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Problem Set 12
discussion session: 29th of January 2013

Automata, Games, and Verification

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1. **Regular Trees** (Group G01, discussion session 12:00 with Felix Klein)

   Given a tree t and word $u \in \{0,1\}^*$, let t^u be the tree defined by $t^u(v) = t(uv)$ for all $v \in \{0,1\}^*$. A tree is called *regular* if the set $\{t^u \mid u \in \{0,1\}^*\}$ is finite.

   Prove or give a counterexample for the following statements:

   a) Every run of a tree automaton is regular.

   b) Every run of a deterministic tree automaton is regular.

   c) Every run of a tree automaton on the $\{1\}$-tree is regular.

   d) Every run of a deterministic tree automaton on the $\{1\}$-tree is regular.

2. **S2S** (Group G06, discussion session 12:00 with Hazem Torfah)

   a) Give an S2S formula for the language
   $$L_1 = \{v \in T_{2^{\{a,b\}}} \mid \text{ there is a branch in } v \text{ with infinitely many } a\}$$

   b) Give S2S formula for the language
   $$L_2 = \{v \in T_{2^{\{a,b,c\}}} \mid \text{ each branch in } v \text{ has at least one } a \text{ and the entire tree has at most one } b\}$$

   c) Give an S2S formula for the language
   $$L_3 = \{v \in T_{2^{\{a,b\}}} \mid \text{ each branch in } v \text{ has only finitely many } a\}$$

3. **Muller tree automata and S2S** (Group G13, discussion session 12:20 with Felix Klein)

   a) Construct a Muller tree automaton $\mathcal{A}$ over $\Sigma = \mathbb{B}^2$, such that $t \in \mathcal{L}(A)$ iff $\sigma_1 \models x = y1$, where $\sigma_1(x) = q$ iff $x \in t(q)$ and $\sigma_1(y) = q$ iff $y \in t(q)$.

   b) Construct a Muller tree automaton $\mathcal{A}$ over $\Sigma = \mathbb{B}^2$, such that $t \in \mathcal{L}(A)$ iff $\sigma_1, \sigma_2 \models y \in X$, where $\sigma_1(y) = q$ iff $y \in t(q)$, and $\sigma_2(X) = \{q \in \mathbb{B}^* \mid X \in t(q)\}$.