Prof. Bernd Finkbeiner, Ph.D.
Felix Klein, M.Sc.

Problem Set 3
Discussions: May 13th, 2015

Automata, Games, and Verification

Please send a mail to `agv15@react.uni-saarland.de` if you can't make it to the discussion session.

1. **ω-Regular Expressions** (presented by Group 06)

   Represent each of the following ω-regular languages over the alphabet $\Sigma = \{a, b\}$ as deterministic Büchi automata. After that, construct an ω-regular expression from each automaton using the construction in the proof of Büchi's Characterization Theorem.

   a) $L_1 = \{\alpha \in \Sigma^\omega \mid$ the maximal substrings of α consisting of only a's have even length$\}$
   b) $L_2 = \{\alpha \in \Sigma^\omega \mid$ each a in α is preceeded by a $b\}$
   c) $L_3 = \{\alpha \in \Sigma^\omega \mid \alpha$ has no occurrence of $bab\}$

2. **Deterministic Büchi Automata** (presented by Group 11)

   Prove or give a counterexample to the following statements:

   a) Deterministic Büchi automata are closed under $\cap$.
   b) Deterministic Büchi automata are closed under $\cup$.
   c) Deterministic Büchi automata are closed under pr_1.

3. **Limit Operation** (presented by Group 05)

   a) Let $V, W \subseteq \Sigma^*$ be two regular languages. Prove or give a counterexample to the following equation:
   $$\overrightarrow{(V \cdot W)} = V \cdot \overrightarrow{W}$$
   b) Let $\mathcal{A} = (\Sigma, Q, \{q_0\}, T, F)$ be an automaton on finite words. Let $L_* = \mathcal{L}(\mathcal{A})$ be the language of $\mathcal{A}$ and let L_ω be the language of $\mathcal{A}$ when it is regarded as a Büchi automaton, i.e., $L_\omega = \mathcal{L}(\mathcal{A}')$ for $\mathcal{A}' = (\Sigma, Q, \{q_0\}, T, \text{BÜCHI}(F))$. Prove or give a counterexample for the following equation:
   $$L_\omega = \overrightarrow{L_*}$$

4. **Universal Projection** (presented by Group 16)

   We define the following "universal flavor" of projection for $i \in \{1, 2\}$ and $L \subseteq (\Sigma_1 \times \Sigma_2)^\omega$:
   $$rp_i(L) = \{\alpha \in \Sigma_i^\omega \mid \forall \beta \in (\Sigma_1 \times \Sigma_2)^\omega.\ pr_i(\{\beta\}) = \{\alpha\} \Rightarrow \beta \in L\}$$
   Show that if L is Büchi-recognizable, then so is $rp_i(L)$.

   Hint: You might want to use that the complement of a Büchi-recognizable language is Büchi-recognizable.

5. **Ultimately Periodic Words (Challenge)**

   An infinite word $\alpha \in \Sigma^\omega$ is *ultimately periodic* if there exist finite words $w_*, w_\omega \in \Sigma^*$ such that $\alpha = w_* w_\omega w_\omega w_\omega \ldots$. We define the following language over the alphabet $\Sigma = \{a, b\}$:
   $$L = \{\alpha \in \Sigma^\omega \mid \alpha \text{ is ultimately periodic}\}$$
   Prove or disprove that L is Büchi-recognizable.