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Problem Set 9
Discussions: June 24th, 2015

Automata, Games, and Verification

Please send a mail to `agv15@react.uni-saarland.de` if you can't make it to the discussion session.

1. **S1S and Büchi** (presented by Group 13)

   Apply the construction introduced in the lecture to build a complete Büchi automaton recognizing the language defined by the following S1S formula:

   $$\forall x.\, x \in P \leftrightarrow x = 0$$

   Hint: Minimize intermediate automata resulting from intersection.

2. **WS1S** (presented by Group 01)

   Give a WS1S formula φ_i for each of the following languages L_i over $\Sigma = 2^{\{a,b\}}$ such that $L_i = \mathcal{L}(\varphi_i)$.

   a) $L_1 = \{\alpha \in \Sigma^\omega \mid \alpha \text{ contains infinitely many } a\text{'s}\}$

   b) $L_2 = \{\alpha \in \Sigma^\omega \mid \alpha \text{ contains only finitely many } b\text{'s}\}$

   c) $L_3 = \{\alpha \in \Sigma^\omega \mid \alpha \text{ contains no more than 3 } a\text{'s and at least one } b\}$

   d) $L_4 = \{\alpha \in \Sigma^\omega \mid \alpha$ contains only finitely many positions $n \in \mathbb{N}$ such that $\alpha[0, n]$ contains an even number of a's and an odd number of $b's\}$

3. **Complete Alternating Büchi Automata** (presented by Group 04)

   An alternating automaton is called *complete* iff neither $true$ nor $false$ are in the mapping of δ (run trees of complete alternating automata have only infinite branches and every input word has a run tree).

   Prove or give a counter-example to the following: Every language recognizable by an alternating Büchi automaton is also recognizable by a complete alternating Büchi automaton.

4. **Alternating Parity Automata** (presented by Group 03)

   Let $\mathcal{A}_1 = (\Sigma, Q_1, q_0^1, \delta_1, \text{PARITY}(c_1))$ and $\mathcal{A}_2 = (\Sigma, Q_2, q_0^2, \delta_2, \text{PARITY}(c_2))$ be two alternating parity automata with disjoint sets of states $Q_1 \cap Q_2 = \emptyset$. Prove or give a counter-example for each of the following statements:

   a) The language $\mathcal{L}(\mathcal{A}_1) \cup \mathcal{L}(\mathcal{A}_2)$ is recognizable by an alternating parity automaton.

   b) The language $\mathcal{L}(\mathcal{A}_1) \cap \mathcal{L}(\mathcal{A}_2)$ is recognizable by an alternating parity automaton.

   c) The language $\Sigma^\omega \setminus \mathcal{L}(\mathcal{A}_1)$ is recognizable by an alternating parity automaton.

5. **Alternating vs. Deterministic Automata (Challenge)**

   Consider the following family of languages L_n:

   $$L_n = \{v_1 \# \, w \, v_2 \, \$ \, w \, \alpha \mid v_1, v_2 \in \{0, 1, \#\}^*,\ w \in \{0, 1\}^n,\ \alpha \in \{0, 1, \#, \$\}^\omega\}$$

   a) Construct a family $\mathcal{A}_n$ of alternating Büchi automata with $\mathcal{L}(\mathcal{A}_n) = L_n$ such that each automaton $\mathcal{A}_n$ has only $\mathcal{O}(n)$ states.

   b) Show that any deterministic Muller automaton that recognizes L_n has at least 2^{2^n} states.