Automata, Games & Verification

Summary #14

Tuesday next week

End-of-term exam

July 26, 9:00–12:00, E1.3 / HS 002. Start: 9:15 End: 11:45 (2.5 hours)
open book (no computers)

If you wish to take the end-of-semester exam (Sept 27),
please register by email within two weeks after the end-of-term exam.
(The best grade counts.)

Alternating tree automata

Definition 1. *An alternating tree automaton over Σ-trees is a tuple $\mathcal{A} = (S, s_0, \delta, \varphi)$:*

- S*: finite set of states*
- $s_0 \in S$
- $\delta : S \times \Sigma \times \{1, \ldots k\} \rightarrow \mathbb{B}^+(\{0, 1, \ldots k-1\} \times S)$ *is the transition function.*
- φ*: acceptance condition (Büchi, parity, ...)*

CTL

- S =closure(φ):= set of all subformulas and their negations
- $\delta(p, \sigma, k) = \mathit{true}$ if $p \in \sigma$
- $\delta(\varphi \wedge \psi, \sigma, k) = \delta(\varphi, \sigma, k) \wedge \delta(\psi, \sigma, k)$
- $\delta(\mathsf{AX}\varphi, \sigma, k) = \bigwedge_{c=0}^{k-1}(c, \varphi)$
- $\delta(\mathsf{EX}\varphi, \sigma, k) = \bigvee_{c=0}^{k-1}(c, \varphi)$
- $\delta(\mathsf{A}\varphi\ \mathcal{U}\ \psi, \sigma, k) = \delta(\psi, \sigma, k) \vee (\delta(\varphi, \sigma, k) \wedge \bigwedge_{c=0}^{k-1}(c, \mathsf{A}\varphi\ \mathcal{U}\ \psi)$
- $\delta(\neg\varphi, \sigma, k) = \overline{\delta(\varphi, \sigma, k)}$

μ-calculus

Theorem 1. *For every μ calculus formula in normal form there is an equivalent guarded formula.*

Closure $cl(\varphi)$ of a μ-calculus formula φ:

- $\varphi \in cl(\varphi)$
- if $\psi \vee \eta \in cl(\varphi)$ then $\psi, \eta \in cl(\varphi)$
- if $\Diamond\psi \in cl(\varphi)$ then $\psi \in cl(\varphi)$
- if $\Box\psi \in cl(\varphi)$ then $\psi \in cl(\varphi)$
- if $\mu y.\psi(y) \in cl(\varphi)$ then $\psi(\mu y.\psi(y)) \in cl(\varphi)$
- if $\nu y.\psi(y) \in cl(\varphi)$ then $\psi(\nu y.\psi(y)) \in cl(\varphi)$

Alternation-free μ-calculus

- $\delta(p, \sigma, k) = \mathit{true}$ if $p \in \sigma$
- $\delta(\varphi \wedge \psi, \sigma, k) = \delta(\varphi, \sigma, k) \wedge \delta(\psi, \sigma, k)$
- $\delta(\Box\varphi, \sigma, k) = \bigwedge_{c=0}^{k-1}(c, \varphi)$
- $\delta(\Diamond\varphi, \sigma, k) = \bigvee_{c=0}^{k-1}(c, \varphi)$
- $\delta(\mu y.\psi(y), \sigma, k) = \delta(\psi(\mu y.\psi(y)), \sigma, k)$
- $\delta(\nu y.\psi(y), \sigma, k) = \delta(\psi(\nu y.\psi(y)), \sigma, k)$

$\varphi \approx \psi$ if $\varphi \in cl(\psi)$ and $\psi \in cl(\varphi)$.

$F = \{$set of formulas that are equivalent to some formula $\nu y.\psi(y) \in cl(\varphi)\}$