# Automata, Games & Verification

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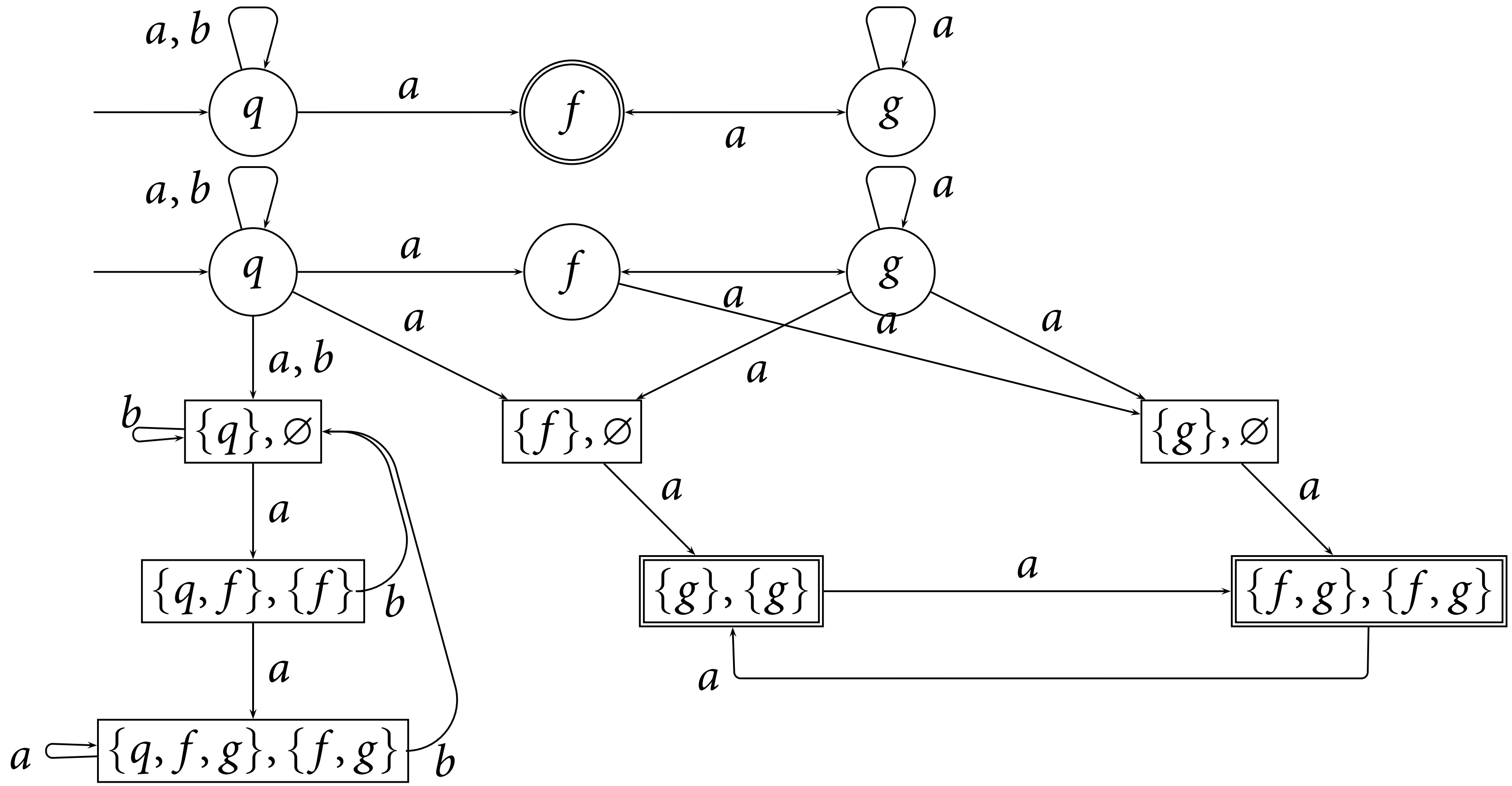
# Determinization

**Theorem 1. [McNaughton's Theorem (1966)]** *Every Büchi recognizable language is recognizable by a deterministic Muller automaton.*

**Definition 1.** *A Büchi automaton* $(S, I, T, F)$ *is semi-deterministic if* $S = N \uplus D$ *is a partition of* $S$, $F \subseteq D$, $pr_3(T \cap (D \times \Sigma \times S)) \subseteq D$, *and* $(D, \{d\}, T \cap (D \times \Sigma \times D), F)$ *is deterministic for every* $d \in D$.

***Lemma 1.***

*For every Büchi automaton $\mathcal{A}$ there exists a semi-deterministic Büchi automaton $\mathcal{A}'$ with $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}')$.*



Given $\mathcal{A} = (S, I, T, F)$, we construct $\mathcal{A}' = (S', I', T', F')$:

- $S' = S \uplus (2^S \times 2^S)$;
- $I' = I$;
- $$\begin{aligned} T' = T \;\cup\; & \{(s, \sigma, (\{s'\}, \emptyset)) \mid (s, \sigma, s') \in T\} \\ & \cup \{((L_1, L_2), \sigma, (L_1', L_2')) \mid L_1 \neq L_2 \\ & \qquad L_1' = pr_3(T \cap L_1 \times \{\sigma\} \times S), \\ & \qquad L_2' = pr_3(T \cap L_1 \times \{\sigma\} \times F) \cup pr_3(T \cap L_2 \times \{\sigma\} \times S)\} \\ & \cup \{((L, L), \sigma, (L_1', L_2')) \mid L_1' = pr_3(T \cap L \times \{\sigma\} \times S), \\ & \qquad L_2' = pr_3(T \cap L \times \{\sigma\} \times F)\} \end{aligned}$$
- $F' = \{(L, L) \mid L \neq \emptyset)\}$