Prof. Bernd Finkbeiner, Ph.D.
Markus Rabe, M.Sc.
Hazem Torfah, B.Sc.
Felix Klein, B.Sc.

Problem Set 8
discussion session: 18th of December 2012

Automata, Games, and Verification

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1. **LTL-to-Alternating-Büchi** (Group G10, discussion session 12:00 with Felix Klein)

   Construct an alternating Büchi automaton $\mathcal{A}$ such that

   $$\mathcal{L}(\mathcal{A}) = models((\mathsf{F}p)\ \mathcal{U}\ (\mathsf{G}q)).$$

   Use the construction from the lecture to obtain $\mathcal{A}$.

2. **Complete Alternating Büchi Automata** (Group G11, discussion session 12:20 with Felix Klein)

   An alternating automaton is called *complete* iff neither *true* nor *false* are in the mapping of δ (run trees of complete alternating automata have only infinite branches and every input word has a run tree).

   Prove or give a counter-example to the following statement:
   Every language that is recognized by an alternating Büchi automaton is recognized by a complete alternating Büchi automaton.

3. **Alternating Parity Automata** (Group G14, discussion session 12:40 with Felix Klein)

   Let $\mathcal{P}_1 = (Q_1, q_0^1, \delta_1, \alpha_1)$ and $\mathcal{P}_2(Q_2, q_0^2, \delta_2, \alpha_2)$ with disjoint sets $Q_1 \cap Q_2 = \emptyset$ of states be two alternating parity automata. Prove or give a counter-example for the general correctness of the following statements:

   a) The language $\mathcal{L}(\mathcal{P}_1) \cup \mathcal{L}(\mathcal{P}_2)$ is recognizable by an alternating parity automaton.
   b) The language $\mathcal{L}(\mathcal{P}_1) \cap \mathcal{L}(\mathcal{P}_2)$ is recognizable by an alternating parity automaton.
   c) The language $\Sigma^\omega \setminus \mathcal{L}(\mathcal{P}_1)$ is recognizable by an alternating parity automaton.

4. **Alternating vs. Deterministic Automata** (challenge problem)

   Consider the following family of languages L_n:

   $$L_n = \{v_1 \# u\, v_2 \$ u\, \beta \mid \begin{array}{l} v_1 \in \{0,1,\#\}^* \\ v_2 \in \{0,1,\#\}^* \\ u \in \{0,1\}^n \\ \beta \in \{0,1,\#,\$\}^\omega\ \}. \end{array}$$

   a) Construct a family $\mathcal{A}_n$ of alternating Büchi automata with $\mathcal{L}(\mathcal{A}_n) = L_n$ such that each automaton $\mathcal{A}_n$ has only $O(n)$ states.
   b) Show that any deterministic Muller automaton that recognizes L_n has at least 2^{2^n} states.