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Synthesis of Temporal Causality

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CISPA Helmholtz Center for Information Security

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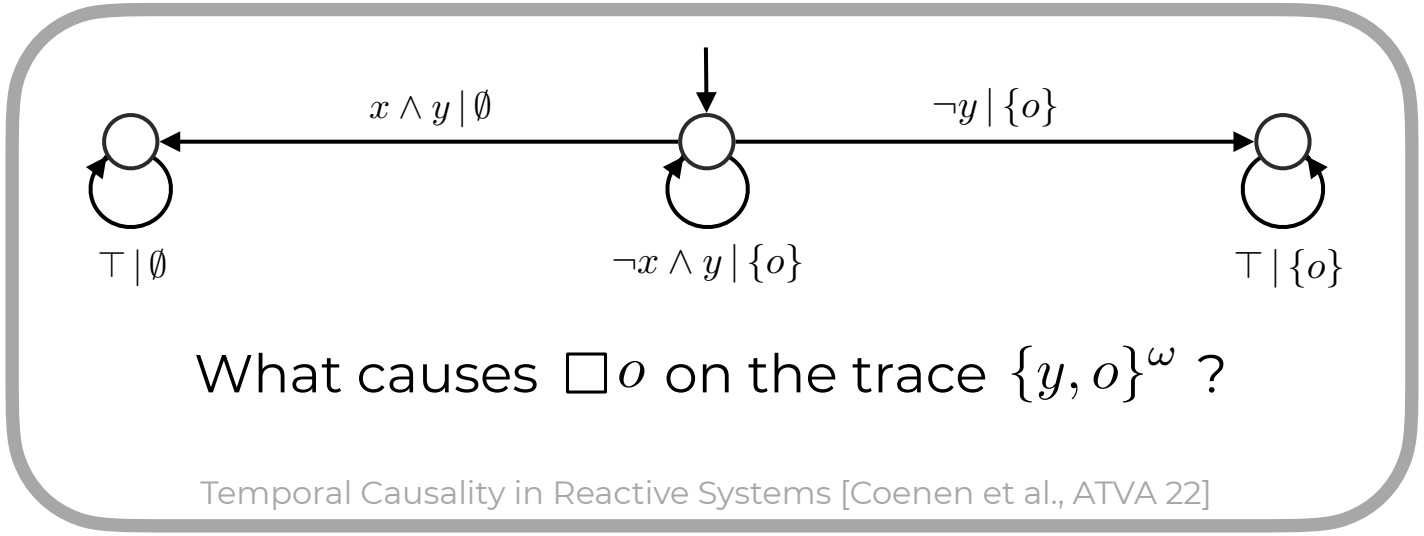
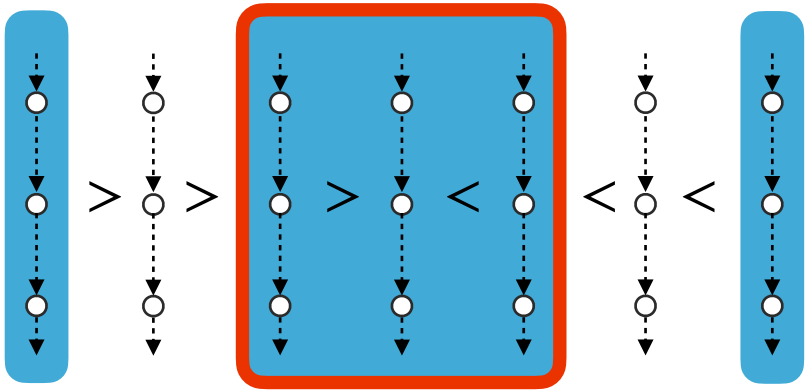
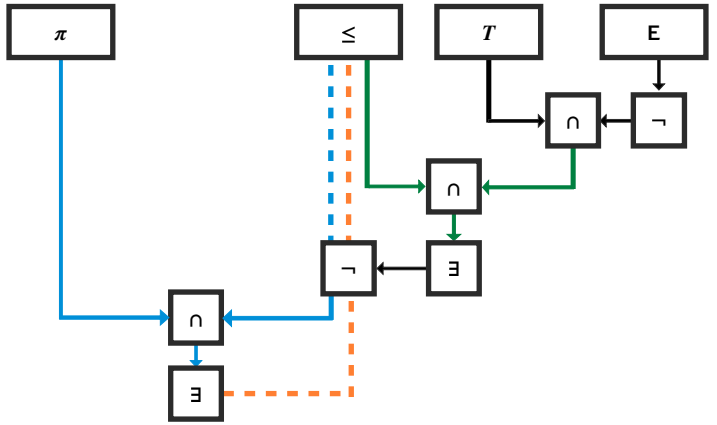
Overview



Temporal Causality in Reactive Systems [Coenen et al., ATVA 22]



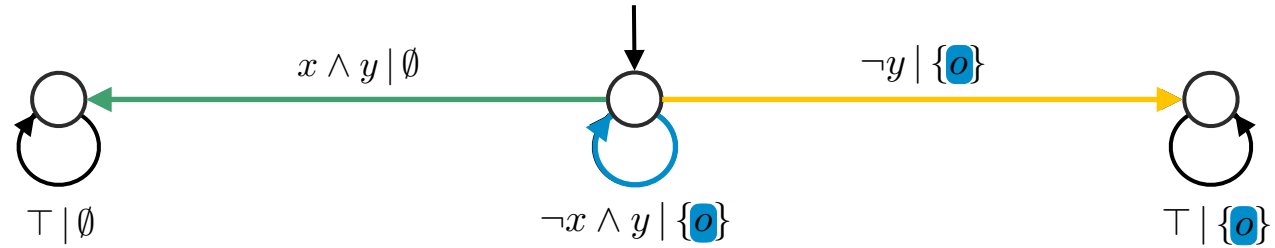
Temporal causes are downward-closed sets



Cause synthesis algorithm for omega-regular effects

Temporal Causality

Causes are temporal properties of the input sequence.



What causes $\square o$ on the trace $\{y, o\}^\omega$?

Temporal Causality in Reactive Systems [Coenen et al., ATVA 22]

SAT: Cause and effect hold on the given trace.

$y \wedge \square \neg x$ ✗ $\quad \square \neg x$ ✓ $\quad \neg y \, \mathcal{R} \, \neg x$ ✗

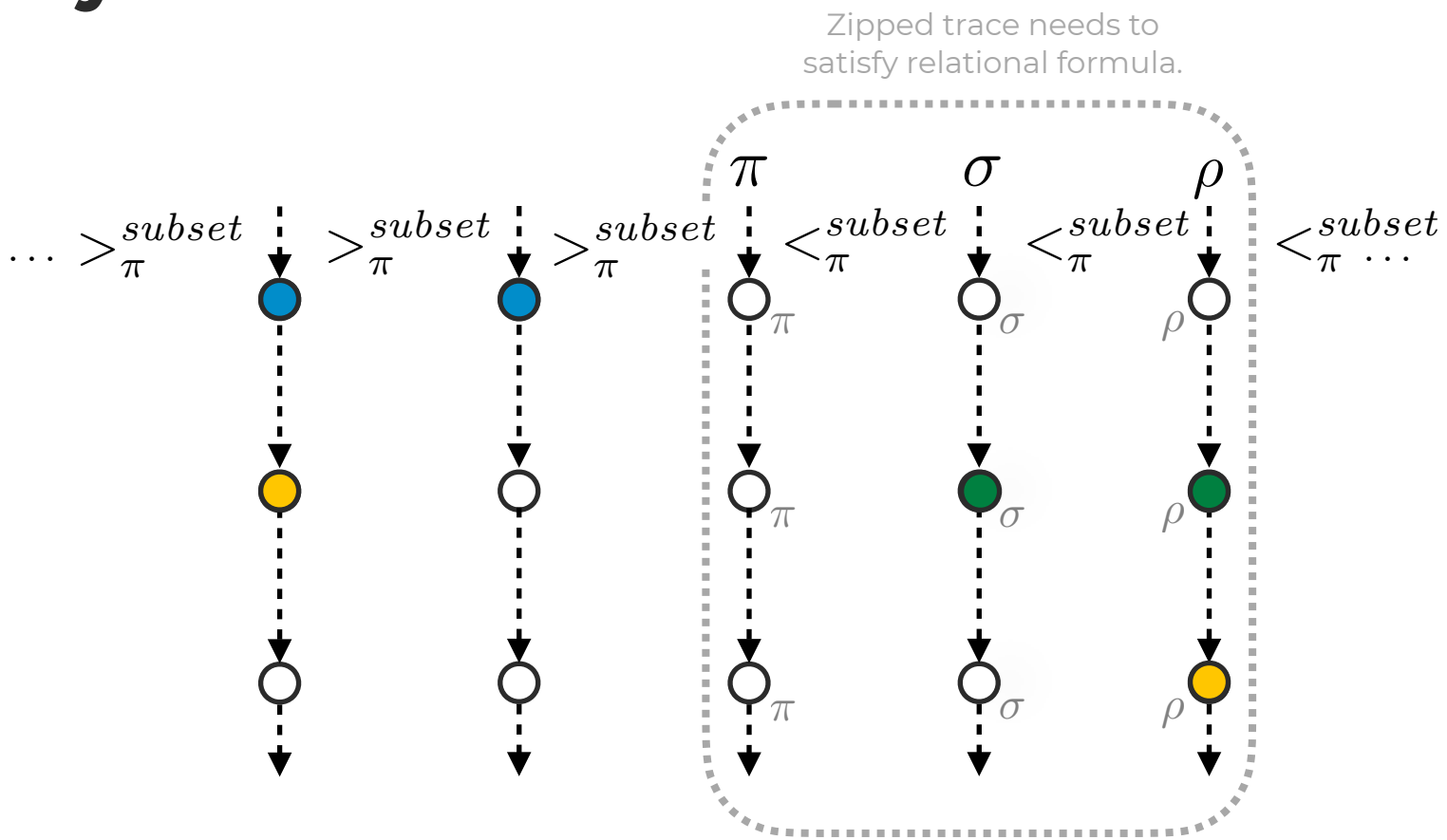
CF: All closest traces that do not satisfy the cause, also do not satisfy the effect.

$\{o\}\{y, o\}^\omega, \ldots \quad \{x, y\}\{y\}^\omega, \ldots$

MIN: No strict subset of the cause satisfies **SAT** and **CF**.

$\mathcal{L}(y \wedge \square \neg x) \subset \mathcal{L}(\square \neg x) \subset \mathcal{L}(\neg y \, \mathcal{R} \, \neg x)$

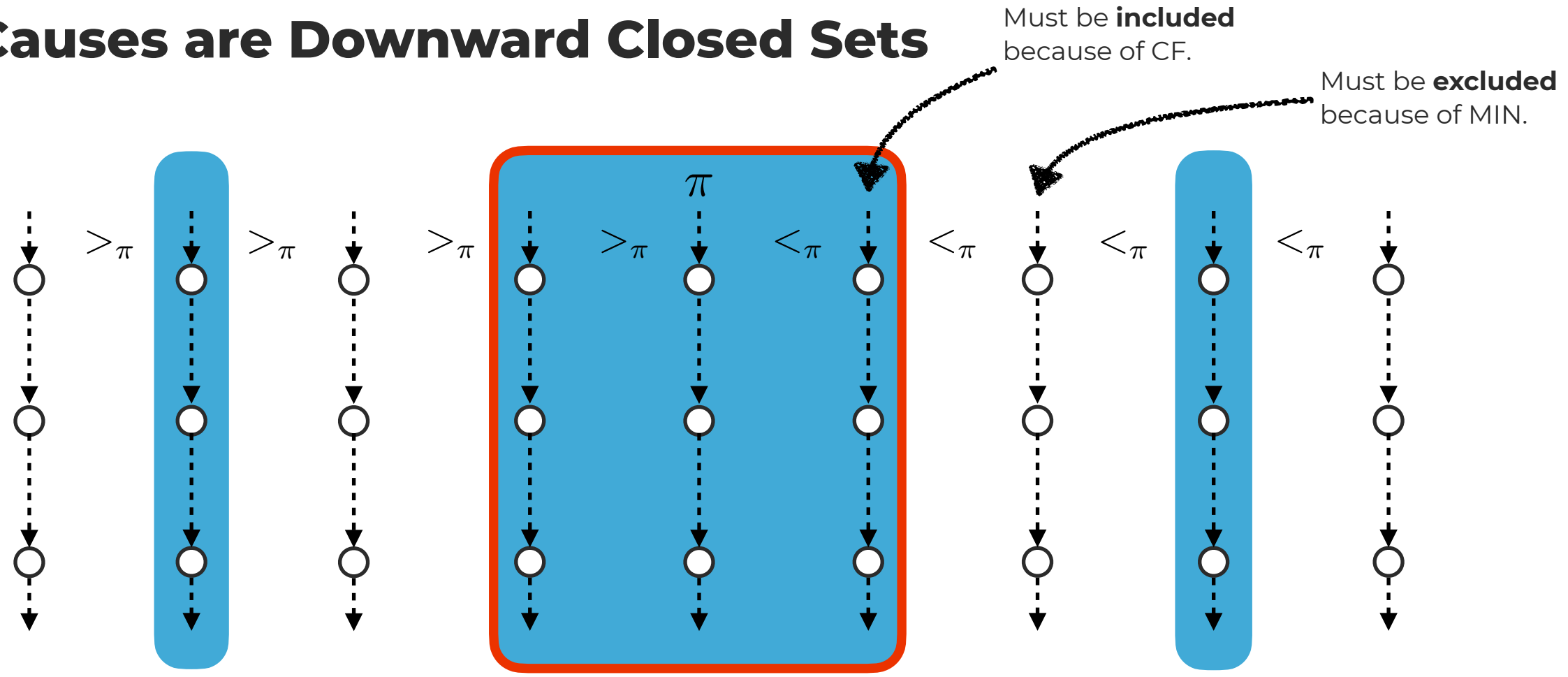
Similarity Relations



The closeness of traces is modeled by a similarity relation.

E.g. $\sigma \leq_\pi^{subset} \rho$ iff $\square \bigwedge_{i \in I} (i_\sigma \not\leftrightarrow i_\pi) \rightarrow (i_\rho \not\leftrightarrow i_\pi)$

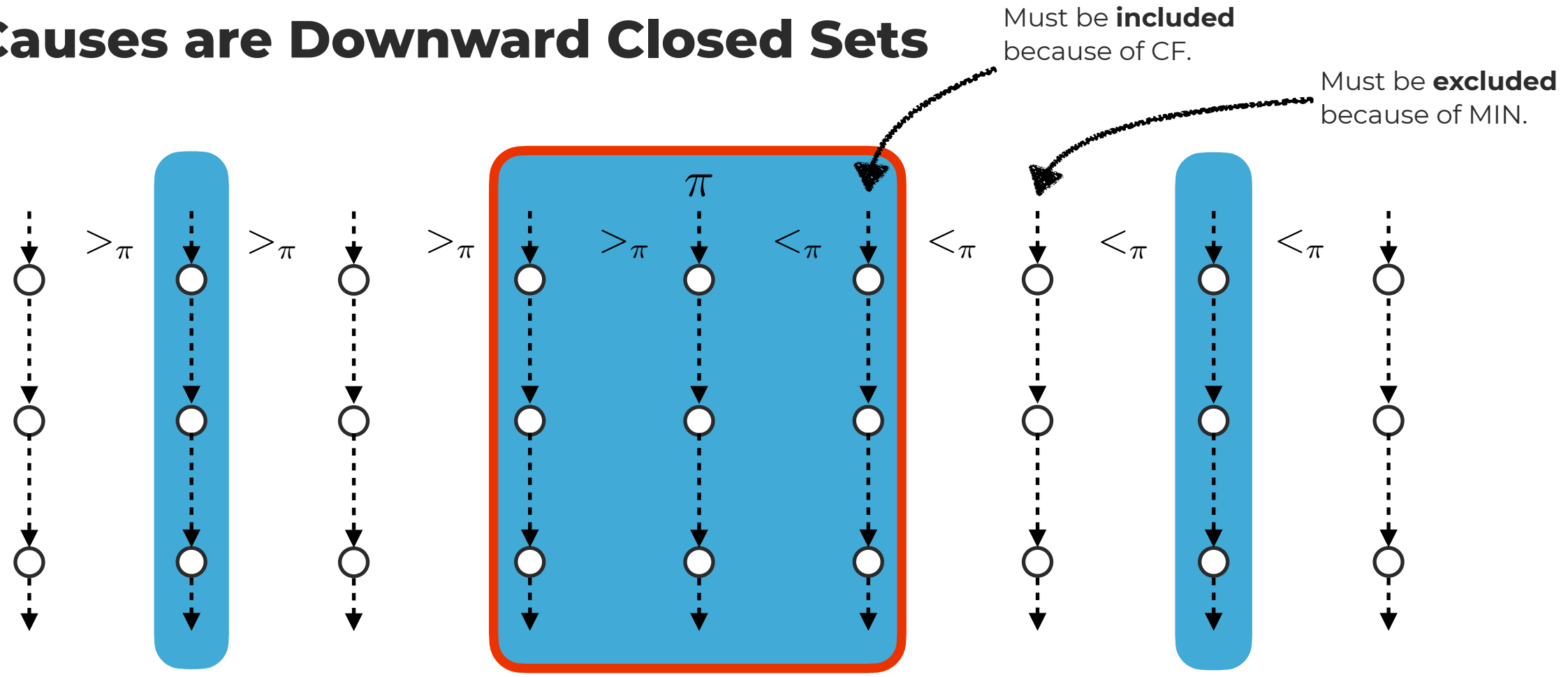
Causes are Downward Closed Sets



$$\text{Cause} = \{\rho \in \text{Effect} \mid \forall \sigma \in T.\, \sigma \leq_\pi \rho \rightarrow \sigma \in \text{Cause}\}$$

= the largest subset of the effect that is downward closed in $(T, \leq_\pi)$

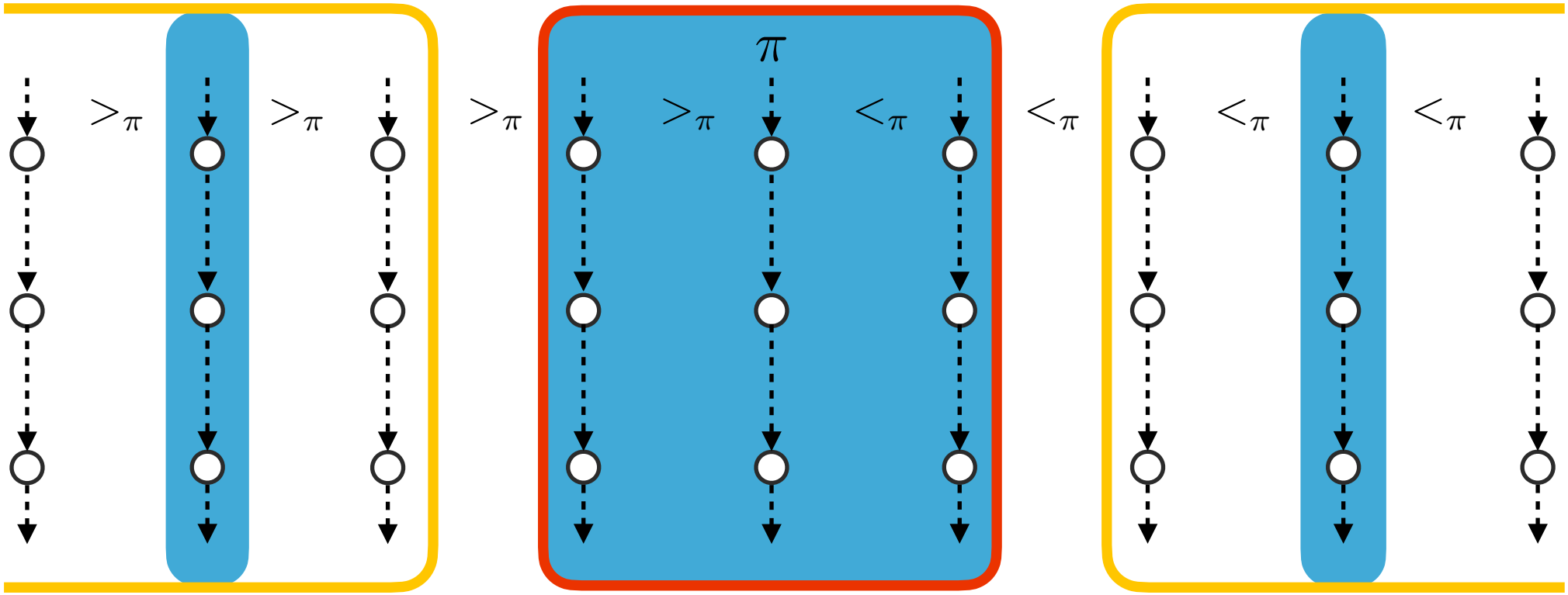
Causes are Downward Closed Sets



Can be rewritten to:

$$\boxed{\text{Cause}} = \{\rho \in T \mid \forall \sigma \in T.\, \sigma \leq_\pi \rho \rightarrow \sigma \in \boxed{\text{Effect}}\}$$

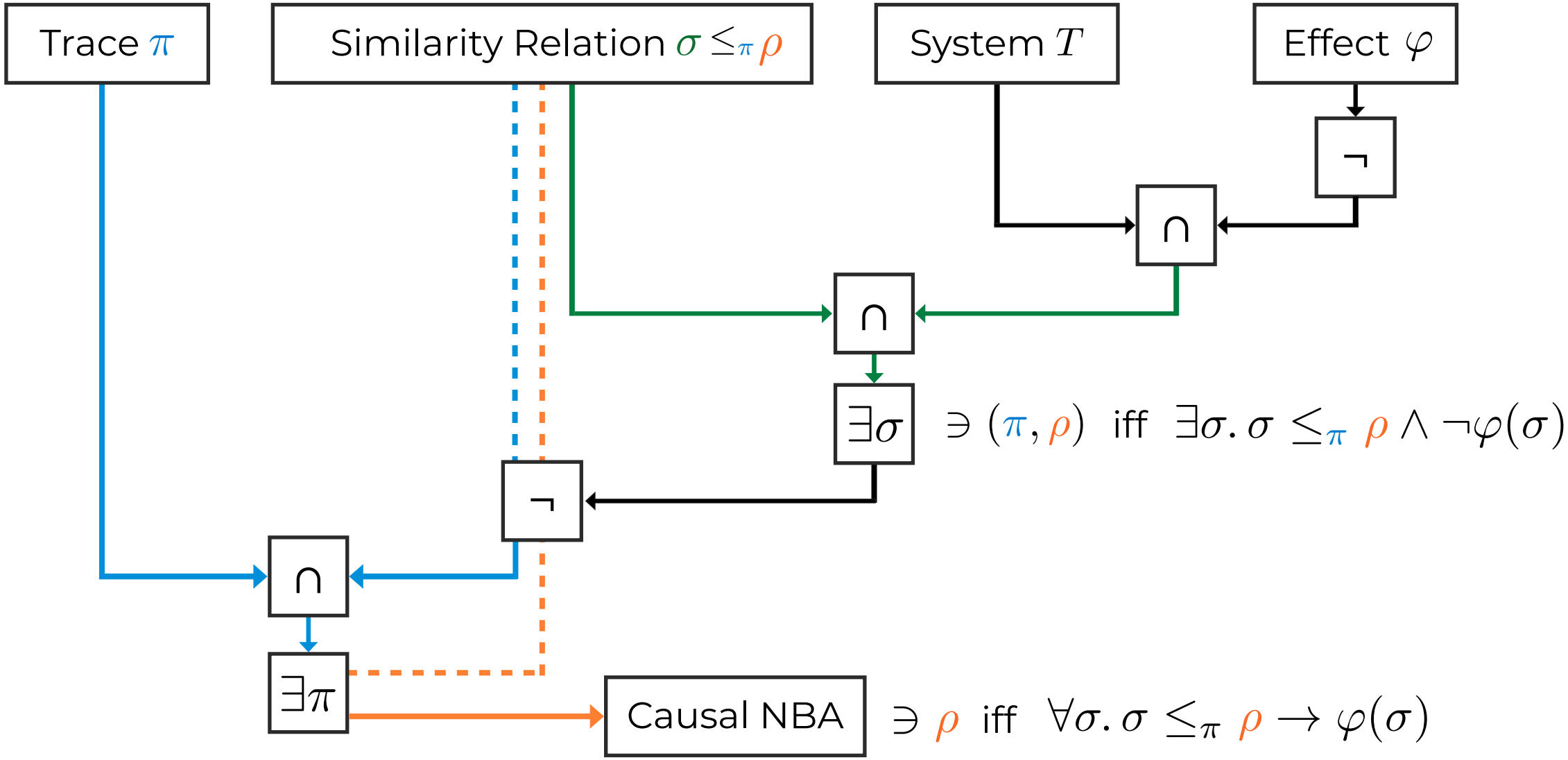
Cause Complements are Upward Closures



$$\overline{\text{Cause}} = \{\rho \in T \mid \exists \sigma \in T.\, \sigma \leq_\pi \rho \wedge \sigma \in \overline{\text{Effect}}\}$$

$$\text{Cause} = \{\rho \in T \mid \forall \sigma \in T.\, \sigma \leq_\pi \rho \rightarrow \sigma \in \text{Effect}\}$$

Automata-based Cause Synthesis



Experiments: Arbiters

CAV Artifact Evaluation ★★★ Reusable

Arbiters grant mutually exclusive access to a resource (e.g., grant g_0).

Instance	$\lvert\mathcal{T}\rvert$	φ_E	$t(\leq^{subset})$	$\lvert\mathcal{A}_C^{subset}\rvert$	φ_C
Full 2	4	$\Diamond g_0$	0.11	2	$\Diamond r_0$
		$\Box\Diamond g_0$	0.11	4	$\Box\Diamond r_0$
Full 3	11	$\Diamond g_0$	0.11	2	$\Diamond r_0$
		$\Box\Diamond g_0$	0.16	24	$\Box\Diamond r_0$
Full 4	46	$\Diamond g_0$	0.11	2	$\Diamond r_0$
		$\Box\Diamond g_0$	33.22	214	$\Box\Diamond r_0$
Spurious 2	2	$\Diamond g_0$	0.11	1	$true$
Spurious 3	3	$\Diamond g_0$	0.11	1	$true$
Spurious 4	4	$\Diamond g_0$	0.11	1	$true$
Unfair 2	2	$\Box\neg g_0$	0.11	1	$\Box r_{prio}$
Unfair 3	4	$\Box\neg g_0$	0.11	1	$\Box r_{prio}$
Unfair 4	6	$\Box\neg g_0$	0.11	1	$\Box r_{prio}$

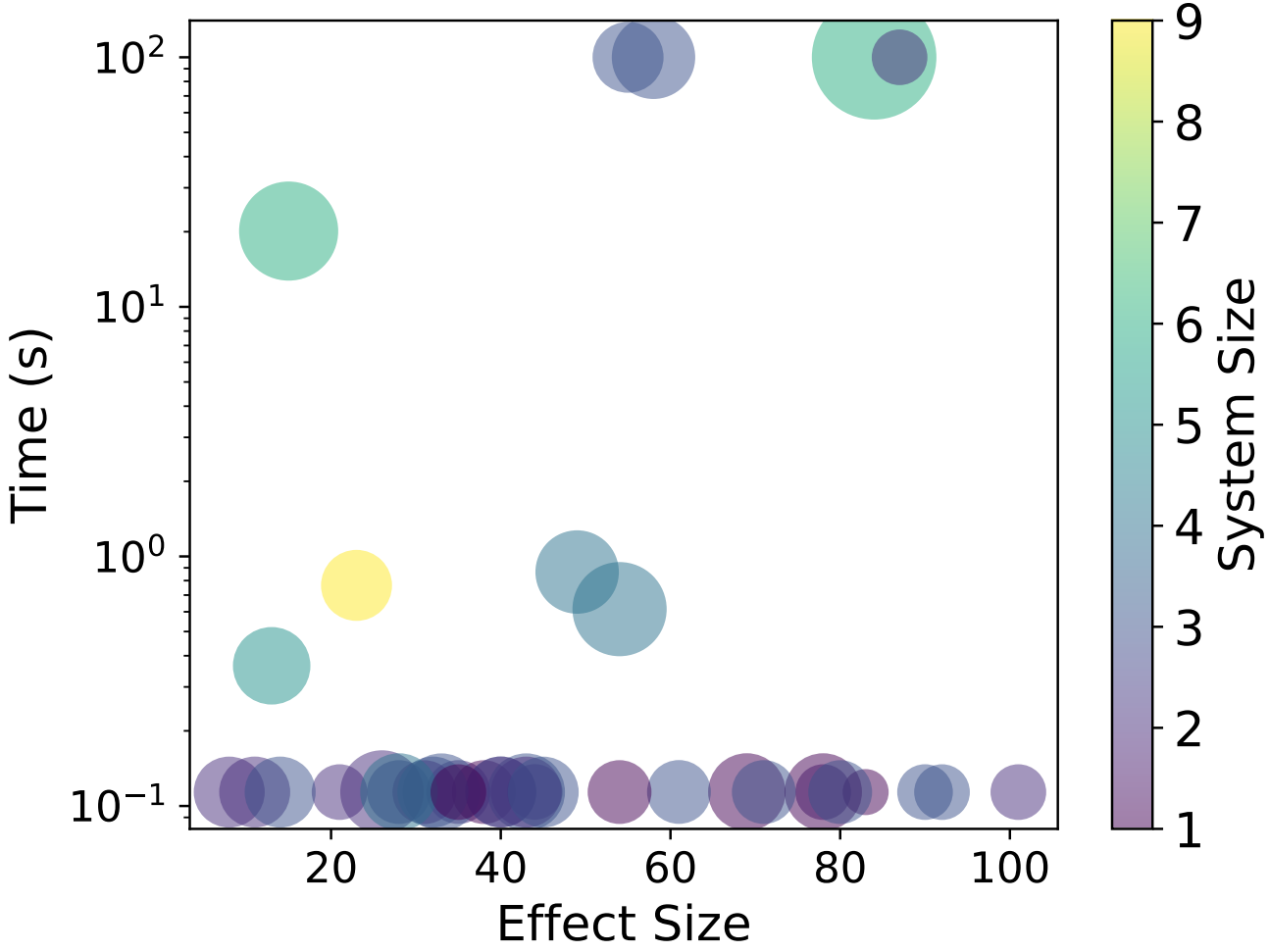
Round-robin strategy

Prioritizes $prio$

We always consider the input sequence where all clients send requests continuously, e.g.:

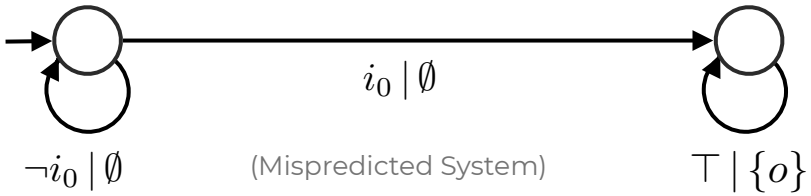
$(\{r_0, r_1, g_0\}\{r_0, r_1, g_1\})^\omega$

Experiments: Neural Synthesis[1]



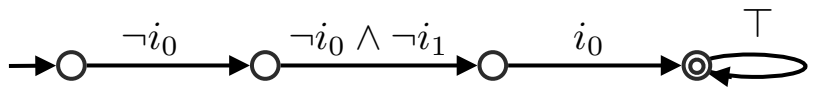
$$\text{Effect} = \neg\big((i_1 \,\mathcal{U}\, i_0) \leftrightarrow (\square\lozenge o)\big)$$

(Negated Specification)



$$\pi = \{i_1\} \cdots \{\} \cdots \{i_0, o\} \cdots (\{i_1, o\})^\omega$$

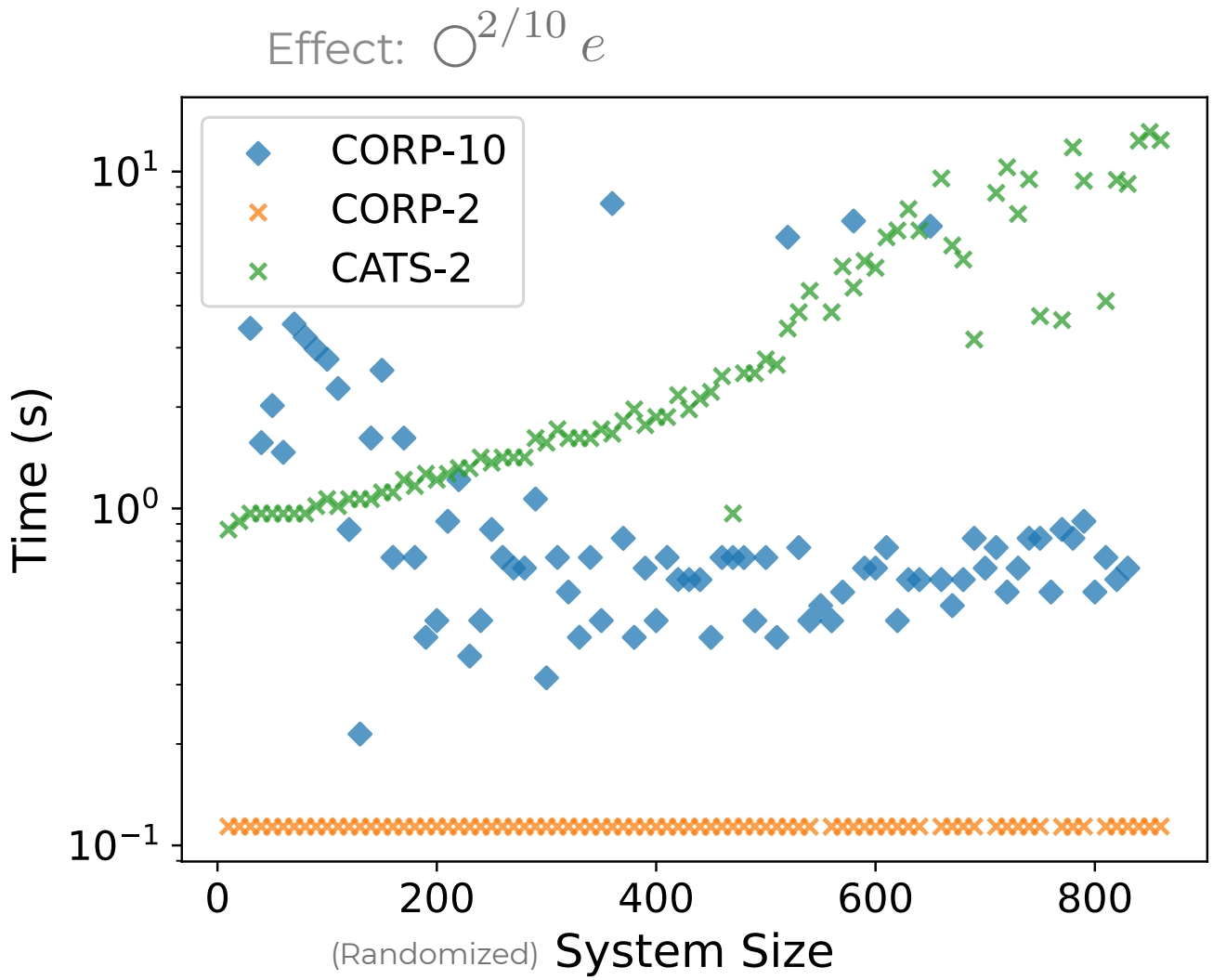
(Counterexample to Specification)



(Causal NBA)

[1] *Neural circuit synthesis from specification patterns*. Schmitt, Hahn, Rabe, and Finkbeiner. NeurIPS 2021.

Experiments: Cause Sketching[1]

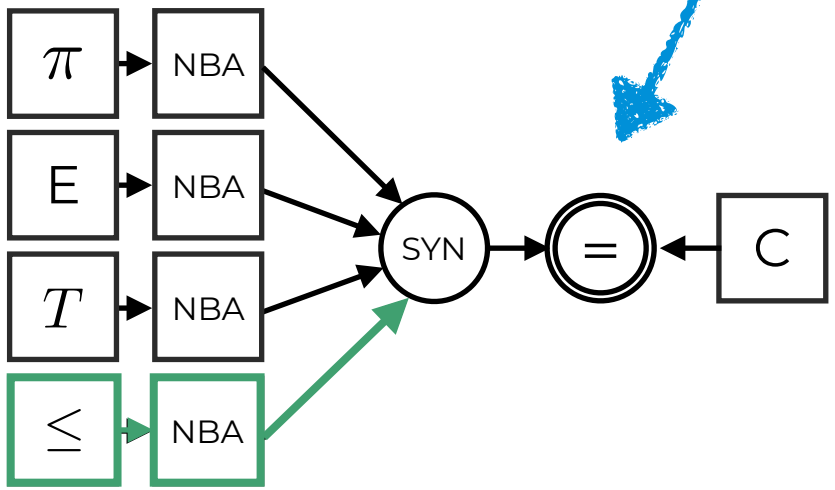


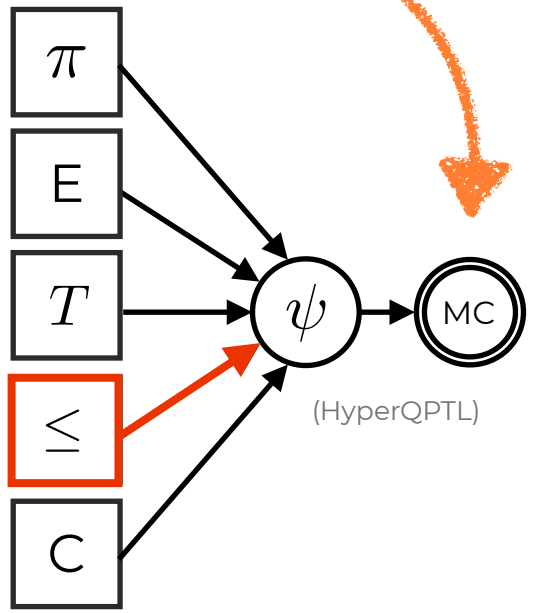
[1] *Checking and Sketching Causes on Temporal Sequences.* Beutner, Finkbeiner, Frenkel, and Siber. ATVA 2023.

Experiments: Cause Checking

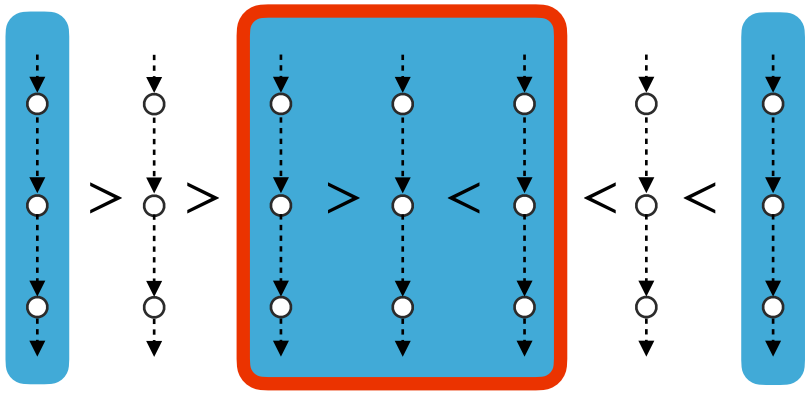
CAV Artifact Evaluation ★ ★ ★ Reusable

Instance	CATS	CORP
Spurious Arbiter	0.72	0.11
Simple Arbiter	0.97	0.11
Arbiter	22.84	0.11
Instance 6, odd	1.81	0.11
Instance 8	0.92	0.11
TP Right	0.87	0.11

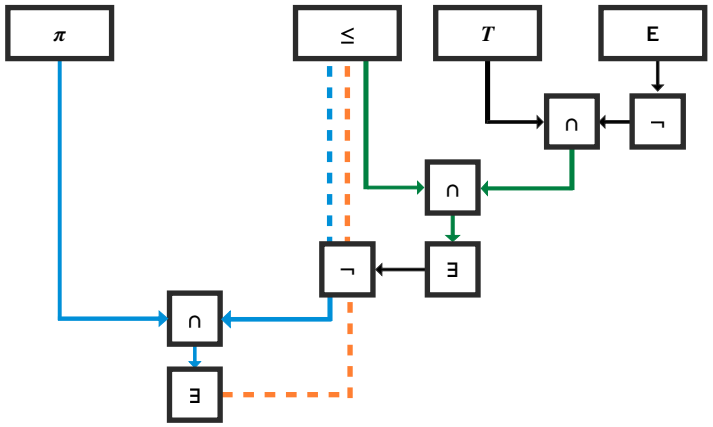




Conclusion



Minimal counterfactual causes are downward-closed sets



Causes can be synthesized under omega-regular assumptions

Are there always closest traces? What about nondeterminism?
See our arXiv version!

CAV Artifact Evaluation ★ ★ ★ Reusable

Want to know what caused the violation on your counterexample?
Check out our artifact!