Bernd Finkbeiner
Rüdiger Ehlers
Andrey Kupriyanov

Problem Set 3
Summer term 2011
Due: May 5, 2011

Automata, Games, and Verification

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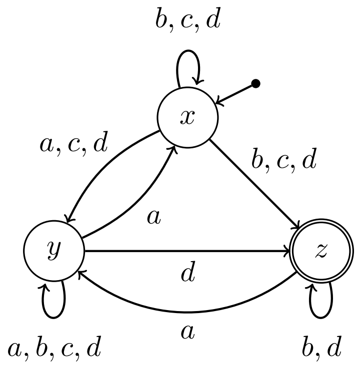
1. **Run DAGs** (tutorial A: group G15, tutorial B: group G16)

   Let $\Sigma = \{a, b, c, d\}$ be an alphabet, $w = ddbac^\omega$ be a word over this alphabet, and $\mathcal{A}$ be the following Büchi automaton over Σ having the states $\{x, y, z\}$:

   a) Draw the run DAG for $\mathcal{A}$ on w. As the DAG is infinite, you only need to sketch it in a way such that it is, intuitively, clear how it is to be continued after a certain pattern emerges.

   b) Reason whether w is accepted by $\mathcal{A}$.

   c) Finally, write down the sequence of DAGs $G_0 \supseteq G_1 \supseteq G_2 \ldots$ as defined in the proof of Lemma 1 of Section 5 of the lecture.

2. **Strictly Büchi Recognizable Languages** (tutorial A: group G17, tutorial B: group G06)

   A *strict Büchi* automaton $\mathcal{A} = (S, I, T, F)$ is the same as a Büchi automaton except that the definition of an accepting run is changed as follows:

   A run r for $\alpha \in \Sigma^\omega$ is *accepting* on $\mathcal{A}$, when $\mathit{In}(r) = F$.

   Proof or give a counter example to the following statements:

   a) If L is recognizable by a strict Büchi automaton then L is Büchi-recognizable.

   b) If L is recognizable by a strict Büchi automaton then L is recognizable by a deterministic Büchi automaton.

   c) If L is Büchi-recognizable then L is strictly Büchi-recognizable.

   d) If L is recognizable by a deterministic Büchi automaton then L is strictly Büchi-recognizable.

3. **Co-Limit Operation** (tutorial A: group G01, tutorial B: group G08)

   The *co-limit* of W is defined as $\overleftarrow{W} = \{\alpha \in \Sigma^\omega \mid \text{there exist only finitely many } n \in \omega \text{ s.t. } \alpha(0, n) \in W\}$[1].

   Let $V, W \subseteq \Sigma^*$ be two regular languages. Prove or give a counter example to the following statements:

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[1] For a finite word $\alpha \in \Sigma^*$ and two natural numbers $m, n \in \omega$ with $m \leq n$, $\alpha(m, n)$ denotes the substring from m to n: $\alpha(m, n) = \alpha(m)\,\alpha(m+1)\,\ldots\,\alpha(n)$.

a) $\overleftarrow{(V \cdot W)} = V \cdot \overleftarrow{W}$

b) $V \cdot \overleftarrow{W}$ is Büchi-recognizable

c) $V \cdot \overleftarrow{W}$ is recognizable by a deterministic Büchi automaton

4. **co-Büchi Automata** (tutorial A: group G05, tutorial B: group G10)

A co-Büchi automaton $\mathcal{A} = (S, I, T, F)$ is the same as a Büchi automaton except that the definition of an accepting run is changed as follows:

A run r for $\alpha \in \Sigma^\omega$ is *accepting* on $\mathcal{A}$, when $In(r) \cap F = \emptyset$.

Prove or give a counter example to the following statements:

a) co-Büchi automata are closed under $\cap$.

b) co-Büchi automata are closed under $\cup$.

c) co-Büchi automata are closed under pr_1.

5. **Complementation of Büchi automata via Büchi's Characterization Theorem (challenge problem)**

In this problem, we develop an alternative to the complementation construction from Lectures 3 and 4. Let $\mathcal{A}$ be a nondeterministic Büchi automaton over the alphabet Σ.

a) Show that Σ^ω can be represented as a finite union $\bigcup_{i=1,\ldots,n} U_i \cdot V_i^\omega$ such that

- for all $i = 1, \ldots, n$, U_i and V_i are regular languages $U_i, V_i \subseteq \Sigma^*$, and
- for all $i = 1, \ldots, n$, either $U_i \cdot V_i^\omega \cap \mathcal{L}(\mathcal{A}) = \emptyset$ or $U_i \cdot V_i^\omega \subseteq \mathcal{L}(\mathcal{A})$.

(Suggestion: For a finite word w, consider (1) the pairs of states of $\mathcal{A}$ that are connected by a path labeled with w, and (2) the pairs of states of $\mathcal{A}$ that are connected by a path that visits an accepting state and that is labeled with w. Let two finite words be equivalent if they agree on these pairs. Show that the equivalence classes can be represented as finite-word automata.)

b) Use Büchi's characterization theorem to argue that there exists a nondeterministic Büchi automaton $\mathcal{A}'$ such that $\mathcal{L}(\mathcal{A}') = \Sigma^\omega \setminus \mathcal{L}(\mathcal{A})$.

c) Prove or disprove the following claim for regular languages $U, V \subseteq \Sigma^*$:
$U, V \subseteq \Sigma^*$: $\Sigma^\omega \setminus (U \cdot V^\omega) = (\Sigma^* \setminus U) \cdot (\Sigma^* \setminus V)^\omega$