Automata, Games & Verification

Summary $\#6$

Today at 2:15pm in SR 016
Seminar "Games, Synthesis, and Robotics"
Tutorial: A Theory of Timed Automata

End-of-Term Exam:
July 26, 9-12, HS 002, Building E 1.3

Linear-Time Temporal Logic (LTL)

Syntax:

- Given a set of atomic propositions *AP*.
- Any atomic proposition $p \in AP$ is an LTL formula
- If φ, ψ are LTL formulars then so are
  - $\neg\varphi,\ \varphi \wedge \phi,$
  - $\bigcirc\varphi,\ \varphi\,\mathcal{U}\,\psi$

Abbreviations:

$\Diamond\varphi \equiv$ true $\mathcal{U}\,\varphi;$

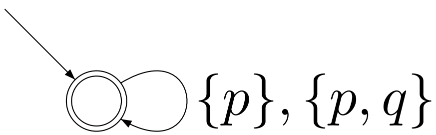
$\Box\,\varphi \ \equiv\ \neg(\Diamond\neg\varphi);$

$\varphi\,\mathcal{W}\,\psi \ \equiv\ (\varphi\,\mathcal{U}\,\psi) \vee \Box\varphi;$

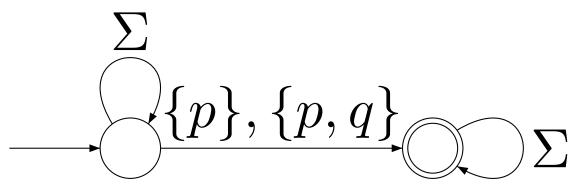
Semantics:

- $\alpha, i \models p$ if $p \in \alpha(i)$;
- $\alpha, i \models \neg\varphi$ if $\alpha, i \not\models \varphi$;
  $\alpha, i \models \varphi \wedge \psi$ if $\alpha, i \models \varphi$ and $\alpha, i \models \psi$;
- $\alpha, i \models \bigcirc\varphi$ if $\alpha, i+1 \models \varphi$
- $\alpha, i \models \varphi\,\mathcal{U}\,\psi$ if there is some $j \geq i$ s.t. $\alpha, j \models \psi$ and for all $i \leq k < j$: $\alpha, k \models \varphi$

$\Box p$:



$\Diamond p$:



Examples:

- Invariant: $\Box p$
- Guarantee: $\Diamond p$
- Recurrence: $\Box\Diamond p$
- Request-Response: $\Box(p \rightarrow \Diamond q)$
- Fairness: $(\Box\Diamond p) \rightarrow (\Box\Diamond q)$

Definition 1.

- *models*(φ)=$\{\alpha \in (2^{AP})^w \mid \alpha \models \varphi\}$
- *an LTL formula* φ *is* satisfiable *if models*$(\varphi) \neq \varnothing$
- *an LTL formula* φ *is* valid *if models*$(\varphi) = (2^{AP})^\omega$

There are Büchi-recognizable languages that are not LTL-definable!
Example: $(\varnothing\varnothing)^*\{p\}^\omega$

Definition 2. *A language* $L \subseteq \Sigma^\omega$ *is* *non-counting* *iff*
$\exists n_0 \in \omega \;.\; \forall n \geqslant n_0 \;.\; \forall u, v \in \Sigma^*, \gamma \in \Sigma^\omega \;.$
$uv^n\gamma \in L \;\Leftrightarrow\; uv^{n+1}\gamma \in L$

Example: $L = (\varnothing\varnothing)^*\{p\}^\omega$ is counting.
For every $\varnothing^n\{p\}^\omega \in L$, $\varnothing^{n+1}\{p\}^\omega \notin L$.

Theorem 1. *For every LTL-formula* φ*, models*(φ) *is non-counting.*

QPTL

Syntax: LTL formula $\mid \varphi \wedge \varphi \mid \neg\varphi \mid \exists p.\ \varphi$

Semantics:

$\alpha, i \models \exists q.\varphi$ iff there is an α' with
$\alpha'(j) \cap (AP \setminus \{q\}) = \alpha(j) \cap (AP \setminus \{q\})$ for all $j \in \omega$,
s.t. $\alpha', i \models \varphi$.