Automata, Games & Verification

Summary #6

Today at 4:00pm in SR 014
Games in Verification and Synthesis
Andreas Augustin: *Synthesis under Incomplete Information*
Michael Maurer: *Simulation Games*

Linear-Time Temporal Logic (LTL)

Syntax:

- Given a set of atomic propositions *AP*.
- Any atomic proposition $p \in AP$ is an LTL formula
- If φ, ψ are LTL formulars then so are
  - $\neg\varphi,\ \varphi \wedge \phi,$
  - $\bigcirc\varphi,\ \varphi\,\mathcal{U}\,\psi$

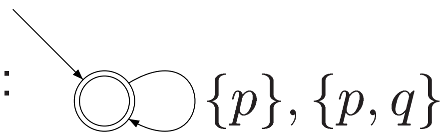
Abbreviations:

$\Diamond\,\varphi \equiv \text{true}\,\mathcal{U}\,\varphi;$
$\Box\,\varphi \equiv \neg(\Diamond\,\neg\varphi);$
$\varphi\,\mathcal{W}\,\psi \equiv (\varphi\,\mathcal{U}\,\psi) \vee \Box\,\varphi;$

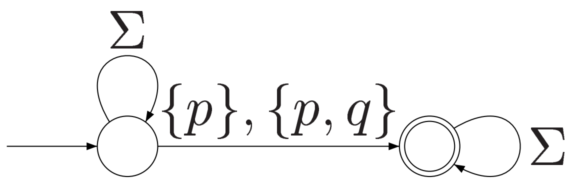
Semantics:

- $\alpha, i \vDash p$ if $p \in \alpha(i)$;
- $\alpha, i \vDash \neg\varphi$ if $\alpha, i \nvDash \varphi$;
  $\alpha, i \vDash \varphi \wedge \psi$ if $\alpha, i \vDash \varphi$ and $\alpha, i \vDash \psi$;
- $\alpha, i \vDash \bigcirc\varphi$ if $\alpha, i+1 \vDash \varphi$
- $\alpha, i \vDash \varphi\, \mathcal{U}\, \psi$ if there is some $j \geq i$ s.t.
  $\alpha, j \vDash \psi$ and for all $i \leq k < j$: $\alpha, k \vDash \varphi$

$\Box\, p$:



$\Diamond\, p$:



Examples:

- Invariant: $\Box\, p$
- Guarantee: $\Diamond\, p$
- Recurrence: $\Box \Diamond\, p$
- Request-Response: $\Box\, (p \rightarrow \Diamond\, q)$
- Until-Property: $\Box\, (p \rightarrow \bigcirc(q\, \mathcal{U}\, p))$
- Fairness: $(\Box \Diamond\, p) \rightarrow (\Box \Diamond\, q)$

Definition 1.

- *models*(φ)=$\{\alpha \in (2^{AP})^w \mid \alpha \models \varphi\}$
- *an LTL formula* φ *is* satisfiable *if models*$(\varphi) \neq \emptyset$
- *an LTL formula* φ *is* valid *if models*$(\varphi) = (2^{AP})^\omega$

There are Büchi-recognizable languages that are not LTL-definable!
Example: $(\emptyset\emptyset)^*\{p\}^\omega$

Definition 2. *A language $L \subseteq \Sigma^\omega$ is non-counting iff*
$\exists n_0 \in \omega \;.\; \forall n \geqslant n_0 \;.\; \forall u, v \in \Sigma^*, \gamma \in \Sigma^\omega \;.$
$uv^n\gamma \in L \;\Leftrightarrow\; uv^{n+1}\gamma \in L$

Example: $L = (\varnothing\varnothing)^*\{p\}^\omega$ is counting.
For every $\varnothing^n\{p\}^\omega \in L$, $\varnothing^{n+1}\{p\}^\omega \notin L$.

Theorem 1. *For every LTL-formula φ, models(φ) is non-counting.*