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Problem Set 13
Summer term 2011
Due: July 21, 2011

Automata, Games, and Verification

Important note

As the tutorial on the 21st of July is the last one in this semester, we will have a joint question & answer session in addition to the discussion of the following two problems. To aid this, the tutorials will be merged for the very last session. The joint tutorial will take place in the seminar room 0.01, building E 2 1 (bioinformatics).

Also, if you have any questions or wish to request additional explanation to some of the topics, please drop your tutor an e-mail (preferably at least two days before the tutorial) so that an answer can be prepared. The end-of-semester exam will take place on Tuesday, the 26th of July, 9am-12 noon, Hörsaal 2, building E1.3.

Problems

1. **Alternating tree automata - part one** (group G01)

   Describe alternating parity tree automata for the following tree languages:

   a) $L_1 = \{(T, \tau) \mid T \subseteq \{0, \ldots, 3\}^*, \tau : T \rightarrow 2^{\{a,b,c\}}$, whenever for a tree node t in (T, τ) we have $c \in \tau(t)$, then (1) there exists a branch in the tree on which a is contained infinitely often in the label of the nodes and the branch contains t, and (2) there exists a branch in the tree on which b is contained only finitely often in the label of the nodes and the branch contains $t\}$.

   b) $L_2 = \{(T, \tau) \mid T \subseteq \{0, \ldots, 1\}^*, \tau : T \rightarrow \{a, b, c\}$, for every node t in the tree and $x \in \{a, b, c\}$, if there is some $t' \in \{0, 1\}^*$ with $t1t' \in T$ and $\tau(t1t') = x$, then there also exists some $t'' \in \{0, 1\}^*$ with $t0t'' \in T$ and $\tau(t0t'') = x\}$

2. **Alternating tree automata - part two** (group G10)

   Let a deterministic parity word automaton $\mathcal{A} = (S, I, T, c)$ over some alphabet Σ be given, and let $k = |\Sigma|$. Take for granted that all words in the language of $\mathcal{A}$ start with the letter $a \in \Sigma$. Construct an alternating parity tree automaton over Σ-labeled trees that accepts precisely the trees over the set of directions $D = \{0, \ldots, k - 1\}$ for which the set of its infinite branches represents (by their label sequences) precisely the set of words accepted by $\mathcal{A}$.

   More formally, we search for an alternating tree automaton $\mathcal{A}'$ over the set of directions $D = \{0, \ldots, k - 1\}$ such that $\mathcal{A}'$ accepts precisely the Σ-labeled D-trees (T, τ) for which $\{\tau(\epsilon)\tau(t_0)\tau(t_0t_1)\tau(t_0t_1t_2)\ldots \mid t_0t_1t_2\ldots \in D^\omega \wedge \forall i \in \mathbb{N} : t_0t_1\ldots t_i \in T\}$ is the set of words accepted by $\mathcal{A}$.

   Provide a procedure to construct such an automaton $\mathcal{A}'$ from $\mathcal{A}$. Is it possible that $\mathcal{A}'$ has an empty language even though $\mathcal{A}$ does not?