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Automata, Games, and Verification

Please send a mail to `agv15@react.uni-saarland.de` if you can't make it to the discussion session.

1. **Complementation of Büchi Automata** (presented by Group 07)

   In class, we proved a theorem stating that the complement of an ω-language over alphabet Σ, that is recognizable by a deterministic Büchi automaton, is Büchi recognizable. Unfortunately, this construction does not work for all Büchi-recognizable languages.

   a) Provide a Büchi automaton $\mathcal{A}$, for which the construction generates a new Büchi automaton $\mathcal{A}'$, where $\mathcal{L}(\mathcal{A}') \neq \Sigma^\omega \setminus \mathcal{L}(\mathcal{A})$.

   b) Your counter example is a counter example for one of the following relationships:

   - $\mathcal{L}(\mathcal{A}') \subseteq \Sigma^\omega \setminus \mathcal{L}(\mathcal{A})$
   - $\mathcal{L}(\mathcal{A}') \supseteq \Sigma^\omega \setminus \mathcal{L}(\mathcal{A})$

   Prove or provide a new counter example for the other relationship not refuted by your first answer.

2. **Strictly Büchi-Recognizable Languages** (presented by Group 13)

   A *strict Büchi* automaton $\mathcal{A} = (\Sigma, Q, I, T, \text{STRICTBÜCHI}(F))$ is an automaton over finite words, where the acceptance condition $\text{STRICTBÜCHI}(F)$ is defined by

   $$\text{STRICTBÜCHI}(C) = \{\alpha \in Q^\omega \mid \text{Inf}(\alpha) = F\}.$$

   Proof or provide a counter example to the following statements:

   a) If L is recognizable by a strict Büchi automaton then L is Büchi-recognizable.

   b) If L is recognizable by a strict Büchi automaton then L is recognizable by a deterministic Büchi automaton.

   c) If L is Büchi-recognizable then L is strictly Büchi-recognizable.

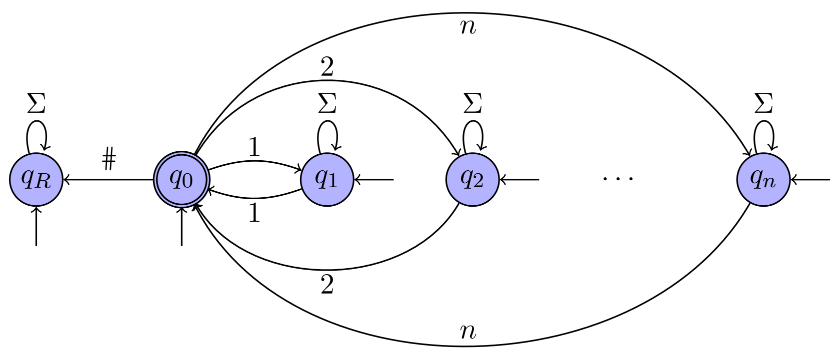
   d) If L is recognizable by a deterministic Büchi automaton then L is strictly Büchi-recognizable.

3. **Universal Automata** (presented by Group 12)

   A *universal* automaton $\mathcal{A} = (\Sigma, Q, I, T, Acc)$ accepts a word $\alpha \in \Sigma^\omega$ if *all* runs of $\mathcal{A}$ on α are accepting. Compare the expressive power of deterministic, nondeterministic, and universal Büchi automata.

4. **Lower Bounds on Complementation** (presented by Group 03)

   a) Consider the following family of Büchi automata $\mathcal{A}_n$ over the alphabet $\Sigma = \{1, \ldots, n, \#\}$, where all states are initial states. Describe $\mathcal{L}(\mathcal{A}_n)$.



b) Prove: For all $n \geq 1$, there is no Büchi automaton $\mathcal{A}'_n$ with $\mathcal{L}(\mathcal{A}'_n) = \Sigma^\omega \setminus \mathcal{L}(\mathcal{A}_n)$ which has less than $n!$ states.

Suggestion: Suppose $\mathcal{A}'_n$ exists. Consider two distinct permutations $(a_1, \ldots, a_n)$ and $(b_1, \ldots, b_n)$ of $(1, \ldots, n)$. Let r_1 be an accepting run of $\mathcal{A}'_n$ on $(a_1 \ldots a_n \, \#)^\omega$ and let r_2 be an accepting run of $\mathcal{A}'_n$ on $(b_1 \ldots b_n \, \#)^\omega$. Show that $\mathrm{Inf}(r_1) \cap \mathrm{Inf}(r_2) = \emptyset$.

5. **co-Büchi Automata (challenge question)**

A co-Büchi automaton $\mathcal{A} = (\Sigma, Q, I, T, \text{COBÜCHI}(C))$ is an automaton over infinite words, where the acceptance condition $\text{COBÜCHI}(C)$ is defined by

$$\text{COBÜCHI}(C) = \{\alpha \in Q^\omega \mid \mathrm{Inf}(\alpha) \cap C = \emptyset\},$$

i.e., a run on $\mathcal{A}$ is accepting, if every state in C is visited only finitely often.

a) Prove or disprove: an ω-regular language is co-Büchi-recognizable if and only if it is Büchi-recognizable.

b) Prove or disprove: an ω-regular language is co-Büchi-recognizable if and only if it is recognizable by a deterministic co-Büchi automaton.