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Problem Set 2
Discussions: May 6th, 2015

Automata, Games, and Verification

Please send a mail to agv15@react.uni-saarland.de if you can't make it to the discussion session.

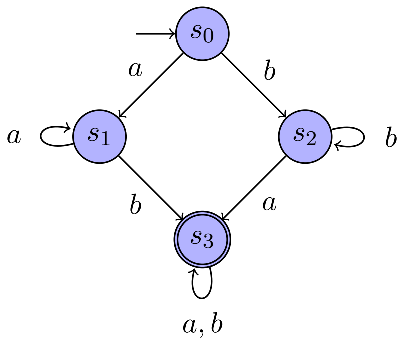
1. **Language Emptiness** (presented by Group 10)

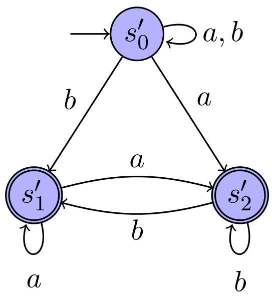
   For each of the following languages over $\Sigma = \{a, b\}$ determine whether it is empty. For each nonempty language, describe an ω-word in the language.

   a) $L_1 = \{\alpha \in \Sigma^\omega \mid$ each occurrence of an a in α is followed immediately by a b and there are infinitely many a's in $\alpha\}$
   b) $L_2 = \{\alpha \in \Sigma^\omega \mid$ each occurrence of a b in α is followed immediately by two occurrences of an $a\}$
   c) $L_3 = \{\alpha \in \Sigma^\omega \mid \alpha \in L_1$ implies $\alpha \in L_2\}$
   d) $L_4 = \{\alpha \in \Sigma^\omega \mid \alpha \in L_2$ implies $\alpha \in L_3\}$
   e) $L_5 = L_1 \cap L_2$
   f) $L_6 = \overline{L_1 \cup L_2}$

2. **Büchi automata** (presented by Group 04)

   Consider the following nondeterministic Büchi automata over $\Sigma = \{a, b\}$:





   a) Which of the automata are deterministic? Which are complete?
   b) For each of the automata, check whether the words aab^ω, a^ω and $(ab)^\omega$ are accepted. If yes, write down an accepting run.
   c) Do the two automata have the same language? Justify your answer informally.

3. **Büchi automata and ω-regular languages** (presented by Group 02)

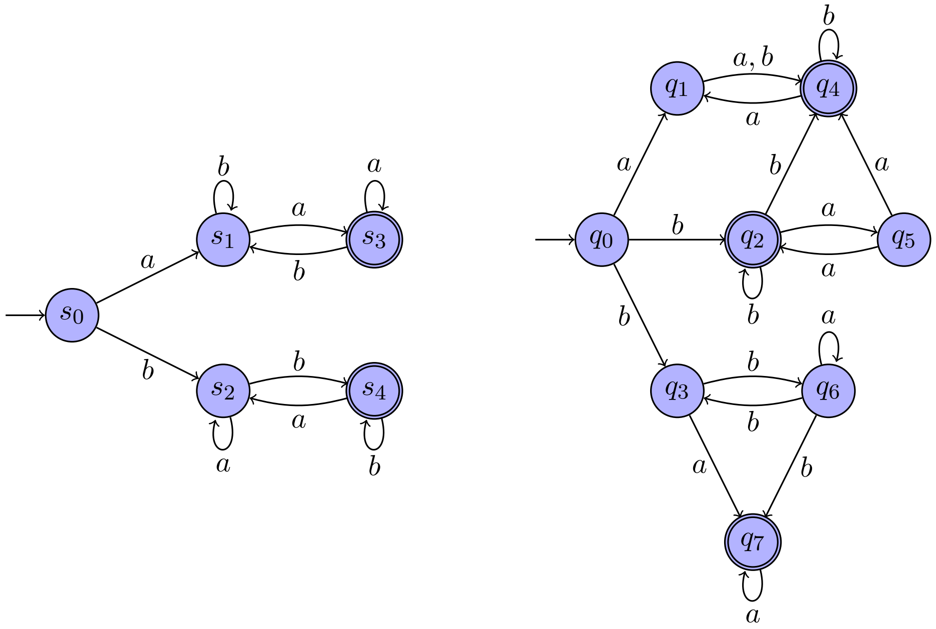
   Build complete Büchi automata and ω-regular expressions for each of the following ω-regular languages over $\Sigma = \{a, b\}$. For L_3, L_4 and L_5 use the constructions of the lecture.

   Hint: It may be useful to simplify intermediate automata first.

   a) $L_1 = \{\alpha \in \Sigma^\omega \mid$ each occurrence of a in α is followed immediately by a $b\}$
   b) $L_2 = \{\alpha \in \Sigma^\omega \mid$ the letter a occurs infinitely often in $\alpha\}$
   c) $L_3 = \{\alpha \in \Sigma^\omega \mid$ the letter b occurs finitely often in $\alpha\}$
   d) $L_4 = L_1 \cap L_2$
   e) $L_5 = L_2 \cup L_3$
   f) $L_6 = L_1 \cap L_2 \cap L_3$

4. **Büchi automata and non-accepting words** (presented by Group 08)

   For each of the following automata, find out whether there exist words that are not accepted by them.



   In case of a positive answer (there is a non-accepted word), state the word and reason informally why it is not accepted. In case of a negative answer, reason informally why there is no word that is not accepted.

5. **Projection and Büchi Recognizable Languages (Challenge)**

   In the following, we suppose that our alphabet Σ is of the form

$$\Sigma = \Sigma_1 \times \Sigma_2 = \{(\sigma_1, \sigma_2) \mid \sigma_1 \in \Sigma_1, \sigma_2 \in \Sigma_2\}$$

   where Σ_1 and Σ_2 are some arbitrary alphabets. Let $L \subseteq \Sigma^\omega$ be a language over the alphabet Σ. We define the projections $pr_1(L)$ and $pr_2(L)$ as follows:

$$pr_1(L) = \{\alpha \in \Sigma_1^\omega \mid (\alpha(0), \beta(0))(\alpha(1), \beta(1))(\alpha(2), \beta(2)) \ldots \in L \text{ for some } \beta \in \Sigma_2^\omega\}$$
$$pr_2(L) = \{\beta \in \Sigma_2^\omega \mid (\alpha(0), \beta(0))(\alpha(1), \beta(1))(\alpha(2), \beta(2)) \ldots \in L \text{ for some } \alpha \in \Sigma_1^\omega\}$$

   a) Prove that the projections $pr_1(L)$ and $pr_2(L)$ of a Büchi recognizable language L on the alphabet $\Sigma_1 \times \Sigma_2$ are Büchi recognizable.

   b) Prove that the converse is false: Construct a non-Büchi recognizable ω-language L such that both $pr_1(L)$ and $pr_2(L)$ are Büchi recognizable.

      Hint: The language $L' = \{a^n b^n \mid n = 1, 2, 3, \ldots\}^\omega$ over the alphabet $\{a, b\}$ is not Büchi-recognizable. Some variation of L' is useful to construct the required language L.