Automata, Games & Verification

Summary #7

Final Exam:
Friday, July 25, 14:00 – 16:00

Today at 4:00pm in SR 014
Games in Verification and Synthesis
Maël Hörz: *Module Checking*
Fabienne Eigner: *Alternating-Time Model Checking*

LTL

Syntax:

- Given a set of atomic propositions *AP*.
- Any atomic proposition $p \in AP$ is an LTL formula
- If φ, ψ are LTL formulars then so are
  - $\neg\varphi,\ \varphi \wedge \phi,$
  - $\bigcirc\varphi,\ \varphi\,\mathcal{U}\,\psi$

Abbreviations:

$\Diamond\,\varphi \ \equiv \text{true}\,\mathcal{U}\,\varphi;$
$\Box\,\varphi \ \equiv\ \neg(\Diamond\,\neg\varphi);$
$\varphi\,\mathcal{W}\,\psi \ \equiv\ (\varphi\,\mathcal{U}\,\psi) \vee \Box\,\varphi;$

QPTL

Syntax: LTL formula | $\varphi \wedge \varphi$ | $\neg\varphi$ | $\exists p.\ \varphi$

Semantics:

$\alpha, i \models \exists q.\varphi$ iff there is an α' with
$\alpha'(j) \cap (AP \setminus \{q\}) = \alpha(j) \cap (AP \setminus \{q\})$ for all $j \in \omega$,
s.t. $\alpha', i \models \varphi$.

Theorem 1. *For every Büchi automaton $\mathcal{A}$ over $\Sigma = 2^{AP}$ there exists a QPTL formula φ such that $models(\varphi) = \mathcal{L}(\mathcal{A})$.*

S1S

Syntax:

- first-order variable set $V_1 = \{x, y, \ldots\}$
- second-order variable set $V_2 = \{X, Y, \ldots\}$
- Terms t:

$$t ::= 0 \mid x \mid S(t)$$

- Formulas φ:

$$\varphi ::= t \in X \mid t_1 = t_2 \mid \neg\varphi \mid \varphi_0 \vee \varphi_1 \mid \exists x.\varphi \mid \exists X.\varphi$$

Examples:

$$
\begin{aligned}
X \subseteq Y &:\equiv \forall z.\ (z \in X \rightarrow z \in Y);\\
X = Y &:\equiv X \subseteq Y \wedge Y \subseteq X;\\
\mathit{Suff}(X) &:\equiv \forall y.\ (y \in X \rightarrow \mathsf{S}(y) \in X);\\
x \leqslant y &:\equiv \forall Z.\ (x \in Z \wedge \mathit{Suff}(Z)) \rightarrow y \in Z;\\
\mathit{Fin}(X) &:\equiv \exists Y.\ (X \subseteq Y \wedge \exists z.\ z \notin Y \wedge \forall z.\ (z \notin Y \rightarrow \mathsf{S}(z) \notin Y));
\end{aligned}
$$

Theorem 2. *Every QPTL-definable language is S1S-definable.*

Theorem 3. *Every S1S-definable language is Büchi-recognizable.*

Hence:

$$\text{LTL} \subsetneq \text{QPTL} \subseteq \text{S1S} \subseteq \text{Büchi} \subseteq \text{QPTL}.$$