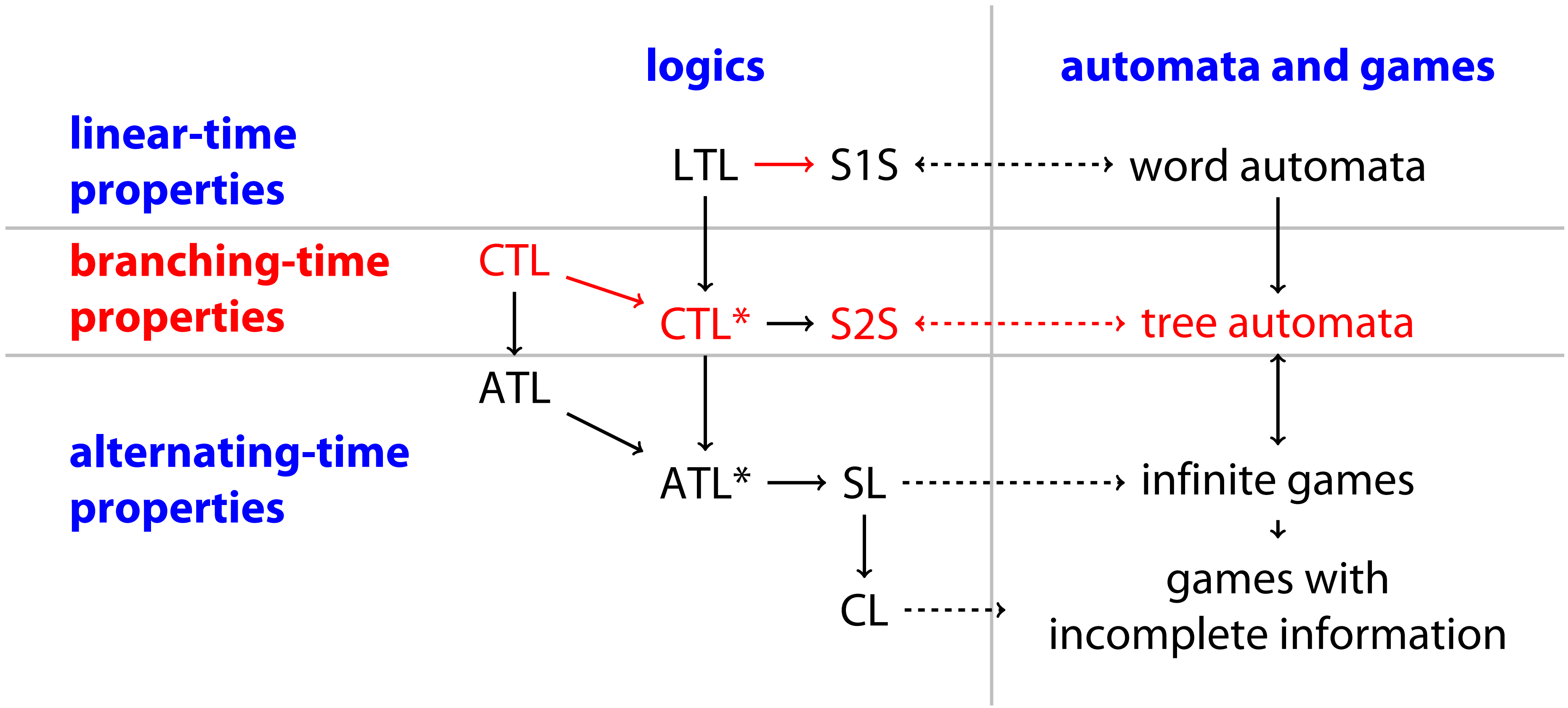
Automata, Games & Verification

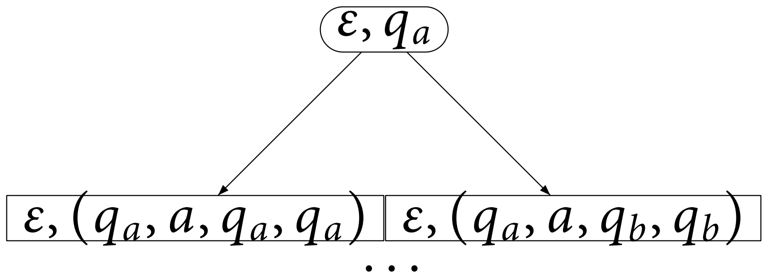
#13



logics
automata and games
linear-time
properties
LTL
S1S
word automata
branching-time
properties
CTL
CTL*
S2S
tree automata
ATL
alternating-time
properties
ATL*
SL
infinite games
CL
games with
incomplete information

Theorem 1. [Acceptance Game] *A parity tree automaton* $\mathcal{A} = (S, s_0, M, c)$ *accepts an input tree* t *iff Player 0 wins the parity game* $\mathcal{G}_{\mathcal{A},t} = (V_0, V_1, E, c')$ *from position* (ε, s_0).

- $V_0 = \{(w, q) \mid w \in \{0,1\}^*, q \in S\}$;
- $V_1 = \{(w, \tau) \mid w \in \{0,1\}^*, \tau \in M\}$;
- $E = \{((w,q),(w,\tau)) \mid \tau = (q, t(w), q_0', q_1'), \tau \in M\}$
  $\cup \{((w,\tau),(w',q')) \mid \tau = (q, \sigma, q_0', q_1')$ *and*
  $((w' = w0$ *and* $q' = q_0')$ *or* $(w' = w1$ *and* $q' = q_1'))\}$;
- $c'(w, q) = c(q)$ *if* $q \in S$;
- $c'(w, \tau) = 0$ *if* $\tau \in M$.



Theorem 2. [Emptiness Game] *The language of a parity tree automaton $\mathcal{A} = (S, s_0, M, c)$ is non-empty iff Player 0 wins the parity game $\mathcal{G}_{\mathcal{A},t} = (V_0, V_1, E, c')$ from position s_0.*

- $V_0 = S$;
- $V_1 = M$;

$\leftarrow$ *V is finite!*

- $E = \{(q, \tau) \mid \tau = (q, 1, q'_0, q'_1), \tau \in M\}$
  $\cup \{(\tau, q') \mid \tau = (q, 1, q'_0, q'_1)$ *and*
  $(q' = q'_0$ *or* $q' = q'_1)\}$;
- $c'(q) = c(q)$ *for* $q \in S$;
- $c(\tau) = 0$ *for* $\tau \in M$.

Complementation of Parity Tree Automata

Theorem 3. *For each parity tree automaton $\mathcal{A}$ over Σ there is a parity tree automaton $\mathcal{A}'$ with $\mathcal{L}(\mathcal{A}') = T_\Sigma - \mathcal{L}(\mathcal{A})$.*

$\mathcal{A}$ does not accept t iff

(1) there is a $(M \to \{0,1\})$-tree v such that

(2) for all $i_0, i_1, i_2, \ldots \in \{0,1\}^\omega$

(3) for all $\tau_0, \tau_1, \ldots \in M^\omega$

(4) if

- for all j,
  $\tau_j = (q, a, q'_0, q'_1)$
  $\Rightarrow a = t(i_0, i_1, \ldots, i_j)$ and
- $i_0 i_1 \ldots = v(\varepsilon)(\tau_0) v(i_0)(\tau_1) \ldots$

then the generated state sequence $q_0 q_1 \ldots$
with $q_0 = s_0$, $(q_j, a, q^0, q^1) = \tau_j$,
$q_{j+1} = q^{v(i_0,\ldots,i_{j-1})(\tau_j)}$
violates c.