Automata, Games & Verification

Summary #12

Today at 2:15pm in SR 016

Seminar "Games, Synthesis, and Robotics"

Design and Synthesis of Synchronization Skeletons using Branching Time Temporal Logic

Complementation of Parity Tree Automata

Theorem 1. *For each parity tree automaton $\mathcal{A}$ over Σ there is a parity tree automaton $\mathcal{A}'$ with $\mathcal{L}(\mathcal{A}') = T_\Sigma - \mathcal{L}(\mathcal{A})$.*

$\mathcal{A}$ does not accept t iff

(1) there is a $(M \to \{0,1\})$-tree v such that

(2) for all $i_0, i_1, i_2, \ldots \in \{0,1\}^\omega$

(3) for all $\tau_0, \tau_1, \ldots \in M^\omega$

(4) <u>if</u>

- for all j,
  $\tau_j = (q, a, q'_0, q'_1)$
  $\Rightarrow a = t(i_0, i_1, \ldots, i_j)$ and
- $i_0 i_1 \ldots = v(\varepsilon)(\tau_0) v(i_0)(\tau_1) \ldots$

<u>then</u> the generated state sequence $q_0 q_1 \ldots$
with $q_0 = s_0$, $(q_j, a, q^0, q^1) = \tau_j$,
$q_{j+1} = q^{v(i_0,\ldots,i_{j-1})(\tau_j)}$
violates c.

Monadic Second-Order Theory of Two Successors (S2S)

Syntax:

- first-order variable set $V_1 = \{x_0, x_1, \ldots\}$
- second-order variable set $V_2 = \{X_0, X_1, \ldots\}$
- Terms: $t ::= \epsilon \mid x \mid t0 \mid t1$
- Formulas $\varphi ::= t \in X \mid t_1 = t_2 \mid \neg\varphi \mid \varphi_0 \vee \varphi_1 \mid \exists x.\varphi \mid \exists X.\varphi$

Semantics:

- first-order valuation $\sigma_1 : V_1 \rightarrow \mathbb{B}^*$
- second-order valuation $\sigma_2 : V_2 \rightarrow 2^{\mathbb{B}^*}$
- terms: $[\![\epsilon]\!] = \epsilon$, $[\![t0]\!]_{\sigma_1} = [\![t]\!]_{\sigma_1} 0$, etc.
- formulas: $\sigma_1, \sigma_2 \models \exists x_i.\varphi$ iff there is a $a \in \mathbb{B}^*$ s.t.

$$\sigma_1'(y) = \begin{cases} \sigma_1(y) & \text{if } x \neq y, \\ a & \text{otherwise;} \end{cases} \qquad \text{and } \sigma_1', \sigma_2 \models \varphi$$

etc.

Theorem 2. *For each Muller tree automaton* $\mathcal{A} = (S, s_0, M, \mathcal{F})$ *over* $\Sigma = 2^{V_2}$ *there is a S2S formula* φ *over* V_2 *s.t.* $t \in \mathcal{L}(\mathcal{A})$ *iff* $\sigma_2 \models \varphi$ *where* $\sigma_2(P) = \{q \in \{0,1\}^* \mid P \in t(q)\}$.

Theorem 3. *For every S2S formula* φ *over* V_1, V_2 *there is a Muller tree automaton* $\mathcal{A}$ *over* $\Sigma = 2^{V_1 \cup V_2}$ *such that* $t \in \mathcal{L}(\mathcal{A})$ *iff* $\sigma_1, \sigma_2 \models \varphi$ *where*

$$\begin{aligned} \sigma_1(x) &= q \text{ iff } x \in t(q); \\ \sigma_2(X) &= \{q \in \{0,1\}^* \mid X \in t(q)\}. \end{aligned}$$

Corollary 1.

S2S is decidable.

- SnS is the monadic second order theory of n successors.

  Corollary 2.

  SnS is decidable.

- SωS is the monadic second order theory of ω successors.

  Theorem 4. *SωS is decidable.*

- WS2S is the weak monadic second order theory of two successors.

  $\sigma_1, \sigma_2 \models \exists X.\varphi$ iff there is a **finite** $A \subseteq \mathbb{B}^*$ s.t.

$$\sigma_2'(Y) = \begin{cases} \sigma_2(Y) & \text{if } X \neq Y \\ A & \text{otherwise} \end{cases} \qquad \text{and} \qquad \sigma_1, \sigma_2' \models \varphi.$$

Corollary 3.

WS2S is decidable.

Theorem 5. *For a language $L \subseteq T_\Sigma$, the following are equivalent:*

1. *Both L and its complement are recognizable by a Büchi tree automaton.*
2. *L is WS2S-definable.*

Corollary 4.

WS2S is strictly weaker than S2S.