Automata, Games & Verification

Summary #11

Today at 2:15pm in SR 016

Seminar "Games, Synthesis, and Robotics"

A new representation and associated algorithms for generalized planning

Nondeterministic Tree Automata

Definition 1. *A nondeterministic tree automaton (over binary Σ-trees) $\mathcal{A} = (S, s_0, M, \varphi)$ consists of*

- S*: finite set of states;*
- $s_0 \in S$*;*
- $M = S \times \Sigma \times S \times S$*;*
- φ*: acceptance condition (Büchi, parity, . . .).*

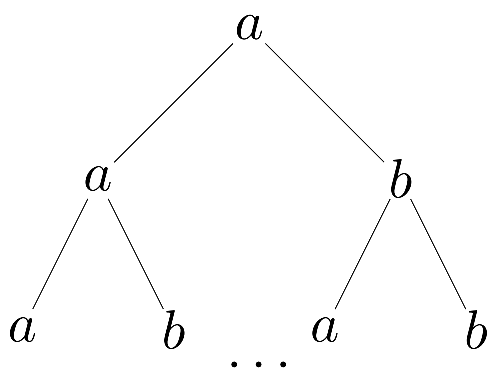
Definition 2. *A run of a nondeterministic tree automaton $\mathcal{A}$ on a Σ-tree v is a S-tree (T, r), s.t.*

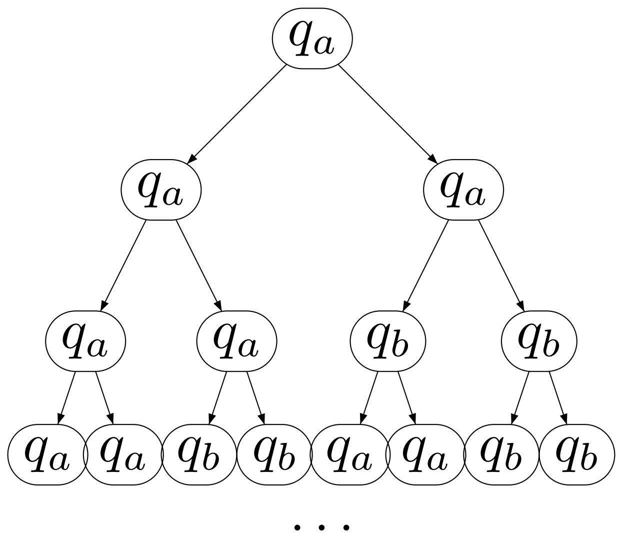
- $r(\epsilon) = s_0$*;*
- $(r(q), v(q), r(q0), r(q1)) \in M$ *for all* $q \in \{0, 1\}^*$*;*

A run is accepting if every branch is accepting.

Example: $\{a, b\}$-trees with infinitely many bs on each path.

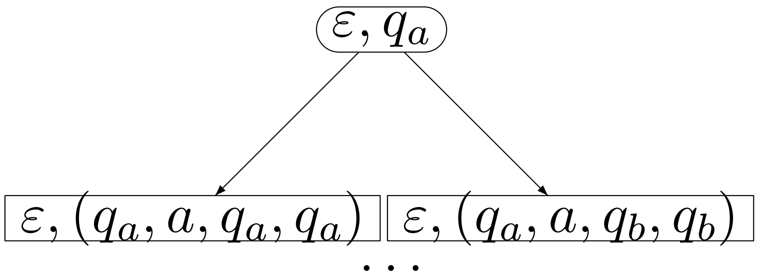
$\mathcal{A} = (S, s_0, M, c); \Sigma = \{a, b\};$
$S = \{q_a, q_b\}; s_0 = q_a;$
$M = \{(q_a, a, q_a, q_a), (q_b, a, q_a, q_a), (q_a, b, q_b, q_b), (q_a, a, q_b, q_b), \ldots\};$
Büchi $F = \{q_b\}$.





Theorem 1. [Acceptance Game] *A parity tree automaton* $\mathcal{A} = (S, s_0, M, c)$ *accepts an input tree* t *iff Player 0 wins the parity game* $\mathcal{G}_{\mathcal{A},t} = (V_0, V_1, E, c')$ *from position* (ε, s_0)*.*

- $V_0 = \{(w, q) \mid w \in \{0,1\}^*, q \in S\}$;
- $V_1 = \{(w, \tau) \mid w \in \{0,1\}^*, \tau \in M\}$;
- $E = \{((w, q), (w, \tau)) \mid \tau = (q, t(w), q_0', q_1'), \tau \in M\}$
  $\cup \{((w, \tau), (w', q')) \mid \tau = (q, \sigma, q_0', q_1')$ *and*
  $((w' = w0$ *and* $q' = q_0')$ *or* $(w' = w1$ *and* $q' = q_1'))\}$;
- $c'(w, q) = c(q)$ *if* $q \in S$;
- $c'(w, \tau) = 0$ *if* $\tau \in M$.



Theorem 2. [Emptiness Game] *The language of a parity tree automaton* $\mathcal{A} = (S, s_0, M, c)$ *is non-empty iff Player 0 wins the parity game* $\mathcal{G}_{\mathcal{A},t} = (V_0, V_1, E, c')$ *from position* s_0.

$$\left.\begin{array}{l} \bullet\ V_0 = S; \\[1em] \bullet\ V_1 = M; \end{array}\right\} \leftarrow V \text{ is finite!}$$

- $E = \{(q, \tau) \mid \tau = (q, 1, q'_0, q'_1), \tau \in M\}$
  $\cup \{(\tau, q') \mid \tau = (q, 1, q'_0, q'_1)$ *and*
  $(q' = q'_0$ *or* $q' = q'_1)\}$;
- $c'(q) = c(q)$ *for* $q \in S$;
- $c(\tau) = 0$ *for* $\tau \in M$.

Theorem 3. *Büchi tree automata are strictly weaker than parity tree automata.*

recall from Lectures 2 & 3:

Theorem 4. *Deterministic Büchi word automata are structly weaker than nondeterministic Büchi word automata.*

Theorem 5. [Büchi's Characterization Theorem (1962)] *An ω-language is recognizable by a Büchi word automaton iff it is ω-regular.*