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Problem Set 6
Discussions: June 3th, 2015

Automata, Games, and Verification

Please send a mail to `agv15@react.uni-saarland.de` if you can't make it to the discussion session.

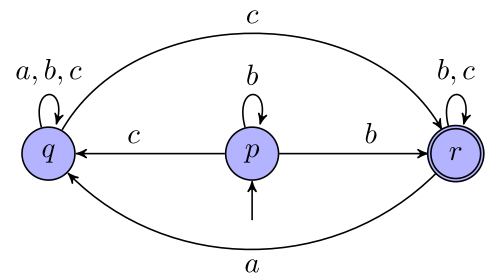
1. **Language Emptiness** (presented by Group 09)

   An automaton is called *empty* if its language is empty. Describe a method to test language emptiness for

   - nondeterministic Büchi automata $\mathcal{B} = (\Sigma, Q, I, T, \text{BÜCHI}(F))$,
   - nondeterministic Rabin automata $\mathcal{R} = (\Sigma, Q, I, T, \text{RABIN}(\{(A_j, R_j) \mid j \in J\})$, and
   - nondeterministic Muller automata $\mathcal{M} = (\Sigma, Q, I, T, \text{MULLER}(\mathcal{F}))$.

2. **Semi-deterministic automata** (presented by Group 06)

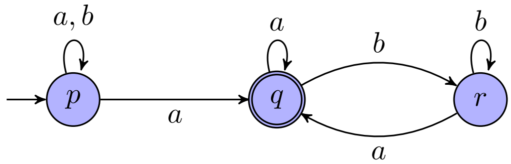
   Let $\Sigma = \{a, b, c\}$ be an alphabet and $\mathcal{A}$ be the following Büchi automaton over Σ having the states $\{p, q, r\}$:



   Construct an equivalent semi-deterministic automaton using the construction of the lecture.

3. **McNaughton's Theorem** (presented by Group 12)

   Consider the following semi-deterministic Büchi automaton over $\Sigma = \{a, b\}$:



   Construct an equivalent deterministic Muller automaton using the construction from the lecture.

4. **From Deterministic Muller to Deterministic Parity** (presented by Group 07)

   Let $\mathcal{A} = (\Sigma, Q, \{q_0\}, T, \text{MULLER}(\mathcal{F}))$ be a complete and deterministic Muller automaton. For a finite set $S = \{s_1, s_2, \ldots, s_n\}$ we denote by $\mathcal{P}(S)$ the set of permutations over S, i.e., $(s_1, s_2, s_3, \ldots, s_n) \in \mathcal{P}(S)$ is a such permutation, $(s_2, s_1, s_3, \ldots, s_n) \in \mathcal{P}(S)$ is another one, and so forth. Show that $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}')$ for the deterministic parity automaton $\mathcal{A}' = (\Sigma, Q, \{q_0'\}, T, \text{PARITY}(c))$ with

   - $Q' = \mathcal{P}(Q) \times \{1, \ldots, 2|Q|\}$,
   - $q_1' = (p, 1)$, where p is some permutation that starts with q_1, i.e., $pr_0(p) = q_1$,
   - $T' = \{((p, j), \sigma, (p', j')) \mid p'$ results from $p = (q_1, q_2, \ldots, q_n)$ by moving q_j with $(q_1, \sigma, q_j) \in T$ to the first position, i.e., $p' = (q_j, q_1, q_2, \ldots, q_{j-1}, q_{j+1}, \ldots, q_n)$,
   $$j' = \begin{cases} 2j & \text{if } \{q_1, q_2, \ldots, q_j\} \in \mathcal{F} \\ 2j - 1 & \text{otherwise} \end{cases}$$
   $\}$, and
   - $c(p, j) = j$ for all $(p, j) \in Q$.

5. **Deterministic Parity Automata (Challenge)**

   Show that deterministic parity automata are closed under negation, union and intersection. Use direct constructions from deterministic parity automata to deterministic parity automata, i.e., you are not allowed to use constructions to other automata models as intermediate steps.