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Problem Set 6
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Automata, Games, and Verification

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1. **Language Emptiness** (tutorial A: group G01, tutorial B: group G12)

   An automaton is called *empty* if its language is empty.

   Describe a method to test language emptiness for

   - nondeterministic Büchi automata $\mathcal{B} = (S, I, T, F)$,
   - nondeterministic Rabin automata $\mathcal{R} = (S, I, T, \{(N_j, R_j) \mid j \in J\})$, and
   - nondeterministic Muller automata $\mathcal{M} = (S, I, T, \mathcal{F})$.

2. **Universal Automata** (tutorial A: group G07, tutorial B: group G04)

   A *universal* Büchi automaton $\mathcal{A} = (S, I, T, F)$ is defined as a nondeterministic Büchi automaton, with the exception that a universal automaton accepts a word $\alpha \in \Sigma^\omega$ iff *all* runs r of $\mathcal{A}$ on α are accepting.

   Compare the expressive power of deterministic, nondeterministic, and universal Büchi automata.

3. **LTL & Determininistic Automata** (tutorial A: group G05, tutorial B: group G16)

   - Compare the expressive power of linear-time temporal logic and deterministic Büchi automata.
   - Compare the expressive power of linear-time temporal logic and deterministic co-Büchi automata.

4. **Temporal Operators** (tutorial A: group G13, tutorial B: group G02)

   Show that $\{\neg, \wedge, \mathrm{X}, \mathcal{W}\}$ is an operator basis for LTL, i.e., that you can express every LTL formula ψ as an equivalent LTL formula ψ', in which apart from atomic propositions, only the operators $\neg$, $\wedge$, X and $\mathcal{W}$ are used.

5. **Temporal Operators** (challenge problem)

   Show that $\{\neg, \wedge, \mathrm{X}, \mathrm{F}, \mathrm{G}\}$ is *not* an operator basis for LTL.

   Suggestion: Find two families of label sequences u_n, v_n, and an LTL formula φ, such that

   - for every n, $u_n \vDash \varphi$ and $v_n \nvDash \varphi$,
   - but for every LTL formula ψ *without* Until, there is an n, such that
     either $u_n \vDash \psi$ and $v_n \vDash \psi$, or $u_n \nvDash \psi$ and $v_n \nvDash \psi$.