Automata, Games & Verification

#7

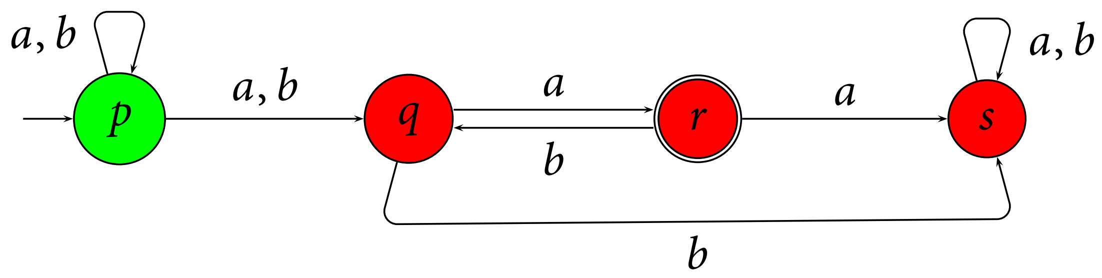
Theorem 1. [McNaughton's Theorem (1966)] *Every Büchi recognizable language is recognizable by a deterministic Muller automaton.*

Lemma 1.

For every Büchi automaton $\mathcal{A}$ there exists a semi-deterministic Büchi automaton $\mathcal{A}'$ with $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}')$.

Lemma 2.

For every semi-deterministic Büchi automaton $\mathcal{A}$ there exists a deterministic Muller automaton $\mathcal{A}'$ with $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}')$.


a, b
p
a, b
q
a
b
r
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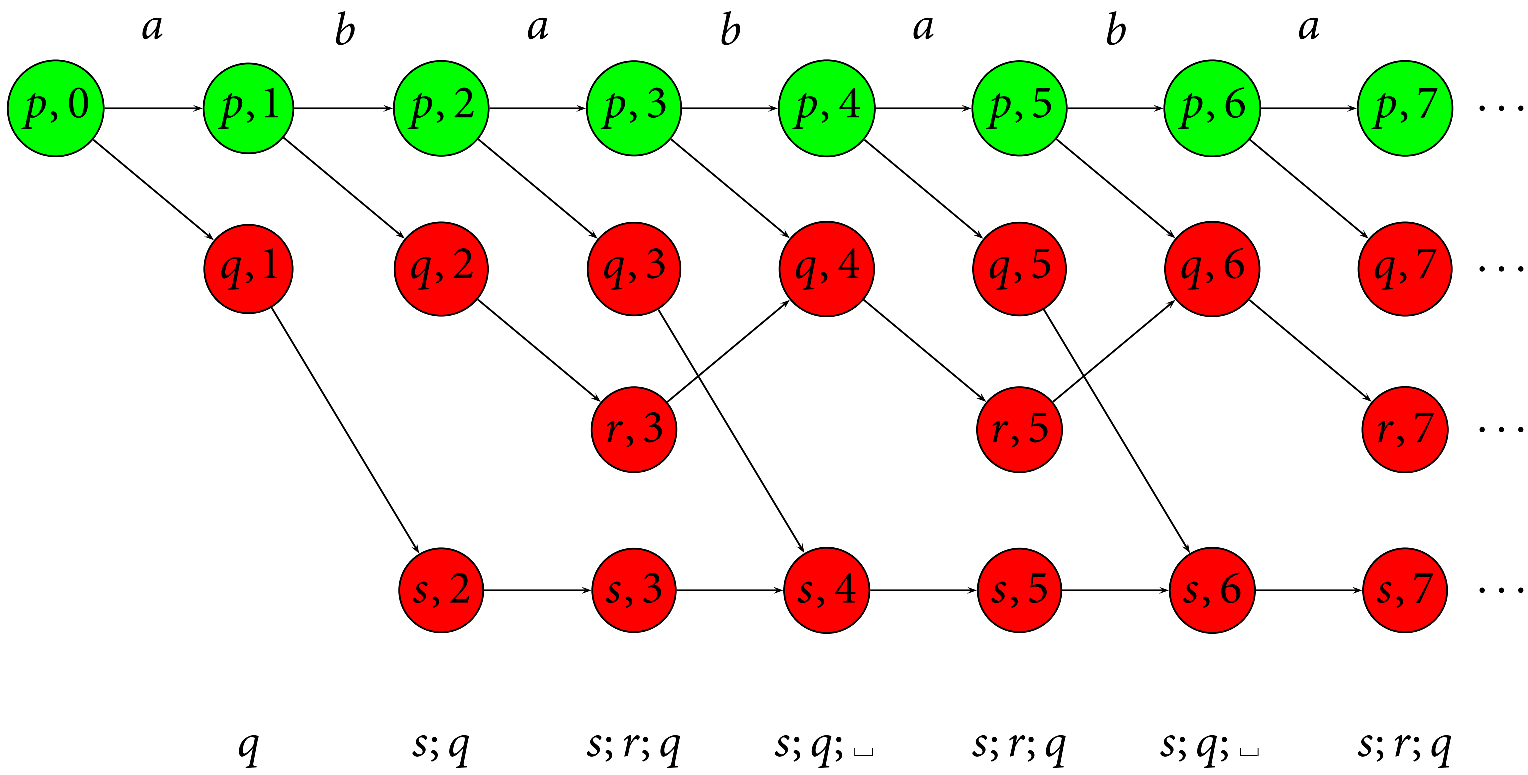
s
a, b
b


a b a b a b a
p, 0 p, 1 p, 2 p, 3 p, 4 p, 5 p, 6 p, 7 ...
q, 1 q, 2 q, 3 q, 4 q, 5 q, 6 q, 7 ...
r, 3 r, 5 r, 7 ...
s, 2 s, 3 s, 4 s, 5 s, 6 s, 7 ...
q s; q s; r; q s; q; ␣ s; r; q s; q; ␣ s; r; q

Let $\mathcal{A} = (N \uplus D, I, T, F)$, $d = |D|$, and let D be ordered by $<$. We construct the DMA $(S', \{s'_0\}, T', \mathcal{F})$:

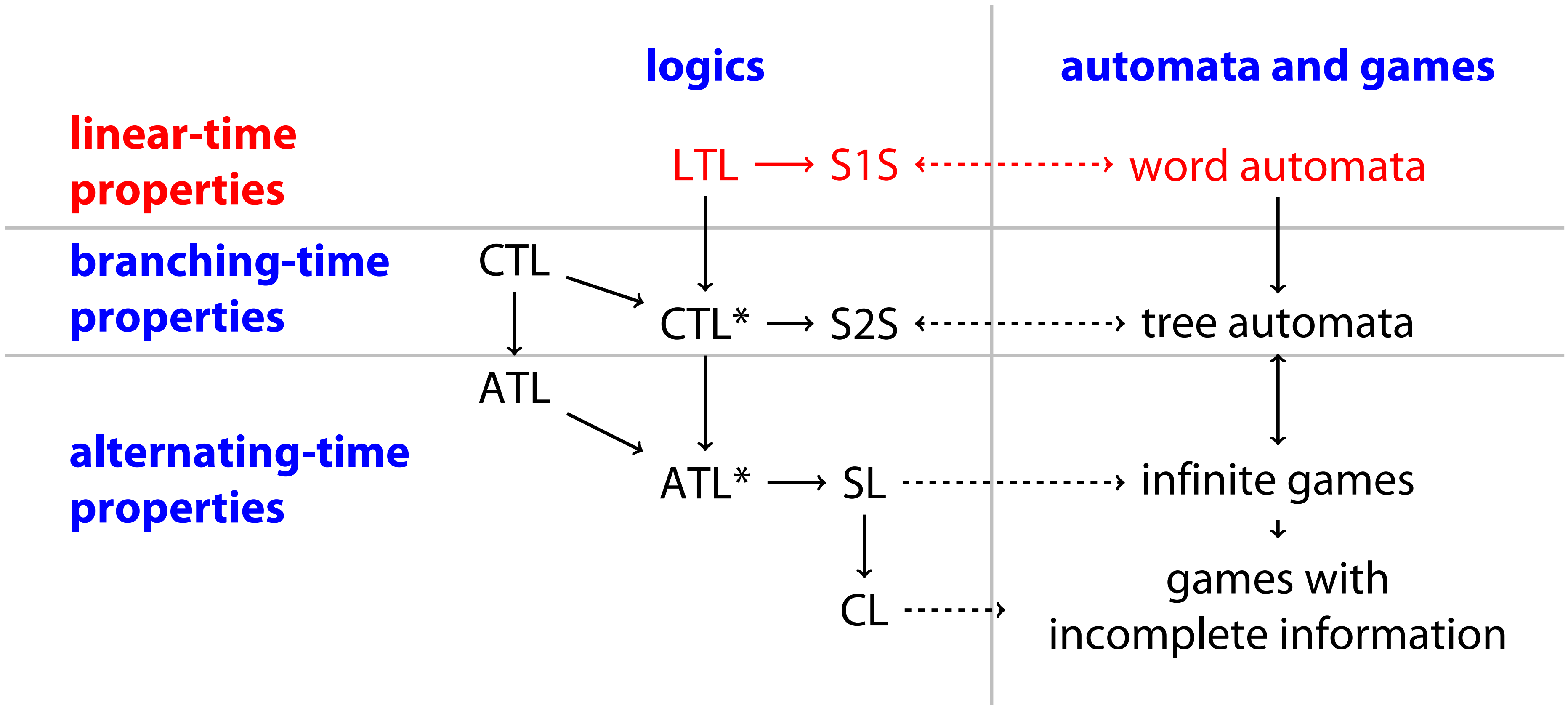
- $S' = 2^N \times \{0, \ldots, 2d\} \to D \cup \{\sqcup\}$

- $s'_0 = (\{N \cap I\}, (d_1, d_2, \ldots, d_n, \sqcup, \ldots, \sqcup))$,
  where $d_i < d_{i+1}$, $\{d_1, \ldots, d_n\} = D \cap I\}$.

- $T' = \{((N_1, f_1), \sigma, (N_2, f_2)) \mid N_2 = pr_3(T \cap N_1 \times \{\sigma\} \times N)$
  $D' = pr_3(T \cap N_1 \times \{\sigma\} \times D)$
  $g_1 : n \mapsto d_2 \in D \Leftrightarrow f_1 : n \mapsto d_1 \in D \wedge d_1 \to^\sigma d_2$

  g_2: insort the elements of D' in the empty slots of g_1 (using $<$)
  f_2: delete every recurre

- $\mathcal{F} = \{F' \subseteq S' \mid \exists i \in 1, \ldots, 2d$ s.t.
  $f(i) \neq \sqcup$ for all $(N', f) \in F'$ and
  $f(i) \in F$ for some $(N', f) \in F'\}$.


logics
automata and games
linear-time properties
LTL
S1S
word automata
branching-time properties
CTL
CTL*
S2S
tree automata
ATL
alternating-time properties
ATL*
SL
infinite games
CL
games with incomplete information

LTL Syntax

- Propositional logic
  - -- $a \in AP$ — atomic proposition
  - -- $\neg\varphi$ and $\varphi \wedge \psi$ — negation and conjunction
- Temporal operators
  - -- $X\varphi$ — next state fulfills φ
  - -- $\varphi \, U \, \psi$ — φ holds Until a ψ-state is reached
- Derived operators
  - -- $F\,\varphi \equiv true\, U\, \varphi$ — "some time in the future"
  - -- $G\,\varphi \equiv \neg F \neg \varphi$ — "from now on forever"

LTL Semantics

An LTL formula φ over *AP* defines the linear-time property

$$\mathcal{L}(\varphi) = \left\{\sigma \in \left(2^{AP}\right)^{\omega} \mid \sigma \models \varphi\right\},$$

where $\models$ is the smallest relation satisfying:

σ	$\models$	a	iff	$a \in \sigma(0)$ (i.e., $\sigma(0) \models a$)
σ	$\models$	$\varphi_1 \wedge \varphi_2$	iff	$\sigma \models \varphi_1$ and $\sigma \models \varphi_2$
σ	$\models$	$\neg \varphi$	iff	$\sigma \not\models \varphi$
σ	$\models$	$\mathsf{X}\varphi$	iff	$\sigma[1..] = \sigma(1)\sigma(2)\sigma(3)\ldots \models \varphi$
σ	$\models$	$\varphi_1 \,\mathsf{U}\, \varphi_2$	iff	$\exists j \geq 0.\ \sigma[j..] \models \varphi_2$ and $\sigma[i..] \models \varphi_1$ for all $0 \leq i < j$