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Problem Set 4
discussion session: 20th of November 2012

Automata, Games, and Verification

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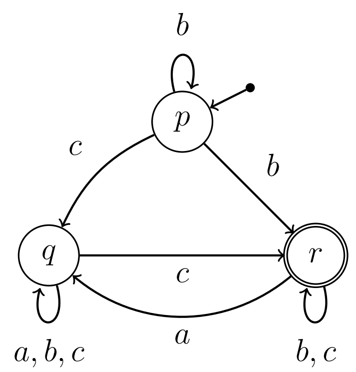
1. **Deterministic Muller Automata** (Group G05, discussion session 12:00 with Felix Klein)

   a) Give an ω-regular expression for which the smallest deterministic Muller automaton recognizing it is larger than the smallest nondeterministic Muller automaton recognizing it, and prove this fact.

   b) For all $i \in \mathbb{N}$, let Z_i describe the set of languages representable by deterministic Muller automata with at most i tables (i.e., for every language in Z_i, there exists a corresponding deterministic Muller automaton $\mathcal{A} = (S, I, T, \mathcal{F})$ with $|\mathcal{F}| \leq i$). Obviously, $Z_1 \subseteq Z_2 \subseteq Z_3 \subseteq \ldots$ holds. Prove that this sequence of inequalities is strict, i.e., $Z_1 \subset Z_2 \subset Z_3 \subset \ldots$ holds as well.

2. **Semi-deterministic automata** (Group G07, discussion session 12:00 with Hazem Torfah)

   Let $\Sigma = \{a, b, c\}$ be an alphabet and $\mathcal{A}$ be the following Büchi automaton over Σ having the states $\{p, q, r\}$:



   Construct an equivalent semi-deterministic automaton using the construction from the proof of Lemma 1 in Section 7 of the lecture (McNaughton's Theorem).

3. **More Acceptance Conditions** (Group G10, discussion session 12:00 with Hazem Torfah)

   Besides Büchi and Muller automata, there are three further important types of ω-automata.

   - A *parity automaton* is a tuple $(S, I, T, c : S \to \mathbb{N})$. A run r of a parity automaton is accepting iff $max\{c(s) \mid s \in In(r)\}$ is even.
   - A *Rabin automaton* is a tuple $(S, I, T, \{(A_i, R_i) \mid i \in J\})$. A run r of a Rabin automaton is accepting iff, for some $i \in J$, $In(r) \cap A_i \neq \emptyset$ and $In(r) \cap R_i = \emptyset$.
   - A *Streett automaton* is a tuple $(S, I, T, \{(A_i, R_i) \mid i \in J\})$. A run r of a Streett automaton is accepting iff, for all $i \in J$, $In(r) \cap A_i \neq \emptyset$ or $In(r) \cap R_i = \emptyset$.

Compare the expressive power of Büchi, Muller, Rabin, Streett and parity automata. Which ones are equally-expressive? Which are less expressive than others? Provide proofs for all your claims.

4. **co-Büchi Automata** (challenge question)

   Prove or disprove the statement: an ω-language is co-Büchi recognizable if and only if it is recognizable by a deterministic co-Büchi automaton.