Automata, Games & Verification

Summary #5

No seminar today. **:-(**
Synthesis of asynchronous systems postponed to July 3, 2008.

Determinization

Theorem 1. [McNaughton's Theorem (1966)] *Every Büchi recognizable language is recognizable by a deterministic Muller automaton.*

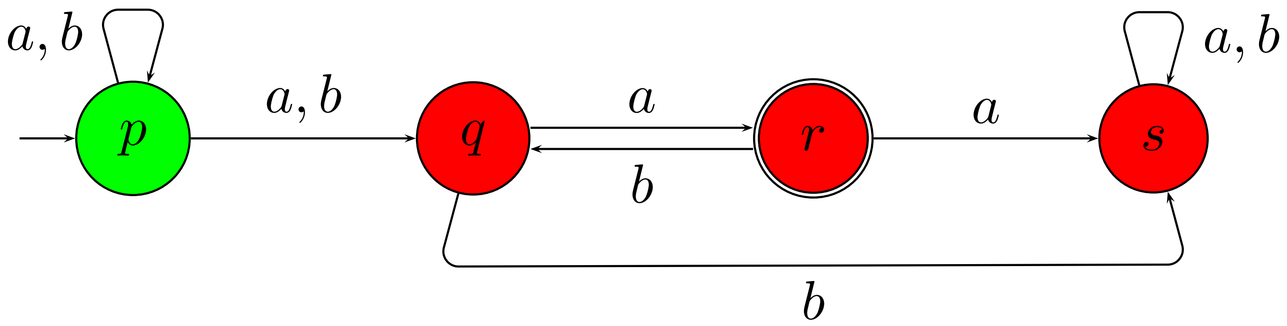
Definition 1. *A Büchi automaton (S, I, T, F) is called semi-deterministic if $S = N \uplus D$ is a partition of S, $F \subseteq D$ and $(D, \{d\}, T, F)$ is deterministic for every $d \in D$.*

Lemma 1.

For every Büchi automaton $\mathcal{A}$ there exists a semi-deterministic Büchi automaton $\mathcal{A}'$ with $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}')$.

Lemma 2.

For every semi-deterministic Büchi automaton $\mathcal{A}$ there exists a deterministic Muller automaton $\mathcal{A}'$ with $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}')$.


a, b
a, b
a
a
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p
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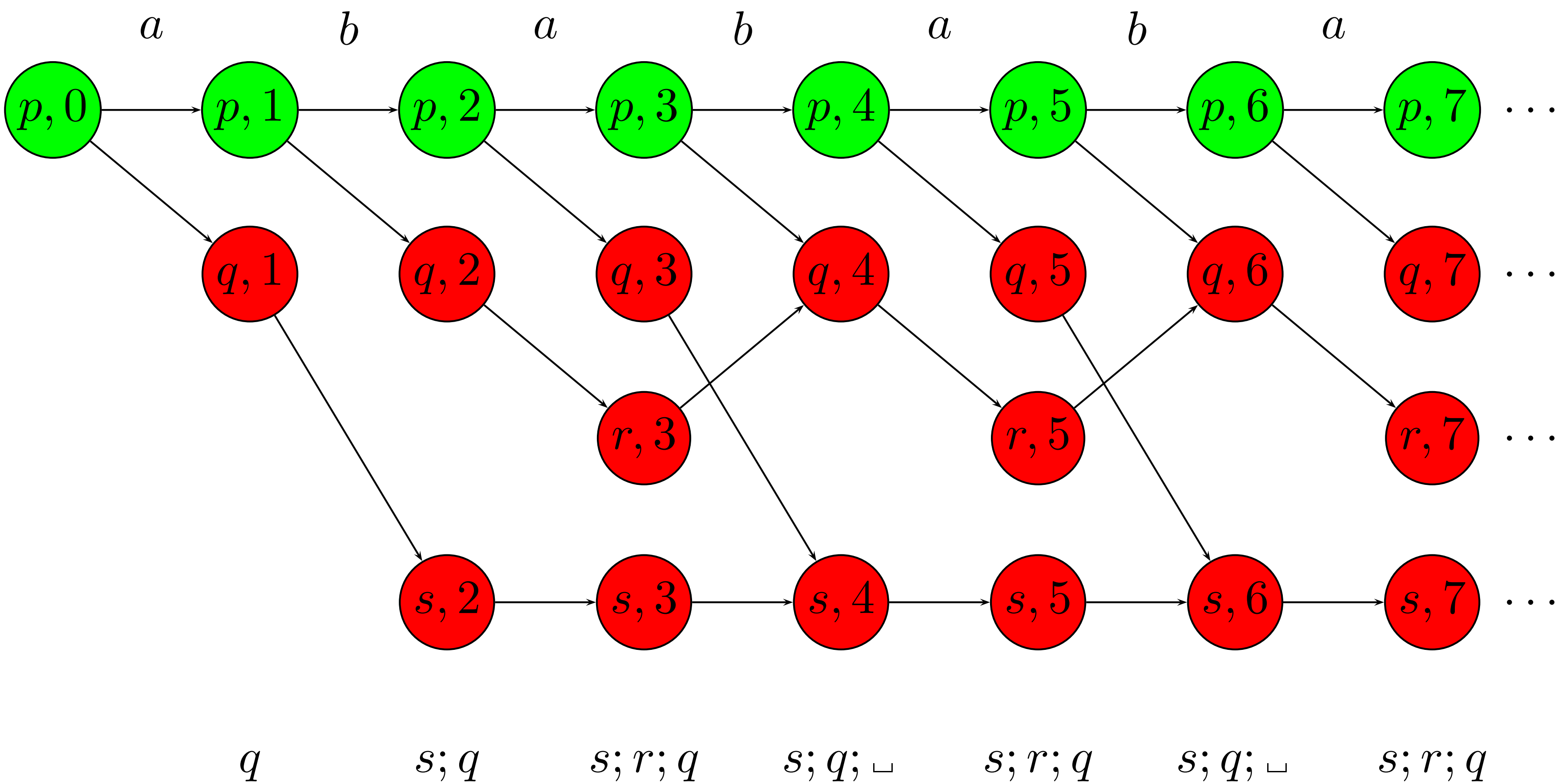
b


a b a b a b a
p, 0
p, 1
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p, 5
p, 6
p, 7
q, 1
q, 2
q, 3
q, 4
q, 5
q, 6
q, 7
r, 3
r, 5
r, 7
s, 2
s, 3
s, 4
s, 5
s, 6
s, 7
. . .
q
s; q
s; r; q
s; q; ␣
s; r; q
s; q; ␣
s; r; q

From SDBA $\mathcal{A} = (N \uplus D, I, T, F)$ to DMA $(S', \{s'_0\}, T', \mathcal{F})$

- $S' = 2^N \times \{0, \ldots, 2d\} \to D \cup \{\llcorner\!\lrcorner\}$, where $d = |D|$;
- $s'_0 = (\{N \cap I\}, (d_1, d_2, \ldots, d_n, \llcorner\!\lrcorner, \ldots, \llcorner\!\lrcorner))$,
  where $d_i < d_{i+1}, \{d_1, \ldots, d_n\} = D \cap I$;
- $T' = \{((N_1, f_1), \sigma, (N_2, f_2)) \mid N_2 = pr_3(T \cap N_1 \times \{\sigma\} \times N)$,
  $D' = pr_3(T \cap N_1 \times \{\sigma\} \times D)$,
  $g_1 : n \mapsto d_2 \in D \Leftrightarrow f_1 : n \mapsto d_1 \in D \wedge d_1 \to^\sigma d_2$,
  g_2: insort the elements of D' in the empty slots of g_1 (using $<$),
  f_2: delete every recurrance (leaving an empty slot) $\}$;
- $\mathcal{F} = \{F' \subseteq S' \mid \exists i \in 1, \ldots, 2d$ s.t.
  $f(i) \neq \llcorner\!\lrcorner$ for all $(N', f) \in F'$ and
  $f(i) \in F$ for some $(N', f) \in F'\}$.