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Problem Set 9
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Automata, Games, and Verification

1. **Index Appearance Record (IAR)** (tutorial A: group G01, tutorial B: group G04)

   Proof that the following construction transforms a deterministic Rabin automaton $\mathcal{A} = (S, \{s_0\}, T, \{(A_i, R_i) \mid i \in J\})$ into a deterministic parity automaton $\mathcal{A}' = (S', \{s'_0\}, T', c)$ with the same language:

   - $S' = S \times P \times \mathbb{N}_{2|J|+1}$, where P is the set of permutations over J
   - $s'_0 = (s_0, p_0, 1)$ where $p_0 \in P$ is an arbitrary (but fixed) permutation of J
   - $T' = \{((s, p, i), \sigma, (s', p', i')) \mid (s, \sigma, s') \in T$

     j is the maximal position in $p = i_1 i_2 i_3 \dots$ s.t. $s' \in A_{i_j} \cup R_{i_j}$

     (and 0 if no such index exists)

     $i' = 2j$ if $s \in A_{i_j} \setminus R_{i_j}$ and $i' = 2j + 1$ otherwise

     p' is obtained from $p = i_1 i_2 \dots$ by moving the indices i with

     $s' \in R_i$ to the front[1]$\}$
   - $c : (s, p, i) \mapsto i$

2. **LTL, QPTL & S1S** (tutorial A: group G05, tutorial B: group G06)

   Let $\mathsf{AP} = \{q, p, r\}$. Given some word $w = w_0 w_1 w_2 \dots \in (2^{\mathsf{AP}})^\omega$, for every $a \in \mathsf{AP}$, we denote $w|_a = (w_0 \cap \{a\})(w_1 \cap \{a\})(w_2 \cap \{a\}) \dots$ and $w(i, j) = w_i w_{i+1} \dots w_j$ for every $i, j \in \mathbb{N}$ with $i \leq j$.

   Given some finite word $w = w_0 w_1 \dots w_n$, we define $f : (2^{\mathsf{AP}}) \to \mathbb{N}$ to denote the number represented by w in binary (with the least significant bit first), where we treat the letter $\emptyset$ as 0 and every other letter in 2^{AP} as 1, i.e., $f(\epsilon) = 0$ and:

   $$f(w_0 w_1 \dots w_n) = \begin{cases} f(w(1, n)) \cdot 2 & \text{if } w_0 = \emptyset \\ f(w(1, n)) \cdot 2 + 1 & \text{if } w_0 \neq \emptyset \end{cases}$$

   Represent the following language L as LTL, QPTL and S1S formulas. You do not need to use the translation algorithms from the lecture and may rather write down the LTL, QPTL and S1S equivalents of the language directly.

   $$L = \{w \in (2^{\mathsf{AP}})^\omega \mid \forall j \in \mathbb{N} : f(w|_r(0, j)) = f(w|_p(0, j)) + f(w|_q(0, j))\}$$

[1] An index $k \in J$ appears earlier than an index $l \in J$ in p' iff $s' \in R_k \setminus R_l$ or k appears earlier than l in p and $s' \in R_k \leftrightarrow s' \in R_l$.

3. **Alternating Parity Automata** (tutorial A: group G07, tutorial B: group G12)

   Let $\mathcal{P}_1 = (Q_1, q_0^1, \delta_1, \alpha_1)$ and $\mathcal{P}_2(Q_2, q_0^2, \delta_2, \alpha_2)$ with disjoint sets $Q_1 \cap Q_2 = \emptyset$ of states be two alternating parity automata. Prove or give a counter-example for the general correctness of the following statements:

   a) The language $\mathcal{L}(\mathcal{P}_1) \cup \mathcal{L}(\mathcal{P}_2)$ is recognizable by an alternating parity automaton.

   b) The language $\mathcal{L}(\mathcal{P}_1) \cup \mathcal{L}(\mathcal{P}_2)$ is recognizable by an alternating parity automaton linear in the size of $\mathcal{P}_1$ and $\mathcal{P}_2$.

   c) The language $\mathcal{L}(\mathcal{P}_1) \cap \mathcal{L}(\mathcal{P}_2)$ is recognizable by an alternating parity automaton.

   d) The language $\mathcal{L}(\mathcal{P}_1) \cap \mathcal{L}(\mathcal{P}_2)$ is recognizable by an alternating parity automaton linear in the size of $\mathcal{P}_1$ and $\mathcal{P}_2$.

   e) The language $\overline{\mathcal{L}(\mathcal{P}_1)}$ is recognizable by an alternating parity automaton.

   f) The language $\overline{\mathcal{L}(\mathcal{P}_1)}$ is recognizable by an alternating parity automaton linear in the size of $\mathcal{P}_1$.

4. **Alternating Büchi vs. Co-Büchi Automata** (challenge problem)

   Prove or give a counter example to the following statement: An ω-language L is recognized by some alternating Büchi automaton iff L is recognized by some alternating co-Büchi automaton. (Hint: Use the results of Problem 3.)