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Problem Set 11
Discussions: July 8th, 2015

**Automata, Games, and Verification**

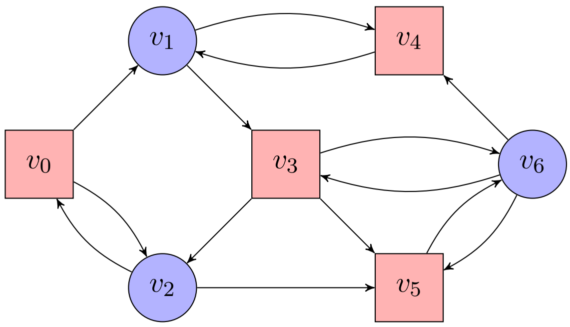
Please send a mail to `agv15@react.uni-saarland.de` if you can't make it to the discussion session.

1. **Games** (presented by Group 11)

   Consider the game $\mathcal{G} = (\mathcal{A}, \mathrm{Win})$ with the arena $\mathcal{A} = (V, V_0, V_1, E)$ depicted below and the winning condition Win defined by

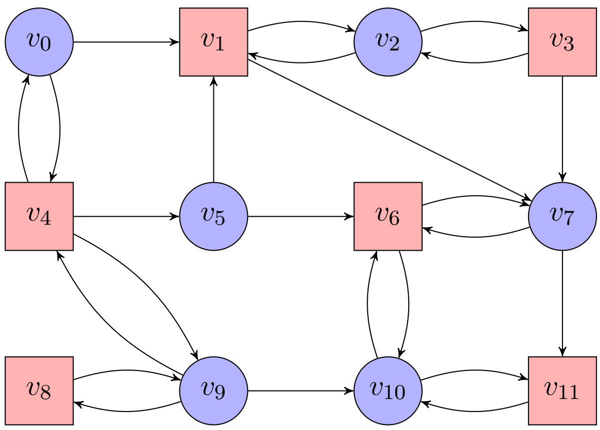
   $$\mathrm{Win} = \{\rho \in V^\omega \mid V = \mathrm{Occ}(\rho)\},$$

   i.e., a play is winning for Player 0 in this game iff all vertices of $\mathcal{G}$ are visited at least once in the play.




   a) Give at least one winning strategy from some vertex for each player. Argue why they are winning.
   b) Determine the winning regions of the game. You do not have to give a justification.
   c) Is the game memoryless determined? Argue formally.

2. **Reachability Games** (presented by Group 04)




   Consider the arena $\mathcal{A}$ depicted above.

   a) Determine the winning regions and the respective winning strategies of both players in the reachability game $\mathcal{G}_1 = (\mathcal{A}, \mathrm{REACH}(\{v_7, v_{10}\}))$.
   b) Determine the winning regions and the respective winning strategies of both players in the reachability game $\mathcal{G}_1 = (\mathcal{A}, \mathrm{REACH}(\{v_1\}))$.
   c) Give a minimal set $R$ of vertices (minimal in the size) such that Player 0 wins the reachability game $\mathcal{G}_R = (\mathcal{A}, \mathrm{REACH}(R))$ from every position of $\mathcal{A}$.

3. **Memoryless Strategies** (presented by Group 13)

   a) Prove or disprove whether Muller games are memoryless determined.

   b) Given a library of procedures for automata, describe a simple algorithm that verifies whether a given memoryless strategy is winning for Player 0 in a Büchi game $\mathcal{G}$ starting at some position $p$.

4. **Nondeterministic Strategies** (presented by Group 01)

   A *nondeterministic memoryless strategy* for Player 0 is a relation $R \subseteq (V_0 \times V) \cap E$. We say that Player 0 follows $R$ in a play $\rho$ if for all $n \in \mathbb{N}$, $\rho(n) \in V_0$ implies $(\rho(n), \rho(n+1)) \in R$. The strategy $R$ is winning for Player 0 if all plays played according to $R$ are winning for Player 0.

   Prove or give a counterexample to the following statement: If Player 0 has two winning nondeterministic memoryless strategies $R_1$ and $R_2$ for a reachability game $\mathcal{G}$ from some position $v$, then $R_1 \cup R_2$ is a winning strategy for Player 0 in the game $\mathcal{G}$ from $p$.

5. **Generalized Reachability (Challenge)**

   Consider the generalized reachability winning condition given by:

   $$\textsc{GenReach}(\mathcal{R}) = \{\rho \in \text{Plays}(\mathcal{A}) \mid \forall R \in \mathcal{R}.\ \text{Occ}(\rho) \cap R \neq \emptyset\}$$

   Prove that solving generalized reachability games is PSPACE-hard. The size of a generalized reachability game $\mathcal{G} = (\mathcal{A}, \textsc{GenReach}(\mathcal{R}))$ is defined to be $|\mathcal{A}| + |\mathcal{R}|$.