the resource. Who?

  - Suppose, it's thread 2
  - Thread 2 should be at m_3

LTL model checking

Thread 1

```
while (true) {
  $l_1$:  noncritical;
  $l_2$:  request r;
  $l_3$:  critical;
  $l_4$:  release r;
}
```

Thread 2

```
while (true) {
  $m_1$:  noncritical;
  $m_2$:  request r;
  $m_3$:  critical;
  $m_4$:  release r;
}
```

Thread 3

```
while (true) {
  $n_1$:  noncritical;
  $n_2$:  request r;
  $n_3$:  critical;
  $n_4$:  release r;
}
```

$$\begin{array}{lllll}
\Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond at_{l_1}) & & \\
\Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond at_{n_1}) & &
\end{array}$$

1. Thread 1 stays at l_1
2. Thread 1 moves to l_2 and stays there
3. Someone should request and hold the resource. Who?
   - Suppose, it's thread 2
   - Thread 2 should be at m_3

LTL model checking

Thread 1

```
while (true) {
  l1:  noncritical;
  l2:  request r;
  l3:  critical;
  l4:  release r;
}
```

Thread 2

```
while (true) {
  m1:  noncritical;
  m2:  request r;
  m3:  critical;
  m4:  release r;
}
```

Thread 3

```
while (true) {
  n1:  noncritical;
  n2:  request r;
  n3:  critical;
  n4:  release r;
}
```

$$
\begin{array}{lllll}
\Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond at_{l_1}) & & \\
\Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond at_{n_1}) & &
\end{array}
$$

1. Thread 1 stays at l_1
2. Thread 1 moves to l_2 and stays there
3. Someone should request and hold the resource. Who?
    - Suppose, it's thread 2
    - Thread 2 should be at m_3
    - Thread 2 should leave the critical section to m_1

LTL model checking

Thread 1

```
while (true) {
  $l_1$:  noncritical;
  $l_2$:  request r;
  $l_3$:  critical;
  $l_4$:  release r;
}
```

Thread 2

```
while (true) {
  $m_1$:  noncritical;
  $m_2$:  request r;
  $m_3$:  critical;
  $m_4$:  release r;
}
```

Thread 3

```
while (true) {
  $n_1$:  noncritical;
  $n_2$:  request r;
  $n_3$:  critical;
  $n_4$:  release r;
}
```

$$\begin{array}{lllll}
\Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond at_{l_1}) & & \\
\Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond at_{n_1}) & &
\end{array}$$

1. Thread 1 stays at l_1
2. Thread 1 moves to l_2 and stays there
3. Someone should request and hold the resource. Who?
   - Suppose, it's thread 2
   - Thread 2 should be at m_3
   - Thread 2 should leave the critical section to m_1

LTL model checking

Thread 1

```
while (true) {
  l1:  noncritical;
  l2:  request r;
  l3:  critical;
  l4:  release r;
}
```

Thread 2

```
while (true) {
  m1:  noncritical;
  m2:  request r;
  m3:  critical;
  m4:  release r;
}
```

Thread 3

```
while (true) {
  n1:  noncritical;
  n2:  request r;
  n3:  critical;
  n4:  release r;
}
```

$$\begin{array}{lclcl} \Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond at_{l_1}) & & \\ \Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\ \Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond at_{n_1}) & & \end{array}$$

❶ Thread 1 stays at l_1

❷ Thread 1 moves to l_2 and stays there

❸ Someone should request and hold the resource. Who?

- Suppose, it's thread 2
- Thread 2 should be at m_3
- Thread 2 should leave the critical section to m_1
- Thread 2 should release the resource

LTL model checking

Thread 1

```
while (true) {
  l1:  noncritical;
  l2:  request r;
  l3:  critical;
  l4:  release r;
}
```

Thread 2

```
while (true) {
  m1:  noncritical;
  m2:  request r;
  m3:  critical;
  m4:  release r;
}
```

Thread 3

```
while (true) {
  n1:  noncritical;
  n2:  request r;
  n3:  critical;
  n4:  release r;
}
```

$$
\begin{array}{lllll}
\Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond at_{l_1}) & & \\
\Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond at_{n_1}) & &
\end{array}
$$

❶ Thread 1 stays at l_1

❷ Thread 1 moves to l_2 and stays there

❸ Someone should request and hold the resource. Who?

- Suppose, it's thread 2
- Thread 2 should be at m_3
- Thread 2 should leave the critical section to m_1
- Thread 2 should release the resource

- Suppose, it's thread 3
- ...

LTL model checking

Thread 1

```
while (true) {
  l1:  noncritical;
  l2:  request r;
  l3:  critical;
  l4:  release r;
}
```

Thread 2

```
while (true) {
  m1:  noncritical;
  m2:  request r;
  m3:  critical;
  m4:  release r;
}
```

Thread 3

```
while (true) {
  n1:  noncritical;
  n2:  request r;
  n3:  critical;
  n4:  release r;
}
```

$$
\begin{array}{lllll}
\Box\Diamond(at_{l_2} \wedge r_{free}) \implies \Box\Diamond at_{l_3} & \wedge & \Box(at_{l_3} \implies \Diamond at_{l_1}) & & \\
\Box\Diamond(at_{m_2} \wedge r_{free}) \implies \Box\Diamond at_{m_3} & \wedge & \Box(at_{m_3} \implies \Diamond at_{m_1}) & \wedge & \Diamond(at_{l_2} \wedge \Box\neg at_{l_3}) \\
\Box\Diamond(at_{n_2} \wedge r_{free}) \implies \Box\Diamond at_{n_3} & \wedge & \Box(at_{n_3} \implies \Diamond at_{n_1}) & &
\end{array}
$$

1. Thread 1 stays at l_1
2. Thread 1 moves to l_2 and stays there
3. Someone should request and hold the resource. Who?
   - Suppose, it's thread 2
   - Thread 2 should be at m_3
   - Thread 2 should leave the critical section to m_1
   - Thread 2 should release the resource

   - Suppose, it's thread 3
   - . . .

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A method that works directly on the LTL formula and provides better scalability

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Thank you for your attention!