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Problem Set 3
Discussion Session: 13th of November 2012

Automata, Games, and Verification

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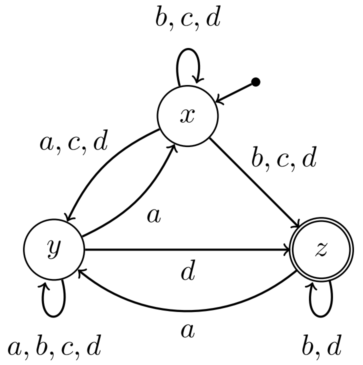
1. **Run DAGs** (Group G03, discussion session 12:00 with Hazem Torfah)

   Let $\Sigma = \{a, b, c, d\}$ be an alphabet, $w = ddbac^\omega$ be a word over this alphabet, and $\mathcal{A}$ be the following Büchi automaton over Σ having the states $\{x, y, z\}$:



   a) Draw the run DAG for $\mathcal{A}$ on w. As the DAG is infinite, you only need to sketch it in a way such that it is, intuitively, clear how it is to be continued after a certain pattern emerges.

   b) Reason whether w is accepted by $\mathcal{A}$.

   c) Finally, write down the sequence of DAGs $G_0 \supseteq G_1 \supseteq G_2 \ldots$ as defined in the proof of Lemma 1 of Section 5 of the lecture.

2. **Strictly Büchi Recognizable Languages** (Group G09, discussion session 12:00 with Felix Klein)

   A *strict Büchi* automaton $\mathcal{A} = (S, I, T, F)$ is the same as a Büchi automaton except that the definition of an accepting run is changed as follows:

   A run r for $\alpha \in \Sigma^\omega$ is *accepting* on $\mathcal{A}$, when $In(r) = F$.

   Proof or give a counter example to the following statements:

   a) If L is recognizable by a strict Büchi automaton then L is Büchi-recognizable.

   b) If L is recognizable by a strict Büchi automaton then L is recognizable by a deterministic Büchi automaton.

   c) If L is Büchi-recognizable then L is strictly Büchi-recognizable.

   d) If L is recognizable by a deterministic Büchi automaton then L is strictly Büchi-recognizable.

3. **Co-Limit Operation** (Group G11, discussion session 12:20 with Felix Klein)

   The *co-limit* of W is defined as $\overleftarrow{W} = \{\alpha \in \Sigma^\omega \mid \text{there exist only finitely many } n \in \omega \text{ s.t. } \alpha(0, n) \in W\}$[1].

   Let $V, W \subseteq \Sigma^*$ be two regular languages. Prove or give a counter example to the following statements:

[1] For a finite word $\alpha \in \Sigma^*$ and two natural numbers $m, n \in \omega$ with $m \leq n$, $\alpha(m, n)$ denotes the substring from m to n: $\alpha(m, n) = \alpha(m)\,\alpha(m+1)\,\ldots\,\alpha(n)$.

a) $\overleftarrow{(V \cdot W)} = V \cdot \overleftarrow{W}$

b) $V \cdot \overleftarrow{W}$ is Büchi-recognizable

c) $V \cdot \overleftarrow{W}$ is recognizable by a deterministic Büchi automaton

4. **co-Büchi Automata** (Group G14, discussion session 12:40 with Felix Klein)

   A co-Büchi automaton $\mathcal{A} = (S, I, T, F)$ is the same as a Büchi automaton except that the definition of an accepting run is changed as follows:

   A run r for $\alpha \in \Sigma^\omega$ is *accepting* on $\mathcal{A}$, when $In(r) \cap F = \emptyset$.

   Prove or give a counter example to the following statements:

   a) co-Büchi automata are closed under $\cap$.

   b) co-Büchi automata are closed under $\cup$.

   c) co-Büchi automata are closed under pr_1.

5.

6. **co-Büchi Automata** (challenge question)

   Prove or provide a counter example to the statement: the co-Büchi recognizable languages and the Büchi recognizable languages are the same.