Automata, Games, and Verification

Please send a mail to agv15@react.uni-saarland.de if you can't make it to the discussion session.

1. Consider Construction 8.2 of the lecture notes. Which of the following alternative definitions for Q' do not affect the correctness of Theorem 8.2?

$$\Box \quad Q' = 2^Q \times 2^{Q \setminus F} \qquad \Box \quad Q' = 2^Q \times 2^Q \setminus \{\emptyset\}$$
$$\Box \quad Q' = 2^{Q \setminus F} \times 2^Q \qquad \Box \quad Q' = 2^Q \setminus \{\emptyset\} \times 2^Q$$

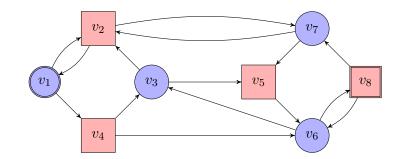
2. Consider the arena $\mathcal{A} = (V, V, \emptyset, V \times V)$ for some arbitrary set V. Does there exist a reachability game using this arena where Player 1 always wins?

$$\Box$$
 Yes \Box No

3. Let $\mathcal{G} = ((V, V_0, V_1, E), \text{REACH}(R))$ be some reachability game. Which of the following are true?

$$\Box W_1(\mathcal{G}) = Attr_1(V \smallsetminus R) \qquad \Box W_1(\mathcal{G}) = V \smallsetminus Attr_0(R) \qquad \Box W_1(\mathcal{G}) = V \smallsetminus Attr_1(R)$$
$$\Box Attr_0^{|V|}(R) = Attr_0^{|V|+1}(R) \qquad \Box (v, v') \in E \cap V_0 \times V_0 \Rightarrow (v' \in W_0(\mathcal{G}) \Rightarrow v \in W_0(\mathcal{G}))$$

4. Consider the Büchi game $\mathcal{G} = (\mathcal{A}, BÜCHI(\{v_1, v_8\}))$ given below. Which are the winning regions of the two players?



- $\square W_0(\mathcal{G}) = \{v_3, v_5, v_6, v_7, v_8\} \quad W_1(\mathcal{G}) = \{v_1, v_2, v_4\}$
- $\Box W_0(\mathcal{G}) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_8\} \quad W_1(\mathcal{G}) = \emptyset$
- $\square W_0(\mathcal{G}) = \emptyset \quad W_1(\mathcal{G}) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$
- $\Box W_0(\mathcal{G}) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\} \quad W_1(\mathcal{G}) = \emptyset$
- $\square W_0(\mathcal{G}) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\} \quad W_1(\mathcal{G}) = \{v_2, v_4\}$
- 5. Let $\mathcal{G} = ((V, V_0, V_1, E), \text{REACH}(R))$ be some arbitrary reachability game. Can you always create a Büchi game $\mathcal{G}' = ((V', V'_0, V'_1, E'), \text{BÜCHI}(F))$ such that

$$V_0 \subseteq V'_0, \quad V_1 \subseteq V'_1, \quad W_0(\mathcal{G}) = W_0(\mathcal{G}') \cap V \text{ and } W_1(\mathcal{G}) = W_1(\mathcal{G}') \cap V?$$

 \Box Yes \Box No

6. Let $\mathcal{G} = ((V, V_0, V_1, E), \text{BUCHI}(F))$ be some arbitrary Büchi game. Can you always create a reachability game $\mathcal{G}' = ((V', V'_0, V'_1, E'), \text{REACH}(R))$ such that

$$V_0 \subseteq V'_0, \quad V_1 \subseteq V'_1, \quad W_0(\mathcal{G}) = W_0(\mathcal{G}') \cap V \text{ and } W_1(\mathcal{G}) = W_1(\mathcal{G}') \cap V?$$

 \Box Yes \Box No