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Problem Set 7
Discussions: June 10th, 2015

Automata, Games, and Verification

Please send a mail to `agv15@react.uni-saarland.de` if you can't make it to the discussion session.

1. **Deterministic Automata** (presented by Group 14)

   Compare the expressive power of *deterministic* Muller, Rabin, Streett and parity automata.

2. **LTL & Determininistic Automata** (presented by Group 13)

   a) Compare the expressive power of LTL and deterministic Büchi automata.

   b) Compare the expressive power of LTL and deterministic co-Büchi automata.

3. **Weak Until** (presented by Group 01)

   Consider the new *weak until* operator $\mathcal{W}$, whose semantics are defined as follows:

   $$\alpha \models \varphi_1 \, \mathcal{W} \, \varphi_2 \;\text{ iff }\; \forall i \geq 0.\; \alpha[i,\infty] \models \varphi_1 \text{ or } \exists j < i.\; \alpha[j,\infty] \models \varphi_2$$

   Show that $\{\neg, \wedge, \bigcirc, \mathcal{W}\}$ is an operator basis for LTL, i.e., show that you can express every LTL formula φ as an equivalent LTL formula φ', in which apart from atomic propositions, only the operators $\neg$, $\wedge$, $\bigcirc$ and $\mathcal{W}$ are used.

4. **LTL & Nondeterministic Automata** (presented by Group 11)

   Prove or give a counter-example to the following statements: For every LTL formula φ there exists

   a) a nondeterministic Büchi automaton $\mathcal{A}_\varphi$ with a *single* accepting state such that $\mathcal{L}(\mathcal{A}_\varphi) = \mathcal{L}(\varphi)$.

   b) a nondeterministic co-Büchi automaton $\mathcal{A}'_\varphi$ such that $\mathcal{L}(\mathcal{A}'_\varphi) = \mathcal{L}(\varphi)$.

   c) a nondeterministic co-Büchi automaton $\mathcal{A}''_\varphi$ with a *single* accepting state such that $\mathcal{L}(\mathcal{A}''_\varphi) = \mathcal{L}(\varphi)$.

5. **Temporal Operators (Challenge)**

   Show that $\{\neg, \wedge, \bigcirc, \Diamond, \Box\}$ is *not* an operator basis for LTL.

   Suggestion: Find two families of label sequences α_n, β_n, and an LTL formula φ, such that

   a) for every n, we have that $\alpha_n \models \varphi$ and $\beta_n \not\models \varphi$,

   b) but for every LTL formula ψ *without* $\mathcal{U}$, there is an n, such that either $\alpha_n \models \psi$ and $\beta_n \models \psi$, or $\alpha_n \not\models \psi$ and $\beta_n \not\models \psi$.