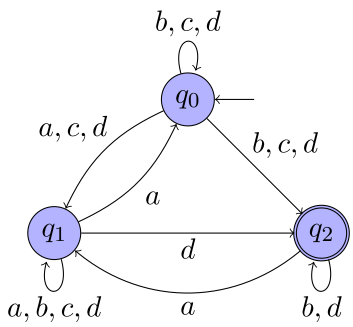
Prof. Bernd Finkbeiner, Ph.D.
Felix Klein, M.Sc.

Automata, Games, and Verification

Please send a mail to `agv15@react.uni-saarland.de` if you can't make it to the discussion session.

1. **Run DAGs** (presented by Group 02)

   Let $\Sigma = \{a, b, c, d\}$ be an alphabet, $\alpha = ddbac^\omega$ be a word over this alphabet, and $\mathcal{A}$ be the following Büchi automaton over Σ:



   a) Draw the run DAG for $\mathcal{A}$ on α. As the DAG is infinite, you only need to sketch it in a way such that it is, intuitively, clear how it is to be continued after a certain pattern emerges.

   b) Reason whether α is accepted by $\mathcal{A}$.

   c) Finally, write down the sequence of DAGs $G_0 \supseteq G_1 \supseteq G_2 \dots$ as defined in the proof of Lemma 5.1 of the lecture.

2. **Co-Limit Operation** (presented by Group 04)

   The *co-limit* of W is defined as $\overleftarrow{W} = \{\alpha \in \Sigma^\omega \mid \text{there exist only finitely many } n \in \mathbb{N} \text{ s.t. } \alpha[0,n] \in W\}$.

   Let $V, W \subseteq \Sigma^*$ be two regular languages. Prove or give a counter example to the following statements:

   a) $\overleftarrow{(V \cdot W)} = V \cdot \overleftarrow{W}$

   b) $V \cdot \overleftarrow{W}$ is Büchi-recognizable

   c) $V \cdot \overleftarrow{W}$ is recognizable by a deterministic Büchi automaton

3. **Deterministic Muller Automata** (presented by Group 10)

   a) Give an ω-regular expression E such that the smallest deterministic Muller automaton recognizing $\mathcal{L}(E)$ is larger than the smallest nondeterministic Muller automaton recognizing $\mathcal{L}(E)$, and prove this fact.

   b) For all $n \in \mathbb{N}$, let Z_n describe the set of languages recognizable by deterministic Muller automata with at most n accepting subsets (i.e., for every language in Z_n, there exists a corresponding deterministic Muller automaton $\mathcal{A} = (\Sigma, Q, I, T, \text{MULLER}(\mathcal{F}))$ with $|\mathcal{F}| \leq n$). Obviously, $Z_1 \subseteq Z_2 \subseteq Z_3 \subseteq \dots$ holds. Prove that this sequence of inequalities is strict, i.e., $Z_1 \subset Z_2 \subset Z_3 \subset \dots$ holds as well.

4. **More Acceptance Conditions** (presented by Group 14)

   Besides Büchi and Muller automata, there are three further important automata types:

   - A *parity automaton* is an automaton $(\Sigma, Q, I, T, \textsc{parity}(c))$ with the *parity acceptance condition* $\textsc{parity}(c)$ defined for a coloring function $c\colon Q \to \mathbb{N}$ by

     $$\textsc{parity}(c) = \{\alpha \in Q^\omega \mid \max\{c(q) \mid q \in \mathrm{Inf}(\alpha)\} \text{ is even}\}.$$

   - A *Rabin automaton* is an automaton $(\Sigma, Q, I, T, \textsc{rabin}(\{(A_i, R_i) \mid i \in J\}))$ with the *Rabin acceptance condition* $\textsc{rabin}(\{(A_j, R_j) \mid j \in J\})$ defined for a set of indices J and sets A_j, R_j for each $j \in J$ by

     $$\textsc{rabin}(\{(A_j, R_j) \mid j \in J\}) = \{\alpha \in Q^\omega \mid \exists j \in J.\ \mathrm{Inf}(\alpha) \cap A_j \neq \emptyset \text{ and } \mathrm{Inf}(\alpha) \cap R_j = \emptyset\}$$

   - A *Streett automaton* is an automaton $(\Sigma, Q, I, T, \textsc{streett}(\{(A_i, R_i) \mid i \in J\}))$ with the *Streett acceptance condition* $\textsc{streett}(\{(A_j, R_j) \mid j \in J\})$ defined for a set of indices J and sets A_j, R_j for each $j \in J$ by

     $$\textsc{streett}(\{(A_j, R_j) \mid j \in J\}) = \{\alpha \in Q^\omega \mid \forall j \in J.\ \mathrm{Inf}(\alpha) \cap A_j \neq \emptyset \text{ or } \mathrm{Inf}(\alpha) \cap R_j = \emptyset\}$$

   Compare the expressive power of Büchi, Muller, Rabin, Streett and parity automata. Which automata types are equally expressive? Which are less expressive than others? Provide proofs for all your claims.

5. **Complementation of Büchi automata via Büchi's Characterization Theorem (Challenge)**

   In this problem, we develop an alternative to the complementation construction from Lectures 4 and 5. Let $\mathcal{A}$ be a nondeterministic Büchi automaton over the alphabet Σ.

   a) Show that Σ^ω can be represented as a finite union $\bigcup_{i=1,\ldots,n} V_i \cdot W_i^\omega$ such that

      - for all $i = 1, \ldots, n$: V_i and W_i are regular languages $V_i, W_i \subseteq \Sigma^*$, and
      - for all $i = 1, \ldots, n$, either $V_i \cdot W_i^\omega \cap \mathcal{L}(\mathcal{A}) = \emptyset$ or $V_i \cdot W_i^\omega \subseteq \mathcal{L}(\mathcal{A})$.

      (Suggestion: For a finite word w, consider (1) the pairs of states of $\mathcal{A}$ that are connected by a path labeled with w, and (2) the pairs of states of $\mathcal{A}$ that are connected by a path that visits an accepting state and that is labeled with w. Let two finite words be equivalent if they agree on these pairs. Show that the equivalence classes can be represented as finite-word automata.)

   b) Use Büchi's characterization theorem to argue that there exists a nondeterministic Büchi automaton $\mathcal{A}'$ such that $\mathcal{L}(\mathcal{A}') = \Sigma^\omega \setminus \mathcal{L}(\mathcal{A})$.

   c) Prove or disprove the following claim for regular languages $V, W \subseteq \Sigma^*$:

      $$\Sigma^\omega \setminus (V \cdot W^\omega) = (\Sigma^* \setminus V) \cdot (\Sigma^* \setminus W)^\omega$$