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Problem Set 7
Summer term 2011
Due: May 31/June 9, 2011

Automata, Games, and Verification

Note: Due to the public holiday on the 2nd of June, there will be no tutorial next week. The solution of problem 1 will be presented in the lecture on the 31st of May. The second problem will be discussed in the tutorial on the 9th of June.

1. **S1S and LTL**

   Decide for each of the languages over $2^{\{p,q\}}$ described below if they can be defined in S1S and/or LTL. Justify your answer in each case by either providing a formula or an argument why the language is not definable.

   a) $L_1 = \{\alpha \mid p \in \alpha(0), p \notin \alpha(i) \text{ for all } i \geq 1\}$;

   b) $L_2 = \{\alpha \mid p \in \alpha(i) \text{ for exactly two different } i \in \omega\}$;

   c) $L_3 = \{\alpha \mid |\{i \in \omega \mid p \in \alpha(i)\}| \text{ is finite and even}\}$;

   d) $L_4 = \{\alpha \mid |\{i \in \omega \mid p \in \alpha(i)\}| \text{ and } |\{i \in \omega \mid q \in \alpha(i)\}| \text{ are finite and equal}\}$.

2. **S1S and LTL** (tutorial A: group G03, tutorial B: group G06)

   Let $L \subseteq (2^{\mathsf{AP}})^\omega$ be an LTL-definable language and let $\mathsf{AP}' \subsetneq \mathsf{AP}$ be a strict subset of AP. Prove or give a counter example to the following statements:

   a) The (weak) projection $L_w = \{\sigma' \in (2^{\mathsf{AP}'})^\omega \mid \exists \sigma \in L \, \forall i \in \omega . \, \sigma'(i) = \sigma(i) \cap \mathsf{AP}'\}$ of L is LTL-definable.

   b) The (weak) projection L_w of L is S1S-definable.

   c) The strong projection $L_s = \{\sigma' \in (2^{\mathsf{AP}'})^\omega \mid \forall \sigma \in (2^{\mathsf{AP}})^\omega . \, (\forall i \in \omega . \, \sigma'(i) = \sigma(i) \cap \mathsf{AP}') \rightarrow \sigma \in L\}$ of L is LTL-definable.

   d) The strong projection L_s of L is S1S-definable.