On Alternating-Time Temporal Logic, Hyperproperties, and Strategy Sharing

Raven Beutner, Bernd Finkbeiner
CISPA Helmholtz Center for Information Security, Germany

Abstract

Alternating-time temporal logic (ATL*) is a well-established framework for formal reasoning about multi-agent systems. However, while ATL* can reason about the strategic ability of agents (e.g., some coalition $A$ can ensure that a goal $\psi$ is reached eventually), we cannot compare multiple strategic interactions, nor can we require multiple agents to follow the same strategy. For example, we cannot state that coalition $A$ can reach a goal sooner (or more often) than some other coalition $A'$. In this paper, we propose HyperATL$^\mathbb{S}$, an extension of ATL* in which we can (1) compare the outcome of multiple strategic interactions w.r.t. a hyperproperty, i.e., a property that refers to multiple paths at the same time, and (2) enforce that some agents share the same strategy. We show that HyperATL$^\mathbb{S}$ is a rich specification language that captures important AI-related properties that were out of reach of existing logics. We prove that model checking of HyperATL$^\mathbb{S}$ on concurrent game structures is decidable. We implement our model-checking algorithm in a tool we call HyMASMC and evaluate it on a range of benchmarks.

1 Introduction

Logics play a key role in the specification and verification of strategic properties in multi-agent systems (MAS) (Calegarì et al. 2021). One of the most influential temporal logics for MASs is alternating-time temporal logic (ATL*), which extends CTL* with (implicit) quantification over strategies (Alur, Henzinger, and Kupferman 2002). As an example, assume we want to formally verify that a set of agents $A$ can ensure that some temporal objective $\psi$ is ultimately fulfilled. We can express this as the ATL* formula $\langle A \rangle \Box \psi$, stating that the agents in $A$ have a joint strategy that ensures that all compatible executions eventually (Box) satisfy $\psi$. Likewise, we can express that coalition $A$ has no strategy to ensure that $\psi$ is reached as $[A] G \neg \psi$, i.e., for every strategy of $A$, some execution globally (G) satisfies $\neg \psi$.

However, in many situations, we are interested not only in the strategic (in)ability of a coalition but also in comparing the ability of multiple coalitions. For example, we might ask if some coalition $A$ is able to reach some goal $\psi$ strictly sooner (or more often) than some other coalition $A'$. Indeed, important game-theoretic concepts such as Shapley values (Shapley 1953) are inherently based on the relative contribution of individual agents: To compute the Shapley value for some agent $i$, we need to compare the ability of some arbitrary coalitions $A$ with that of $A \cup \{i\}$. Stating such comparison-based properties in ATL* is impossible as ATL* only considers a single path in isolation.

Hyperproperties. In contrast, the formal methods community has extensively studied properties that relate multiple system executions and coined them hyperproperties (Clarkson and Schneider 2008). In this paper, we bring the powerful concept of hyperproperties to the realm of AI and MASs. We introduce HyperATL$^\mathbb{S}$ – a temporal logic that combines strategic reasoning (as found in ATL*), the ability to compare executions w.r.t. a hyperproperty (as, e.g., found in HyperATL), and the possibility of enforcing agents to share strategies. As in HyperATL$^*$ (Beutner and Finkbeiner 2021, 2023b), we bind the outcome of a strategic interaction (resulting from an ATL*–like quantification) to a path variable and can then refer to atomic propositions on multiple paths. This combination of strategic reasoning and hyperproperties is needed for many AI-related properties; not only the information-flow properties envisioned in (Beutner and Finkbeiner 2021, 2023b). For example, HyperATL$^\mathbb{S}$ allows us to express that coalition $A$ can reach $\psi$ strictly sooner than coalition $A'$ as follows.

$$\langle A \rangle \pi_1 \land \langle A' \rangle \pi_2 \land (\neg \psi_{\pi_1}\lor \neg \psi_{\pi_2}) \lor (\neg \psi_{\pi_1}\land \psi_{\pi_2}).$$

This formula states that there exist strategies for the agents in $A$, such that for every path $\pi$ under those strategies, it holds that: under every strategy for the agents in $A'$, there exists some compatible path $\pi'$, such that $\pi$ reaches $\psi$ (denoted $\psi_{\pi}$) before $\pi'$ does (expressed using LTL’s until operator $\lor$). Phrased differently, some strategy $A$ can ensure that $\psi$ is reached strictly faster than any strategy for $A'$ could.

Note that this approach is very flexible, as we can compare $\pi$ and $\pi'$ w.r.t. to an arbitrary temporal property (e.g., $\pi$ reaches $\psi$ more often than $\pi'$). This goes well beyond the capabilities of ATL*, even when extended with quantitative operators (cf. Section 2).

Strategy Sharing and HyperATL$^\mathbb{S}$. HyperATL$^\mathbb{S}$ then extends HyperATL$^*$ with the ability to force agents to follow the same strategy. A sharing constraint $\xi$ is a set of pairs of agents, and the HyperATL$^\mathbb{S}$ formula $\langle A \rangle \xi \pi \varphi$ requires...
that coalition $A$ can satisfy $\varphi$, under the assumption that all agents $(i, j) \in \xi$ play the same strategy; similar to what is possible in strategy logic (Mogavero et al. 2014; Chatterjee, Henzinger, and Piterman 2010) in a non-hyper setting.

**Example 1.** Assume we deal with a MAS modeling a planning task with multiple robots and want to ensure that robots in coalition $A$ can reach some target state. To keep the employment overhead as small as possible, we might ask if the robots can follow some optimal trajectory (i.e., reach the target as fast as possible), despite all using the same strategy. We can express this in HyperATL$^*$ as follows:

$$\langle A \rangle_{\{(i,j) \mid i,j \in A\}} \pi \cdot \langle A \rangle_{\pi'} \neg \text{target}_\pi$$

stating that all robots in $A$ can use a shared strategy (on path $\pi$) that reaches the target at least as fast as they can without the constraint that they must play the same strategy (path $\pi'$). Such sharable strategies are, e.g., key for scalable synthesis (Attie and Emerson 1998).

We provide further HyperATL$^*$ examples (such as determinism and good-enough synthesis) in Section 6.2.

**Model Checking.** We show that model checking (MC) of HyperATL$^*$ on finite-state concurrent game structures (a standard model of MASs) is decidable. As HyperATL$^*$ can relate multiple computation paths, we cannot employ the tree-automaton-based MC approach for ATL$^*$ (Alur, Henzinger, and Kupferman 2002). Instead, we develop a MC algorithm based on alternating word automata. Our algorithm iteratively simulates path quantification within an automaton, while ensuring that the strategy-sharing constraints between agents are fulfilled.

**Implementation.** We implement our model-checking algorithm (for full HyperATL$^*$) in a tool we call HyMASMC. Using HyMASMC, we can, for the first time, automatically check properties beyond the self-composition fragment of HyperATL$^*$ – the largest fragment supported by previous tools (Beutner and Finkbeiner 2021, 2023b) (cf. Section 2). We evaluate HyMASMC by verifying a range of properties in MASs from the literature. Our experiments show that our algorithm performs well on non-hyper instances that could already be handled using existing solvers (Cermák, Lomuscio, and Murano 2015) and can successfully verify hyperproperties that cannot be expressed in any existing logic, let alone checked with any existing tool.

**Supplementary Material.** Detailed proofs and additional material can be found in (Beutner and Finkbeiner 2023c).

## 2 Related Work

Various works have extended ATL$^*$ with abilities to reason about probabilistic systems (Chen and Lu 2007), incomplete information (Belardinelli et al. 2017; Berthon, Maubert, and Murano 2017; Belardinelli, Lomuscio, and Malvone 2019), and finite traces (Belardinelli et al. 2018). All of these extensions refer to individual paths and cannot express properties that relate multiple paths. While resource-aware extensions offer quantitative reasoning (Alechina, Demri, and Logan 2020; Bouyer et al. 2019; Henzinger and Prabhu 2006; Jamroga, Konikowska, and Penczek 2016; Chen and Lu 2007), they still cannot state properties that go beyond computing quantities on individual paths. Strategy logic (SL) treats strategies as first-class objects and can naturally express properties where some agents share the same strategy (Mogavero et al. 2014; Chatterjee, Henzinger, and Piterman 2010). While SL can compare the same strategy in different scenarios, it is limited to a boolean combination of LTL properties on individual paths, i.e., we cannot compare different paths w.r.t. a temporal hyperproperty. All properties we consider in Sections 1 and 6.2 cannot be expressed in SL. Most existing hyperlogics, including HyperLTL and HyperCTL$^*$ (Clarkson et al. 2014), reason about paths in a (non-strategic) transition system. HyperATL$^*$ was the first temporal logic that combined strategic reasoning with the ability to express hyperproperties (Beutner and Finkbeiner 2021, 2023b). This captures strategic information-flow policies such as simulation-based non-interference (Mantel and Sabelfeld 2001) and non-deducibility of strategies (Wittbold and Johnson 1990). HyperATL$^*$ extends HyperATL$^*$ with the ability to force agents to share the same strategy, which is useful for many AI-related properties (cf. Example 1). Moreover, automated verification of HyperATL$^*$ was, so far, only possible for the self-composition fragment (Beutner and Finkbeiner 2023b). In this fragment, all quantifiers are grouped together by constructing the self-composition of a MAS (Barthe, D’Argenio, and Rezk 2011), which reduces verification to a parity game. While this fragment suffices for many security-related properties (which are naturally defined in terms of a self-composition), it does not capture any of the properties discussed in Sections 1 and 6. In contrast, our model-checking algorithm (implemented in HyMASMC) uses iterative quantifier elimination and is applicable to all HyperATL$^*$ formulas. In terms of tool support, the MCMAS tool family (Lomuscio, Qu, and Raimondi 2009) implements a range of model checkers for strategic properties (e.g., specified in ATL$^*$ or SL), often with a strong focus on knowledge (Fagin et al. 1995; van der Hoek and Wooldridge 2003). Generally, knowledge properties are hyperproperties; to “know something” means that it should hold on all indistinguishable paths, effectively relating multiple paths in a system (Bozzelli, Maubert, and Pinchinat 2015; Beutner et al. 2023). However, before HyMASMC, none of the existing verifiers could check general hyperproperties (beyond knowledge) in MASs.

## 3 Preliminaries

For two functions $f : X \rightarrow Z$ and $f' : Y \rightarrow Z$ with $X \cap Y = \emptyset$, we define $f \oplus f' : X \cup Y \rightarrow Z$ as the union of both functions. We let $AP$ be a fixed finite set of atomic propositions and let $Agts$ be a fixed finite set of agents. For a set of agents $A \subseteq Agts$, we define $\overline{A} := Agts \setminus A$. Given some set $X$, we write $X^+$ (resp. $X^*$) for the set of non-empty (resp. infinite) sequences over $X$. For $u \in X^*$ and $k \in \mathbb{N}$, we write $x(k)$ for the $k$th element, $u[k, \infty]$ for the infinite suffix starting at position $k$, and $u[0, k]$ for the finite prefix up to $k$. As the underlying model of MASs, we use concurrent game structures (CGS).
Definition 1 (Alur, Henzinger, and Kupferman (2002)). A concurrent game structure is a tuple \( G = (S, s_0, A, k, L) \) where \( S \) is a finite set of states, \( s_0 \in S \) is an initial state, \( A \) is a finite set of actions, \( k : S \times (Agts \rightarrow k) \rightarrow S \) is a transition function, and \( L : S \rightarrow 2^{AP} \) is a state labeling.

An action vector is a function \( a : Agts \rightarrow A \) assigning an action to each agent. Given a state \( s \) and action vector \( a \), the transition function \( k \) determines the next state \( k(s,a) \). A strategy is a function \( f : S^+ \rightarrow A \), mapping finite paths to actions. We denote the set of all strategies in \( G \) with \( Str(G) \). Given a state \( s \in S \) and strategy vector \( f : Agts \rightarrow Str(G) \) mapping each agent to a strategy, we can construct the path \( Play_G(s, f) \) that results from each agent acting according to the strategy defined by \( f \). Formally, we define \( Play_G(s, f) = \{ \psi \in S^\omega \mid \exists \pi. \phi \} \) where \( \phi \) is a path formula in ATL.

\[ \text{Path and state formulas in ATL}^* \]

Path and state formulas in ATL* are defined as follows:

\[ \psi ::= a | \psi \land \psi | \neg \psi | X \psi | \psi U \psi | \varphi \]
\[ \varphi ::= \langle A \rangle \psi | \langle \emptyset \rangle \psi \]

where \( a \in AP \) and \( A \subseteq Agts \). The temporal \( X \) refers to the next timepoint, and \( \psi_1 U \psi_2 \) holds at some future time and \( \psi_2 \) holds at all times until then. We use the standard Boolean connectives \( \land, \neg, \lor \), and the variables \( a, b, c \). We can simulate the protocol \( \psi \) in the transition function \( k \) by “rerouting” every action that is invalid (according to \( \psi \)) to some allowed action, effectively limiting the available actions of an agent.

\[ A \text{TL}^* \]

We briefly recall the syntax and semantics of ATL*. Path and state formulas in ATL* are defined as follows:

\[ \psi ::= a | \psi \land \psi | \neg \psi | X \psi | \psi U \psi | \varphi \]
\[ \varphi ::= \langle A \rangle \psi | \langle \emptyset \rangle \psi \]

where \( a \in AP \) and \( A \subseteq Agts \). The temporal \( X \) refers to the next timepoint, and \( \psi_1 U \psi_2 \) holds at some future time and \( \psi_2 \) holds at all times until then. We use the standard Boolean connectives \( \land, \neg, \lor \), and the variables \( a, b, c \). We can simulate the protocol \( \psi \) in the transition function \( k \) by “rerouting” every action that is invalid (according to \( \psi \)) to some allowed action, effectively limiting the available actions of an agent.

\[ \text{MVL} \]

Let \( \mathcal{V} = \{ \pi, \pi', \ldots \} \) be a set of path variables. Path and state formulas in \( \text{MVL} \) are generated by the following grammar:

\[ \psi ::= a_\pi | \psi \land \psi | \neg \psi | X \psi | \psi U \psi | \varphi_\pi \]
\[ \varphi ::= \langle A \rangle_\pi \psi | \langle \emptyset \rangle_\pi \psi \]

where \( a_\pi \in AP \), \( \pi \in \mathcal{V} \), \( A \subseteq Agts \), and \( \xi \subseteq Agts \times Agts \) is a sharing constraint. We assume that nested state formulas are closed, i.e., for each atomic formula \( a_\pi \), path variable \( \pi \) is bound by some quantifier.

Similar to ATL*, formula \( \langle A \rangle_\pi \psi \) states that there exists a strategy for coalition \( A \) such that all paths under that strategy satisfy \( \psi \). However, differently from ATL*, we bind this path to the path variable \( \pi \). We can then use path variables to refer to multiple paths via indexed atomic propositions.

Sémantique. We evaluate HyperATL* formulas in the context of a path assignment, which is a partial mapping \( \Pi : \mathcal{V} \rightarrow S^\omega \). We write \( \emptyset \) for the path assignment with an empty domain. Given \( k \in N \), we define \( \Pi[k] \) as the assignment defined by \( \Pi[k,\infty](\pi) := \Pi(\pi)[k,\infty] \), i.e., the assignment where all paths are (synchronously) shifted by \( k \) positions. For a path \( p \in S^\omega \), we define \( \Pi[\pi \rightarrow p] \) as the updated assignment that maps \( \pi \) to \( p \). For path formulas, we define:

\[ \Pi |\models a \quad \text{iff} \quad a \in L(\Pi(\pi))(0) \]
\[ \Pi |\models \psi_1 \land \psi_2 \quad \text{iff} \quad \Pi |\models \psi_1 \quad \text{and} \quad \Pi |\models \psi_2 \]
\[ \Pi |\models \neg \psi \quad \text{iff} \quad \Pi |\not\models \psi \]
\[ \Pi |\models X \psi \quad \text{iff} \quad \Pi[1,\infty] |\models \psi \]
\[ \Pi |\models \psi_1 U \psi_2 \quad \text{iff} \quad \exists k \in N. \Pi[k,\infty] |\models \psi_2 \quad \text{and} \quad \forall 0 \leq m < k. \Pi[m,\infty] |\models \psi_1 \]

That is, \( \langle A \rangle \psi \) holds in state \( s \) if the agents in \( A \) can enforce \( \psi \). Formally, this means that there exists a strategy for each agent in \( A \) (formalized as function \( f \)) such that – no matter what strategy the agents in \( A \) adopt (formalized by \( f^+ \)) – the resulting path satisfies path formula \( \psi \). Conversely, \( \langle A \rangle \psi \) states that coalition \( A \) cannot avoid \( \psi \), i.e., every strategy for \( A \) admits some path that satisfies \( \psi \).
Whenever we check if \( a \) currently holds, we check if \( a \) holds on the path that is bound to \( \pi \). A nested state formula \( \varphi_\pi \) holds iff \( \varphi \) holds in the first state of the path bound to \( \pi \).

Given a set of agents \( A \subseteq \text{Agents} \) and sharing constraints \( \xi \), we define \( \text{shr}_G(A, \xi) := \{ f : A \rightarrow \text{Str}(G) \mid \forall i,j \in A. (i,j) \in \xi \Rightarrow f(i) = f(j) \} \), i.e., all strategy vectors for \( A \) that satisfy the constraints in \( \xi \). HyperATL\( _\pi^* \) state formulas are evaluated in a state \( s \) and path assignments \( \Pi \). For each strategy quantifier, we construct a new path and bind this path to a path variable in \( \Pi \):

\[
\begin{align*}
\text{s, } \Pi &\models_{\xi} \varphi \\
\text{s, } \Pi &\models_{\xi} \langle A \rangle \pi. \varphi \\
\text{s, } \Pi &\models_{\xi} \langle A \rangle \pi. \varphi \\
\end{align*}
\]

\( \forall f' \in \text{shr}_G(A, \xi) \cdot \text{s, } \Pi[\text{Play}_G(s, f \oplus f')] &\models_{\xi} \varphi \\
\exists f' \in \text{shr}_G(A, \xi) \cdot \text{s, } \Pi[\text{Play}_G(s, f \oplus f')] &\models_{\xi} \varphi
\]

Take \( \langle A \rangle \pi. \varphi \) as an example. As in ATL\( ^* \), we existentially quantify over strategies for the agents in \( A \) (subject to the condition that they respect the sharing constraints in \( \xi \)), followed by universal quantification over strategies for agents in \( \overline{A} \) (again, subject to \( \xi \)). The resulting strategy vector \( f \oplus f' \) then yields a unique path \( \text{Play}_G(s, f \oplus f') \), which we bind to path variable \( \pi \) and continue evaluation of \( \varphi \). Note that in case \( \xi = \emptyset \), the quantification behavior is very close to that of ATL\( ^* \) as \( \text{shr}_G(A, \emptyset) \) contains all functions \( A \rightarrow \text{Str}(G) \). The important difference to ATL\( ^* \) is that once we have constructed the path \( \text{Play}_G(s, f \oplus f') \), we do not immediately evaluate a path formula but rather add the path to our current assignment. Without sharing constraints, HyperATL\( _\pi^* \) corresponds to HyperATL\( ^* \) (Beutner and Finkbeiner 2021, 2023b) and is strictly more expressive than ATL\( ^* \).

We say that \( G \) satisfies \( \varphi \), written \( G \models \varphi \), if \( s_0, 0 \models_{\xi} \varphi \).

\textbf{Example 2 (Running Example).} \textit{Let us consider a very simple CGS between agents Agts = \{sched, W1, W2\}, describing a scheduler and two worker agents. The scheduler sched can choose actions \{g, ng\} modeling a grant or no grant, and each of the workers can choose actions \{z, nz\} modeling a request to work or no request to work. We model the dynamics of the CGS in Figure 1. If the scheduler chooses ng or both of the workers do not request to work, we remain in idle state \( s_0 \). If the scheduler grants work and both workers request to work, we directly transition to the working state \( s_2 \) where proposition \( w \in AP \) holds. If only one of the workers requests work, we also transition to \( s_2 \) but pass through \( s_1 \), i.e., the work is delayed by one step.}

\textbf{Let us assume we want to verify that coalition \{sched, W1, W2\} can reach the work state \( s_2 \) (strictly) faster than \{sched, W1\}. As argued in the introduction, we can express this using the following HyperATL\( _\pi^* \) formula \( \langle sched, W1, W2 \rangle \pi. \langle sched, W1 \rangle \varphi' \cup \langle sched, W2 \rangle \varphi'' \cup \langle sched, W1 \rangle \varphi \). This formula holds in the above CGS: \{sched, W1, W2\} can construct a path \( w \) where \( w \) holds in the second step, whereas \{sched, W1\} can, on their own, only ensure that \( w \) holds in the third step on \( \pi' \) (at the earliest).}

\textbf{Proposition 1.} For every ATL\( ^* \) formula \( \varphi \), there exists an effectively computable HyperATL\( _\pi^* \) formula \( \varphi' \) such that for every CGS \( G, G \models_{\xi} \varphi \iff G \models \varphi' \).

\section{Model Checking of HyperATL\( _\pi^* \)}

While the extension of ATL\( ^* \) to reason about hyperproperties required only minor modifications to its syntax, the subtle changes bring major complications in terms of model checking. In particular, the model-checking algorithm for ATL\( ^* \) proposed by Alur, Henzinger, and Kupferman (2002) is no longer applicable: In ATL\( ^* \), checking if \( \langle A \rangle \psi \) holds in some state \( s \) can be reduced to the non-emptiness of the intersection of two tree automata. One accepts all trees that represent possible strategies by the agents in \( A \), and one accepts all trees whose paths satisfy the path formula \( \psi \). In HyperATL\( _\pi^* \), this is not possible: In a formula \( \langle A \rangle \pi. \varphi \), the satisfaction of \( \varphi \) does not only depend on \( \pi \) but also on path variables that are quantified before (outside).

\subsection{Alternating Automata}

Instead, our model-checking algorithm uses automata to “summarize” path assignments that satisfy subformulas, similar to previous hyperlogics such as HyperLTL (Finkbeiner, Rabe, and Sánchez 2015; Beutner and Finkbeiner 2023a), and HyperATL\( ^* \) (Beutner and Finkbeiner 2021, 2023b). To handle the strategic interaction found in MASs, we rely on alternating automata, i.e., automata that alternate between existential (non-deterministic) and universal transitions.

\textbf{Definition 2.} An alternating parity automaton (APA) over alphabet \( \Sigma \) is a tuple \( A = (Q, q_0, \delta, c) \) where \( Q \) is a finite set of states, \( q_0 \in Q \) is an initial state, \( c : Q \rightarrow \mathbb{N} \) is a state coloring, and \( \delta : Q \times \Sigma \rightarrow \mathbb{B}^+ (Q) \) is a transition function that maps pairs of state and letter to a positive boolean formula over \( Q \) (denoted with \( \mathbb{B}^+ (Q) \)).

Formally, we model the alternation in APAs by viewing the transitions as positive boolean formulas over states (i.e., formulas formed using only conjunctions and disjunctions). For example, if \( \delta(q, l) = q_1 \lor (q_2 \land q_3) \), we can – from state \( q \in Q \) and upon reading letter \( l \in \Sigma \) – either move to state \( q_1 \) or move to both \( q_2 \) and \( q_3 \) (i.e., spawn two copies of our automaton, one starting in state \( q_2 \) and one in \( q_3 \)). We write \( \mathcal{L}(A) \subseteq \Sigma^\omega \) for the set of all infinite words that are accepted by \( A \), i.e., all infinite words where we can construct a run tree such that for all paths, the minimal color that occurs infinity many times (as given by \( c \)) is even. See (Vardi 1995; Beutner and Finkbeiner 2023c) for details.
Figure 2: Example APA over alphabet $\Sigma = \{a,b,c\}$.

Example 3. Consider the APA in Figure 2. We display the color of each state and visualize transition formulas using $\land$ and $\lor$ nodes. For example, $\delta(q_0, a) = \delta(q_0, b) = \delta(q_0, c) = q_0 \land (q_1 \lor q_2)$, i.e., whenever reading letter $a$, $b$, or $c$ in $q_0$ we start a fresh run from $q_0$ and at the same time start a run from either $q_1$ or $q_2$. To derive the language of the APA, we first observe that state $q_1$ (resp. $q_2$) accepts all words that contain at least one $a$ (resp. $b$) (note that the color of $q_1, q_2$ is odd). In the initial state $q_0$, we restart a run from $q_0$ and transition to either $q_1$ or $q_2$. The language thus contains exactly those words that contain $a$ or $b$ infinitely often.

Deterministic Automata. Our model-checking algorithm relies on the fact that we can determinize APAs. We say $A$ is a deterministic parity automaton (DPA) if we can view $\delta$ as a function $Q \times \Sigma \to Q$ that assigns a unique successor state to each state, letter pair.

Proposition 2 (Miyan and Hayashi (1984)). For any APA $A$ with $n$ states, we can effectively compute a DPA $A'$ with at most $2^{2^{2^n}}$ states such $L(A) = L(A')$.

5.2 Model Checking Algorithm

We are now in a position to outline our model-checking algorithm. A high-level description is given in Algorithm 1. Here, we write $\langle A \rangle$ as a shorthand for either $\langle A \rangle_l$ or $\langle A \rangle_r$.

Nested State Formulas. Initially, our algorithm recursively checks nested state formulas and replaces them with fresh atomic propositions (Emerson and Halpern 1986). Concretely, given a closed state formula $\varphi = \langle A \rangle_l \psi_1 \ldots \langle A \rangle_r \psi_n$, we first extract all state formulas that are nested in the path formula $\psi$ (line 2). For each nested state formula $\varphi'$, we (1) compute all states in which $\varphi'$ holds (using a recursive call to modelCheck); (2) mark all those states with a fresh atomic proposition $p_{\varphi'}$ by modifying the labeling function $L$ of $G$ (line 4); and (3) replace all occurrences of $\varphi'_n$ within $\psi$ with $p_{\varphi'}$ (line 5).

Eliminating Path Quantification. Afterward, $\psi$ contains no nested state formulas, and we can tackle the strategic quantifiers. For each state $s \in S$, we check if $s, \emptyset \models G \varphi$, and if it does -- add it to the solution set $Sol$ (line 12). Our main idea to check $s, \emptyset \models G \varphi$ is to iteratively eliminate paths $\pi_1, \ldots, \pi_m$ by simulating $G$ using the alternation available in APAs while summarizing path assignments that satisfy the formula from the fixed state $s \in S$. To enable automata-based reasoning about path assignments, i.e., mappings $\Pi : V \rightarrow S^\omega$ for some $V \subseteq V$, we zip such an assignment into an infinite word: Given II : $V \rightarrow S^\omega$ we define $\text{zip}(\Pi) \in (V \rightarrow S)^\omega$ as the infinite word over functions $V \rightarrow S$, defined by $\text{zip}(\Pi)(k)(\pi) := \Pi(\pi)(k)$ for $k \in \mathbb{N}$.

Algorithm 1: Model-checking algorithm for HyperATL$^\omega_S$.

<table>
<thead>
<tr>
<th>Line</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\text{def modelCheck}(G, \varphi = \langle A_1 \rangle_{\xi_1} \pi_1 \ldots \langle A_n \rangle_{\xi_n} \pi_n, \psi)$:</td>
</tr>
<tr>
<td>2</td>
<td>for $\varphi'$ in nestedStateFormulas($\psi$) do</td>
</tr>
<tr>
<td>3</td>
<td>$S_{\varphi'} = \text{modelCheck}(G, \varphi')$</td>
</tr>
<tr>
<td>4</td>
<td>$L = \lambda s. {L(s)}$ if $s \notin S_{\varphi'}$</td>
</tr>
<tr>
<td>5</td>
<td>$\psi = psi[\varphi_{s_1}/(p_{\varphi'})<em>{s_1}] \ldots [\varphi</em>{s_n}/(p_{\varphi'})_{s_n}]$</td>
</tr>
<tr>
<td>6</td>
<td>$Sol = \emptyset$</td>
</tr>
<tr>
<td>7</td>
<td>for $s \in S$ do</td>
</tr>
<tr>
<td>8</td>
<td>$A = \text{LTLtoAPA}(\psi)$</td>
</tr>
<tr>
<td>9</td>
<td>for $j$ from $n$ to $1$ do</td>
</tr>
<tr>
<td>10</td>
<td>$A = \text{product}(G, \dot{s}, A, {A_j}_{\xi_j, \pi_j})$</td>
</tr>
<tr>
<td>11</td>
<td>if $\text{zip}(\emptyset) \in L(A)$ then</td>
</tr>
<tr>
<td>12</td>
<td>$Sol = Sol \cup {s}$</td>
</tr>
<tr>
<td>13</td>
<td>return $Sol$</td>
</tr>
</tbody>
</table>

Definition 3. Assume $\varphi$ is a HyperATL$^\omega_S$ formula with free path variables $V \subseteq V$. We say an automaton $A$ over $V \rightarrow S$ is $(G, \dot{s})$-equivalent to $\varphi$ if for every path assignment $\Pi : V \rightarrow S^\omega$ we have $\text{zip}(\Pi) \in L(A)$ if and only if $s, \Pi \models G \varphi$.

Now assume that $\varphi = \langle A \rangle_l \pi_1 \ldots \langle A \rangle_r \pi_n$, $\psi$ is the state formula we want to check in state $s$. If we could compute a $(G, \dot{s})$-equivalent automaton $A_\varphi$ for $\varphi$, we could immediately check whether $s, \emptyset \models G \varphi$ by testing if $\text{zip}(\emptyset) \in L(A_\varphi)$. Our main theoretical result is that we can construct such an automaton incrementally. We begin with a $(G, \dot{s})$-equivalent automaton $A_\varphi$ for the body $\psi'$; then we use $A_\psi$ to construct a $(G, s \dot{s})$-equivalent automaton $A_{\langle A_n \rangle_{e \Pi, \pi_n, \psi}}$ for $\langle A_n \rangle_{e \Pi, \pi_n, \psi}$; and so forth, finally yielding the desired automaton $A_\varphi$ that is $(G, \dot{s})$-equivalent to $\varphi$. In each step, we apply the construction from the following theorem:

Theorem 1. Assume that $\varphi = \langle A \rangle_{l \pi_1} \pi_2 \ldots \langle A \rangle_{l \pi_n} \pi_n$, and let $A_{\varphi'}$ be an APA over alphabet $V \cup \{\pi\} \rightarrow S$ that is $(G, \dot{s})$-equivalent to $\varphi$. We can effectively construct an APA $A_{\varphi'}$ over alphabet $V \rightarrow S$ that is $(G, \dot{s})$-equivalent to $\varphi$. The size of $A_{\varphi'}$ is at most double exponential in the size of $A_{\varphi'}$.

Proof. Let $A_{\varphi'} = (Q, q_0, \delta, c)$ be a DPA equivalent to $A_{\varphi'}$ (cf. Proposition 2). We define $A_{\varphi} = (Q \times S, (q_0, S), B', c')$ where $c'(q, s) := c(q)$ and $B'$ is defined as follows: If $\varphi = \langle A \rangle_{l \pi_1} \pi_2 \ldots \langle A \rangle_{l \pi_n} \pi_n$, we define $B'(q, s, l)$ for $l : V \rightarrow S$ as

$$\bigvee_{i \leq m, j \leq n} \left( \delta(q, l[\pi_i \rightarrow s]), \kappa(s, a + a') \right)$$

Conversely, if $\varphi = \langle A \rangle_{l \pi_1} \pi_2 \ldots \langle A \rangle_{l \pi_n} \pi_n$, we define $B'(q, s, l)$ as

$$\bigvee_{i \leq m, j \leq n} \left( \delta(q, l[\pi_i \rightarrow s]), \kappa(s, a + a') \right)$$

The intuition behind our construction is that we can simulate the strategic quantification at the level of states, similar to what is possible in HyperATL$^\omega$ (Beutner and Finkbeiner 2021, 2023b). Let us take $\varphi = \langle A \rangle_{l \pi_1} \pi_2 \ldots \langle A \rangle_{l \pi_n} \pi_n$ as an example.
The desired automaton $A_\varphi$ should accept a word $u \in (V \to S)^\omega$ if there exists a strategy vector $f : A \to Str(G)$ that respects $\xi$ and for all paths $\pi$ compatible with $f$, the extended zipped path assignment (a word in $(V \cup \{\pi\} \to S)^\omega$) is accepted by $A_{\varphi}^{det}$. In our constructions, we track the current state $q$ of $A_{\varphi}^{det}$ and simulate $G$ by keeping track of the current state $s$. When in state $(q,s)$, we update the automaton state according to the transition function of $A_{\varphi}^{det}$ using the current state $s$ for path variable $\pi$. To update the state of $G$, we simulate the strategic behavior: (1) we disjunctive fix actions for each agent in $A$ via a function $a$ and ensure that all sharing constraints hold; (2) we conjunctively choose actions for $A$ as a function $a'$ (subject to the sharing constraints); and (3) we update the system state to $\kappa(s,a \oplus a')$.

Arguing that $A_\varphi$ is $(G,s)$-equivalent to $\varphi$ is based on the determination of the underlying game. As we work in a setting of complete information (where all agents observe the state of the CGS), we can replace the existential quantification over strategies for $A$ (as in the semantics of $(A_\varphi)[i] \pi, \varphi'$) with existential quantification over actions for $A$ in each step (as used in the disjunctive choice in the definition of $A_\varphi$). We give a formal proof in (Beutner and Finkbeiner 2023c).

The size of $A_\varphi$ is linear in the size of $G$ and $A_{\varphi}^{det}$ (which itself is doubly exponential in $A_\varphi$, cf. Proposition 2).

For a formula $\varphi = (A_\varphi)[i] \pi, \varphi'$ and APA $A$ that is $(G,s)$-equivalent to $\varphi'$, let $\text{product}(G,s,A,(A_\varphi)[i] \pi)$ be the APA that is $(G,s)$-equivalent to $\varphi$ constructed using Theorem 1. In Algorithm 1, we start with an APA that is equivalent to $\psi$ (line 8), and apply $\text{product}$ to iteratively compute $(G,s)$-equivalent automata for subformulas $\langle A_j \rangle_{i \leq j} \pi_n, \psi$ for $j$ from $n$ to 1 (line 10). After the loop, we are left with an APA $A$ over singleton alphabet $\emptyset \to \{s_0,s_1,s_2\}$ constructed using Theorem 1 that is $(G,s_0)$-equivalent to $\langle \text{sched}, W1, W2 \rangle \pi, \langle \text{sched}, W1 \rangle \pi'. (\neg w_{\pi'} \land w_\pi$).

**Proposition 3.** For every CGS $G = (S,s_0,\kappa,\lambda)$ and closed HyperATL$_S^*$ formula $\varphi$, we have

\[
\text{modelCheck}(G, \varphi) = \{ s \in S \mid s, \emptyset \models G \varphi \}.
\]

**Complexity.** Each application of $\text{product}$ increases the size of $A$ by (in the worst case) two exponents. Checking a HyperATL$_S^*$ formula with $n$ nested quantifiers is thus in $2n$-EXPTIME, and MC for general formulas is non-elementary. As HyperATL$_S^*$ subsumes HyperATL$^*$, we get a matching non-elementary hardness (Beutner and Finkbeiner 2023b). HyperATL$_S^*$ is thus more expressive (and also much harder to model-check) than ATL$^*$. We stress that the non-elementary complexity of HyperATL$_S^*$ stems from its ability to quantify over arbitrarily many paths. In most properties of interest, we quantify over few paths (cf. Section 6), which results in a much lower (elementary) complexity. In particular, if we apply Algorithm 1 to an ATL$^*$-equivalent formula (cf. Proposition 1), we deal with a single nested quantifier $(n = 1)$ and thus match the 2EXPTIME MC complexity known for ATL$^*$ (Alur, Henzinger, and Kupferman 2002).
5.3 Model Checking the Running Example

We illustrate our MC construction on the formula from Example 2. In a first step, we translate the body $(−w_{π'}) ∪ (−w_r) \land w_r)$ to a DPA over alphabet $\{\pi, \pi'\} \rightarrow (s_0, s_1, s_2)$, depicted in Figure 3a. Afterward, we can follow the construction from Theorem 1 to obtain an APA over $(\{\pi\} \rightarrow \{s_0, s_1, s_2\})$ that is $(G, s_0)$-equivalent to subformula $[\text{sched}, W1]^{\pi'} (−w_{π'}) ∪ (−w_r) \land w_r)$. We depict a sketch in Figure 3b. To illustrate our MC construction on the formula from 5.3 Model Checking the Running Example

6 Implementation and Experiments

We have implemented our algorithm in a tool we call HyMASMC. As input, our tool reads MASM in the form of ISPL models (Lomuscio, Qu, and Raimondi 2009). For automata operations (in particular, the translation from alternating to deterministic automata), we use Spot; an actively-maintained automata library (Duret-Lutz et al. 2022). To check APAs over the singleton alphabet $(\emptyset \rightarrow S)$ for emptiness, we use the parity-game solver Oink (van Dijk 2018). All results were obtained on a 3.60GHz Xeon® CPU (E3-1271) with 32GB of memory running Ubuntu 20.04.

| n  | |S| | |S|reach | tHyMASMC  |
|----|----|----|----|----|--------|
| 2  | 72 | 9  | 0.11 | 0.41 |
| 3  | 432| 21 | 6.64 | 2.06 |
| 4  | 2592 | 49 | 322.7 | 24.3 |
| 5  | 15552 | 113 | TO   | 347.1 |

Table 1: We compare HyMASMC and MCMAS-SL[1G]. We give the size of the system (|S|), the size of the reachable fragment (|Sreach|), and the verification times in seconds. The timeout (TO) is set to 1h.

6.1 Model Checking ATL*

In our first experiment, we want to compare the performance of HyMASMC against existing tools for strategic properties. This requires us to consider non-hyper properties in the form of ATL* specification (as no existing tool can handle hyperproperties). Concretely, we compare with MCMAS-SL[1G], a solver for a fragment of strategy logic (Cermá, Lomuscio, and Murano 2015). We use the same benchmark family used by Cermá, Lomuscio, and Murano (2015), describing a parametric scheduling problem consisting of agents $\text{Agts} = \{\text{sched}, y_1, \ldots, y_n\}$ for $n \in \mathbb{N}$. We check the following HyperATL* formula

$$\:\forall \langle \text{sched} \rangle \pi, \bigwedge_{i=1}^{n} G \left(\langle \text{wt}, i \rangle_{\pi} \rightarrow F \neg \langle \text{wt}, i \rangle_{\pi} \right)$$

The formula states that whenever agent $y_i$ waits (modeled by $\text{AP}(\langle \text{wt}, i \rangle)$), it will eventually not wait anymore, i.e., the scheduler has a strategy that avoids starvation of all agents. This formula is equivalent to the strategy logic specification used by Cermá, Lomuscio, and Murano (2015).

We check the HyperATL* formulas (and the equivalent strategy logic specifications) with HyMASMC and MCMAS-SL[1G] for varying values of $n$. We use the “optimized” algorithm in MCMAS-SL[1G] that decomposes the formula as much as possible (Cermá, Lomuscio, and Murano 2015, §4). The results are given in Table 1. We observe that HyMASMC performs much faster than MCMAS-SL[1G], which we largely accredit to the very efficient backend solvers in Spot and Oink.

6.2 Model Checking Hyperproperties

In this section, we challenge HyMASMC with interesting hyperproperties. As underlying MAS models, we use the ISPL models used by MCMAS (Cermá, Lomuscio, and Murano 2015; Lomuscio, Qu, and Raimondi 2009) and design a range of specification templates that model interesting use cases of HyperATL*. We emphasize that not every template models realistic properties in each of the ISPL instances. However, our evaluation (1) demonstrates that HyperATL* can express interesting properties, and (2) empirically shows that HyMASMC can check such properties in existing ISPL models (confirming this via further real-world scenarios is interesting future work). Note that none of the properties falls in the self-composition fragment of HyperATL* (Beutner and Finkbeiner 2021, 2023b), which formed the largest fragment supported by previous tools.
### Optimality I
As argued in Section 1 and Example 2, a particular strength of HyperATL is the ability to compare the power of different coalitions. For $A, A' \subseteq \text{Agts}$ and $\text{tgt} \in AP$ (modeling the target), we check

$$\langle A \rangle \pi . \langle A' \rangle \pi' . ( - \text{tgt}_\pi ) \cup ( - \text{tgt}_{\pi'} \land \text{tgt}_\pi ) .$$

### Optimality II
Using the strategy sharing in HyperATL, we can also check if a coalition can achieve the goal equally fast despite using the same strategy for all agents (cf. Example 1). For a group of agents $A$ and $\text{tgt} \in AP$, we use HyMASMC to check

$$\langle A \rangle \pi . \langle A \rangle \pi' . ( - \text{tgt}_\pi ) \cup \text{tgt}_\pi .$$

### Optimality III
Likewise, we can express that coalition $A$ can reach a target state at strictly more time points than coalition $A'$ as

$$\langle A \rangle \pi . \langle A \rangle \pi' . \text{G}(\text{tgt}_{\pi'} \rightarrow \text{tgt}_\pi) \land \text{F}( - \text{tgt}_{\pi'} \land \text{tgt}_\pi) .$$

### Observational Determinism (OD)
An important property in the context of security in MASs is observational determinism (Zdancewic and Myers 2003). For example, assume we have a system that contains a controller agent $\text{cnt}$ and an AP $h$ that models a high-security value of the system. We want to ensure that the value of $h$ is in control of $\text{cnt}$, which we can express in HyperATL as

$$\langle \text{cnt} \rangle \pi . \langle \text{cnt} \rangle \pi' . \text{G}(h_{\pi} \leftrightarrow h_{\pi'}) .$$

That is, $\text{cnt}$ has a strategy to construct $\pi$ such that in a second execution, $\text{cnt}$ can ensure the same sequence of values for $h$ (despite the other agents potentially acting differently).

### Good-Enough Synthesis (GE)
In many scenarios, there does not exist a strategy that wins in all situations. Instead, we often look for strategies that are good-enough (GE), i.e., strategies that win on every possible input sequence for which a winning output sequence exists (Almagor and Kupferman 2020; Aminof, Giacomo, and Rubin 2021; Li et al. 2021). We can express the existence of a GE strategy for $A \subseteq \text{Agts}$ in HyperATL as

$$\langle A \rangle \pi . \langle \emptyset \rangle \pi' . (\text{G}(\text{in}_\pi \leftrightarrow \text{in}_{\pi'}) \land \text{F}(\text{tgt}_{\pi'}) \rightarrow \text{F}(\text{tgt}_\pi) .$$

### Table 2
For each ISPL model (Lomuscio, Qu, and Raimondi 2009), we display the average time ($t_{avg}$) and the maximal time ($t_{max}$) time (in seconds) needed by HyMASMC across the 20 random instances sampled from each template.

<table>
<thead>
<tr>
<th>ISPL Model</th>
<th>Optimality I</th>
<th>Optimality II</th>
<th>Optimality III</th>
<th>OD</th>
<th>GE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BIT-TRANSMISSION</td>
<td>0.38 0.41</td>
<td>0.39 0.42</td>
<td>0.39 0.44</td>
<td>0.39 0.44</td>
<td>0.38 0.42</td>
</tr>
<tr>
<td>BOOK-STORE</td>
<td>0.39 0.42</td>
<td>0.40 0.47</td>
<td>0.40 0.44</td>
<td>0.39 0.42</td>
<td>0.39 0.43</td>
</tr>
<tr>
<td>CARD-GAME</td>
<td>0.38 0.39</td>
<td>0.38 0.39</td>
<td>0.41 0.48</td>
<td>0.39 0.46</td>
<td>0.36 0.37</td>
</tr>
<tr>
<td>DINING CRYPTOGRAPHERS</td>
<td>0.70 0.77</td>
<td>0.68 0.85</td>
<td>0.70 0.77</td>
<td>0.69 0.74</td>
<td>0.69 1.10</td>
</tr>
<tr>
<td>MUDDY-CHILDREN</td>
<td>0.36 0.44</td>
<td>0.36 0.40</td>
<td>0.36 0.42</td>
<td>0.36 0.42</td>
<td>0.36 0.42</td>
</tr>
<tr>
<td>SIMPLE-CARD-GAME</td>
<td>0.35 0.38</td>
<td>0.35 0.37</td>
<td>0.35 0.38</td>
<td>0.35 0.38</td>
<td>0.34 0.35</td>
</tr>
<tr>
<td>SOFTWARE-DEVELOPMENT</td>
<td>0.35 0.41</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>STRONGLY-CONNECTED</td>
<td>0.35 0.41</td>
<td>0.34 0.37</td>
<td>0.35 0.37</td>
<td>0.37 0.40</td>
<td>0.34 0.38</td>
</tr>
<tr>
<td>TIANJI-HORSE-RACING-GAME</td>
<td>0.38 0.45</td>
<td>0.37 0.39</td>
<td>0.37 0.40</td>
<td>0.37 0.40</td>
<td>0.37 0.42</td>
</tr>
</tbody>
</table>

### Conclusion
Starting with the seminal work on ATL*, the past decade has seen immense progress in (temporal) logic-based frameworks that provide rigorous and formal guarantees in MASs. Thus far, most logics focus on a purely path-based view where we reason about the strategic (in)ability of agents. However, many important properties require reasoning about multiple paths at the same time and investigating scenarios where agents share strategies. We have presented HyperATL, a powerful logic that bridges this gap. Our logic: (1) can express many important properties such as optimality requirements, OD, and GE; (2) admits decidable model checking; and (3) can be checked fully automatically using HyMASMC.

For the future, it is interesting to cast even more properties in a unified framework using (hyper)logics such as HyperATL and explore even more scalable verification approaches for HyperATL using, e.g., symbolic techniques.
Acknowledgments
This work was supported by the European Research Council (ERC) Grant HYPER (101055412), and by the German Research Foundation (DFG) as part of TRR 248 (389792660).

References
Emerson, E. A.; and Halpern, J. Y. 1986. ”Sometimes” and ”Not Never” revisited: on branching versus linear time temporal logic. J. ACM.
In International Joint Conference on Artificial Intelligence, IJCAI 2021.


Shapley, L. S. 1953. A Value for n-Person Games.


