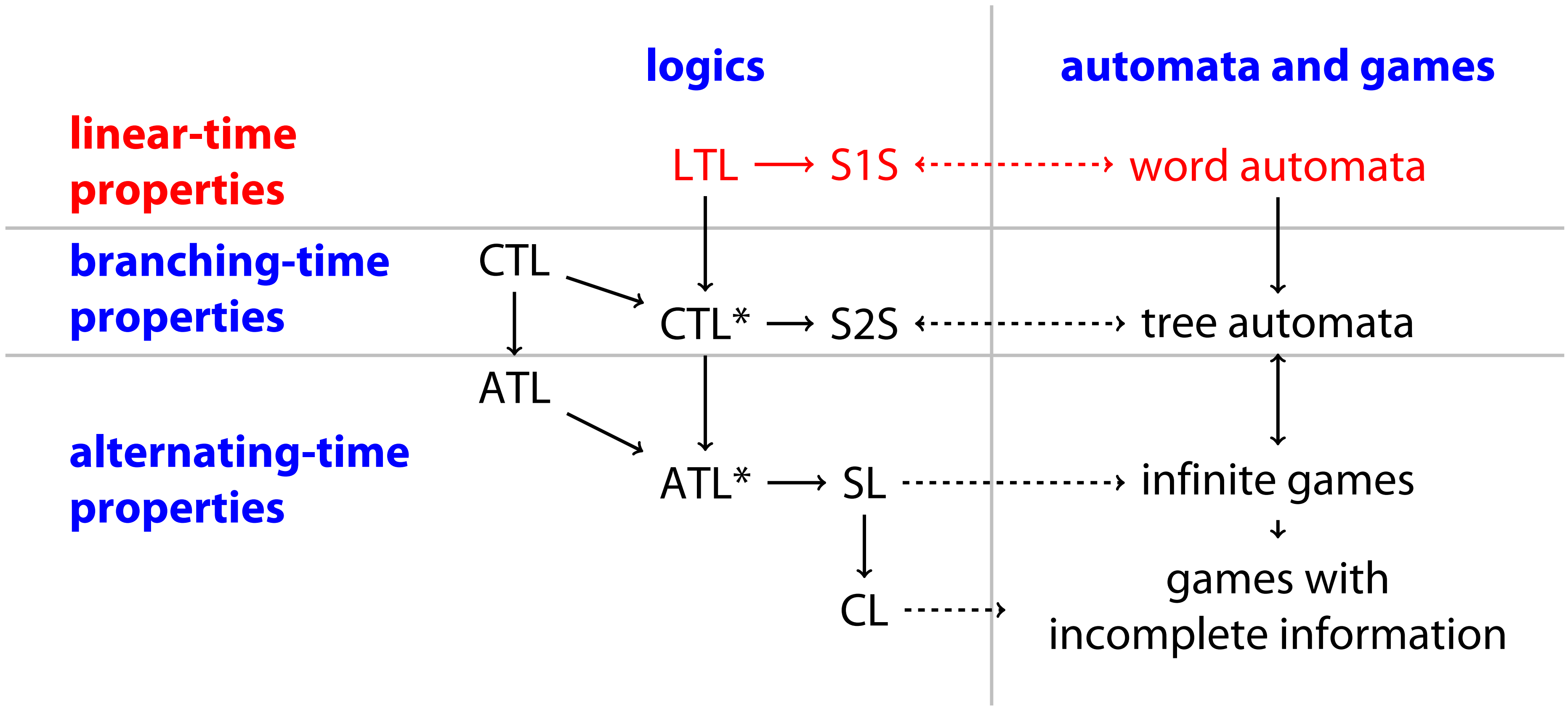
Automata, Games & Verification

#2



logics
automata and games
linear-time properties
LTL
S1S
word automata
branching-time properties
CTL
CTL*
S2S
tree automata
ATL
alternating-time properties
ATL*
SL
infinite games
CL
games with incomplete information

- The set of natural numbers $\{0, 1, 2, 3, \ldots\}$ is denoted by ω.
- An alphabet Σ is a finite set of symbols.
- An infinite sequence/string/word is a function from natural numbers to an alphabet:

  $\alpha : \omega \rightarrow \Sigma$

  Notation: $\alpha = \alpha(0)\alpha(1)\alpha(2)\ldots$
- The set of infinite words over alphabet Σ is denoted Σ^ω.
- An ω-language L is a subset of Σ^ω.

BACKGROUND: The Kleene Theorem

Definition 1. *The regular expressions are defined as follows:*

- *The constants* ε *and* $\emptyset$ *are regular expressions.*
  $\mathcal{L}(\varepsilon) = \{\varepsilon\}, \mathcal{L}(\emptyset) = \emptyset$.
- *If* $a \in \Sigma$ *is a symbol, then* $\mathbf{a}$ *is a regular expression.*
  $\mathcal{L}(\mathbf{a}) = \{a\}$.
- *If* E *and* F *are regular expressions, then* $E + F$ *is a regular expression:*
  $\mathcal{L}(E + F) = \mathcal{L}(E) \cup \mathcal{L}(F)$.
- *If* E *and* F *are regular expressions, then* $E \cdot F$ *is a regular expression:*
  $\mathcal{L}(E \cdot F) = \{uv \mid u \in \mathcal{L}(E), v \in \mathcal{L}(F)\}$.
- *If* E *is a regular expression, then* E^* *is a regular expression.*
  $\mathcal{L}(E^*) = \{u_1 u_2 \ldots u_n \mid n \in \omega, u_i \in \mathcal{L}(E) \; \forall \; 0 \leq i \leq n\}$.

Definition 2. *A language is regular if it is defined by a regular expression.*

Theorem 1. [Kleene Theorem] *A language is regular iff it is recognized by some finite word automaton.*