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Problem Set 1
Due: 21st of April 2011

Automata, Games, and Verification

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1. **Concurrent processes & verification** (tutorial A: group G01, tutorial B: group G02)

   Consider the following set of concurrent processes, communicating using the shared variables t_0 and t_1:

   local t_0, t_1 : boolean where initially $t_0 = 0, t_1 = 0$

   P_0 :: [loop forever do [001 : stuff; 010 : $t_0 := 1$; 011 : await $t_0 = 1 \wedge t_1 = 1$; 100 : synchronized stuff; 101 : $t_0 := 0$; 110 : await $t_0 = 0 \wedge t_1 = 0$;]] || P_1 :: [loop forever do [001 : stuff; 010 : $t_1 := 1$; 011 : await $t_0 = 1 \wedge t_1 = 1$; 100 : synchronized stuff; 101 : $t_1 := 0$; 110 : await $t_0 = 0 \wedge t_1 = 0$;]]

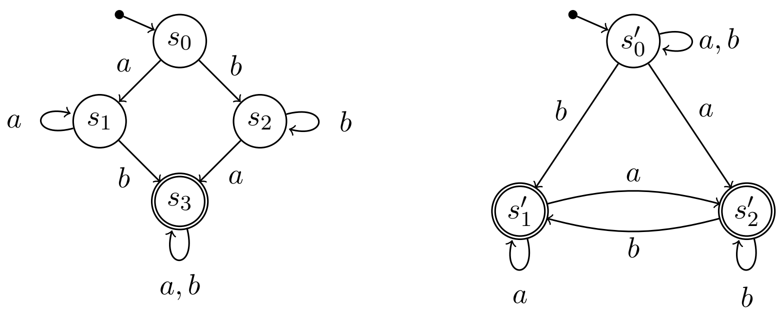
P_0 :: loop forever do		P_1 :: loop forever do	
001 :	stuff;	001 :	stuff;
010 :	$t_0 := 1$;	010 :	$t_1 := 1$;
011 :	await $t_0 = 1 \wedge t_1 = 1$;	011 :	await $t_0 = 1 \wedge t_1 = 1$;
100 :	synchronized stuff;	100 :	synchronized stuff;
101 :	$t_0 := 0$;	101 :	$t_1 := 0$;
110 :	await $t_0 = 0 \wedge t_1 = 0$;	110 :	await $t_0 = 0 \wedge t_1 = 0$;

   a) How many state bits do you need to represent the states of this system?

   b) Reason informally why whenever process P_0 is in location 100, process P_1 can only be in one of the locations 011,100, 101 or 110.

2. **Büchi automata** (tutorial A: group G03, tutorial B: group G04)

   Consider the following nondeterministic Büchi automata over $\Sigma = \{a, b\}$:



   a) Which of the automata are deterministic? Which are complete?

   b) For each of the automata, check if the following words are accepted. If yes, write down an accepting run.

      i. aab^ω

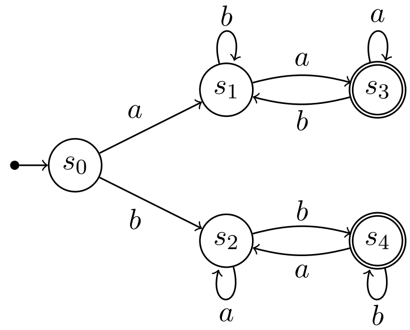
      ii. a^ω

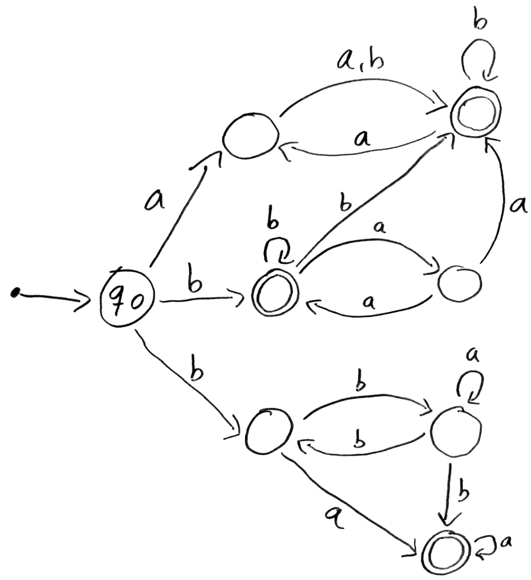
      iii. $(ab)^\omega$

   c) Do the two automata have the same language? Justify your answer informally.

3. **Büchi automata and non-accepting words** (tutorial A: group G05, tutorial B: group G06)

   For each of the following automata, find out whether there exist words that are not accepted by them.





   In case of a positive answer (there is a non-accepted word), state the word and reason informally why it is not accepted. In case of a negative answer, reason informally why there is no word that is not accepted.

4. **Büchi automata** (tutorial A: group G07, tutorial B: group G08)

   Build complete Büchi automata for each of the following ω-languages with alphabet $\Sigma = \{a, b\}$:

   a) $L_1 = \{\alpha \in \Sigma^\omega \mid$ each occurence of a in α is followed immediately by a $b\}$
   b) $L_2 = \{\alpha \in \Sigma^\omega \mid$ the letter a occurs infinitely often in $\alpha\}$
   c) $L_3 = \{\alpha \in \Sigma^\omega \mid$ the letter b occurs finitely often in $\alpha\}$
   d) $L_4 = L_1 \cap L_2$
   e) $L_5 = L_2 \cup L_3$
   f) $L_6 = L_1 \cap L_2 \cap L_3$

5. **Selection on Büchi automata (Challenge)**

   Given a Büchi recognizable language L_{pick} over alphabet $\{1, 2\}$ and two Büchi recognizable languages L_1, L_2 over alphabet Σ, show that the following language L_{choose} (over the alphabet Σ) is also Büchi recognizable:

$$\begin{aligned} L_{choose} \;=\; & \{\delta_0\delta_1\delta_2 \ldots \in \Sigma^\omega \mid \\ & \text{there exists } \sigma_0^1\sigma_1^1\sigma_2^1 \ldots \in L_1, \sigma_0^2\sigma_1^2\sigma_2^2 \ldots \in L_2, \gamma_0\gamma_1\gamma_2 \ldots \in L_{pick} \text{ such that} \\ & \text{for all } i \in \mathbb{N}_0, \delta_i = \sigma_i^{\gamma_i}\} \end{aligned}$$