Prof. Bernd Finkbeiner, Ph.D.
Markus Rabe, M.Sc.
Hazem Torfah, B.Sc.
Felix Klein, B.Sc.

Problem Set 11
discussion session: 22th of January 2013

Automata, Games, and Verification

---

1. **Deterministic tree automata** (Group G08, discussion session 12:00 with Hazem Torfah)

   Compare the expressive power of deterministic and non-deterministic parity tree automata. We call a parity tree automaton $\mathcal{A} = (S, s_0, M, c)$ over the alphabet Σ deterministic if for every $s \in S$ and $x \in \Sigma$, there exists at most one pair $(s_1, s_2) \in S^2$ such that $(s, x, s_1, s_2) \in M$.

2. **Parity tree automata** (Group G03, discussion session 12:00 with Felix Klein)

   Give an algorithm which, given two parity tree automata $\mathcal{A}_1$ and $\mathcal{A}_2$, computes a parity tree automaton $\mathcal{A}$ with $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}_1) \cap \mathcal{L}(\mathcal{A}_2)$.

3. **Cylindrification** (Group G12, discussion session 12:20 with Felix Klein)

   Let L be a language of Σ_1-trees. Let Σ_2 be a new alphabet. The Σ_1-*projection* of a $(\Sigma_1 \times \Sigma_2)$-tree v is a Σ_1-tree v_1 such that for every $n \in \{0,1\}^*$ there exists a $\sigma_2 \in \Sigma_2$ such that $v(n) = (v_1(n), \sigma_2)$. The *cylinder* of L is the set of $(\Sigma_1 \times \Sigma_2)$-trees whose Σ_1-projections belong to L.

   Show that if L is recognized by some Muller tree automaton, then the cylinder of L is recognized by some Muller tree automaton.

4. **From Muller to parity tree automata** (Group G10, discussion session 12:20 with Hazem Torfah)

   Show that for every Muller tree automaton $\mathcal{A}$ there is a parity tree automaton $\mathcal{B}$ with $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{B})$.