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Problem Set 7
discussion session: 11th of December 2012

Automata, Games, and Verification

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1. **LTL-to-Nondeterministic-Büchi** (Group G01, discussion session 12:00 with Felix Klein)

   Prove or give a counter-example the following statements: For every LTL formula φ there exists

   a) a nondeterministic Büchi automaton $\mathcal{A}_\varphi$ with a *single* accepting state such that $\mathcal{L}(\mathcal{A}_\varphi) = \mathit{models}(\varphi)$.

   b) a nondeterministic Co-Büchi automaton $\mathcal{A}'_\varphi$ such that $\mathcal{L}(\mathcal{A}'_\varphi) = \mathit{models}(\varphi)$.

   c) a nondeterministic Co-Büchi automaton $\mathcal{A}''_\varphi$ with a *single* accepting state such that $\mathcal{L}(\mathcal{A}''_\varphi) = \mathit{models}(\varphi)$.

2. **S1S and LTL** (Group G05, discussion session 12:00 with Hazem Torfah)

   Let $L \subseteq (2^{\mathsf{AP}})^\omega$ be an LTL-definable language and let $\mathsf{AP}' \subsetneq \mathsf{AP}$ be a strict subset of AP. Prove or give a counter example to the following statements:

   a) The (weak) projection $L_w = \{\sigma' \in (2^{\mathsf{AP}'})^\omega \mid \exists \sigma \in L \, \forall i \in \omega.\, \sigma'(i) = \sigma(i) \cap \mathsf{AP}'\}$ of L is LTL-definable.

   b) The (weak) projection L_w of L is S1S-definable.

   c) The strong projection $L_s = \{\sigma' \in (2^{\mathsf{AP}'})^\omega \mid \forall \sigma \in (2^{\mathsf{AP}})^\omega.\, (\forall i \in \omega.\, \sigma'(i) = \sigma(i) \cap \mathsf{AP}') \rightarrow \sigma \in L\}$ of L is LTL-definable.

   d) The strong projection L_s of L is S1S-definable.

3. **S1S and Büchi** (Group G09, discussion session 12:20 with Hazem Torfah)

   Apply the construction introduced in the lecture to build a complete Büchi automaton for the following S1S formula:

   $$\forall x. x \in P \leftrightarrow x = 0$$

   Hint: Minimize intermediate automata resulting from intersection.

4. **Presburger Arithmetic** (Challenge Problem)

   Presburger arithmetic is a fragment of natural number arithmetic involving constants, addition, inequalities, and quantification. Since its validity problem is decidable, decision procedures for it can be used in theorem provers to automatically determine whether an arithmetic property is a theorem. The syntax of Presburger arithmetic is defined as follows:

   Terms $\quad t \ ::= \ 0 \,|\, 1 \,|\, v \,|\, t_1 + t_2$,

   where v is a variable $v \in V$ chosen from a set of variables V.

   Formulas $\quad \varphi \ ::= \ t_1 \geq t_2 \,|\, \neg\varphi_1 \,|\, \varphi_1 \vee \varphi_2 \,|\, \exists v.\varphi_1$.

   The semantics is defined in the straightforward way relative to a valuation $\sigma : V \rightarrow \omega$ of the variables. Using a decision procedure for S1S, we develop a decision procedure for Presburger formula satisfiability:

   a) Are Presburger formulas a special case of S1S formulas (disregarding second-order variables)?

   b) Describe a syntactic transformation T which provides an S1S formula given any Presburger formula φ which is satisfiable if and only if the original formula φ is satisfiable.