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Automata, Games, and Verification

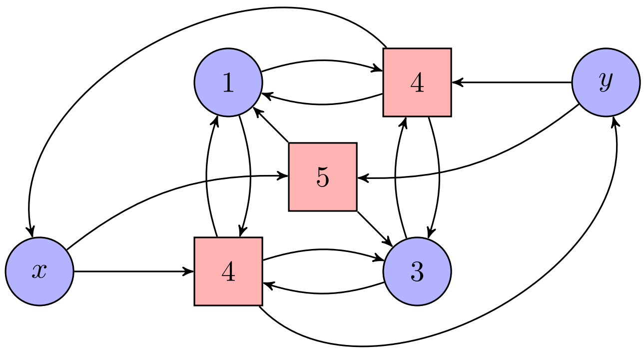
Please send a mail to `agv15@react.uni-saarland.de` if you can't make it to the discussion session.

1. Let $\mathcal{G} = (\mathcal{A}, \text{BÜCHI}(F))$ be a Büchi game and $n \in \mathbb{N}$. Which of the following are true?

   ☐ $\mathit{Recur}^{n+1}(F) \subseteq \mathit{Recur}^{n}(F)$ ☐ $W_1^{n+1} \subseteq W_1^{n}$ ☐ $W_1(\mathcal{G}) = \bigcup_{m\in\mathbb{N}} W_1^m$

   ☐ $\mathit{Recur}^{n}(F) \subseteq \mathit{Recur}^{n+1}(F)$ ☐ $W_1^{n} \subseteq W_1^{n+1}$ ☐ $W_1(\mathcal{G}) = \bigcap_{m\in\mathbb{N}} W_1^m$

2. Consider the parity game given below, where each position is marked with its color. Which values can you choose for x and y such that



Player 0 wins from each position:

$y \backslash x$	0	1	2	3	4	5
0	☐	☐	☐	☐	☐	☐
1	☐	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	☐

Player 1 wins from each position:

$y \backslash x$	0	1	2	3	4	5
0	☐	☐	☐	☐	☐	☐
1	☐	☐	☐	☐	☐	☐
2	☐	☐	☐	☐	☐	☐
3	☐	☐	☐	☐	☐	☐
4	☐	☐	☐	☐	☐	☐
5	☐	☐	☐	☐	☐	☐

3. Which games, equipped with one of the following winning conditions, are memoryless determined?

   ☐ Reachability ☐ Büchi ☐ Street ☐ Rabin ☐ Parity ☐ Muller

4. Consider a modified version of the parity winning condition PARITY^+, which additionally enforces that each play has to visit each successor of a position v if v is visited at all, i.e., we have

   $$\text{PARITY}^+(c) = \text{PARITY}(c) \cap \{\rho \in V^\omega \mid \forall e \in (E \cap (\mathrm{Occ}(\rho) \times V)).\ \exists n \in \mathbb{N}.\ e = (\rho(n), \rho(n+1))\}.$$

   Now let $\mathcal{G} = (\mathcal{A}, \text{PARITY}(c))$ be a normal parity game and $\mathcal{G}' = (\mathcal{A}, \text{PARITY}^+(c))$ be its modified version. Which are valid choices for $\circ_1$ and $\circ_2$ such that $W_0(\mathcal{G}) \circ_1 W_0(\mathcal{G}')$ and $W_1(\mathcal{G}) \circ_2 W_1(\mathcal{G}')$?

   ☐ $\circ_1 \in \{=\}$ ☐ $\circ_1 \in \{\subseteq\}$ ☐ $\circ_1 \in \{\supseteq\}$ ☐ $\circ_1 \in \{\neq\}$

   ☐ $\circ_2 \in \{=\}$ ☐ $\circ_2 \in \{\subseteq\}$ ☐ $\circ_2 \in \{\supseteq\}$ ☐ $\circ_2 \in \{\neq\}$

5. Consider some winning condition Win and some transformation t on Win such that for the games $\mathcal{G} = (\mathcal{A}, \mathit{Win})$ and $\mathcal{G}' = (\mathcal{A}, t(\mathit{Win}))$ we have that $W_0(\mathcal{G}) = W_1(\mathcal{G}')$ and $W_1(\mathcal{G}) = W_0(\mathcal{G}')$. For which of the following types of winning conditions does a transformation t exist such that its type is preserved by t?

   ☐ Reachability ☐ Büchi ☐ Street ☐ Rabin ☐ Parity ☐ Muller