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Warm-up Questions 5
Discussions: May 20th, 2015

Automata, Games, and Verification

Please send a mail to agv15@react.uni-saarland.de if you can't make it to the discussion session.

1. Which of the following are true?

   ☐ Each level ranking is a ranking. ☐ Each ranking is a level ranking.

2. Consider the automaton $\mathcal{A}'$ of Construction 5.1, constructed from a given Büchi automaton $\mathcal{A}$. If $\mathcal{A}$ has n states, then $\mathcal{A}'$ has exactly

   ☐ $2 \cdot n^2$ states. ☐ $2^{n+1} \cdot n^2$ states. ☐ $2^{n+1} \cdot n^2 + 2^{n+1} \cdot n$ states.

   ☐ 2^{n+1} states. ☐ $2^n \cdot n^2$ states. ☐ $2^{n+1} \cdot n^2 + 2^n \cdot n$ states.

3. Let $\mathcal{A} = (\Sigma, Q, I, T, \text{MULLER}(\mathcal{F}))$ be a Muller automaton and $Q = \{q_0\}$ be a singleton set. How many languages can you express by choosing I, T and $\mathcal{F}$?

   ☐ 1 ☐ 2 ☐ $|\Sigma|$ ☐ $2|\Sigma|$ ☐ $2^{|\Sigma|}$ ☐ $2^{|\Sigma|} + 1$

   ☐ $2^{|\Sigma|+1}$ ☐ $2^{|\Sigma|+2}$ ☐ $2^{|\Sigma|} + 2$

4. Which language does the Muller automaton $\mathcal{A} = (\Sigma, Q, I, T, \text{MULLER}(\{\emptyset\}))$ accept?

   ☐ Σ^ω ☐ $\emptyset$ ☐ it depends

5. Let $\mathcal{A}_1 = (\Sigma, Q_1, I_1, T_1, \text{MULLER}(\mathcal{F}_1))$ and $\mathcal{A}_2 = (\Sigma, Q_2, I_2, T_2, \text{MULLER}(\mathcal{F}_2))$ be two Muller automata with $Q_1 \cap Q_2 = \emptyset$ and let the Muller automaton $\mathcal{A}$ be defined by

   $$\mathcal{A} = (\Sigma, Q_1 \cup Q_2, I_1 \cup I_2, T_1 \cup T_2, \text{MULLER}(\mathcal{F}_1 \cup \mathcal{F}_2)).$$

   Does $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{A}_1) \cup \mathcal{L}(\mathcal{A}_2)$ hold?

   ☐ Yes ☐ No

6. Consider Construction 6.2 of the lecture notes. Which of the following alternative definitions for F' are also correct with respect to Theorem 6.2?

   ☐ $F' = \bigcup_{i=1}^{n} \{(i, \max(F_i), \max(F_i))\}$ ☐ $F' = \bigcup_{i=1}^{n} \bigcup_{q \in F_i} \{(i, q, q)\}$

   ☐ $F' = \bigcup_{i=1}^{n} \{(i, \min(F_i), \max(F_i))\}$ ☐ $F' = \bigcup_{i=1}^{n} \bigcap_{q \in F_i} \{(i, q, q)\}$

7. Consider the Büchi automaton $\mathcal{A}'$ of Construction 6.2, constructed from the Muller automaton $\mathcal{A}$. Is every run of $\mathcal{A}$ also a run of $\mathcal{A}'$?

   ☐ Yes ☐ No

8. Consider the following alternative method to complement a deterministic Büchi automaton $\mathcal{A}$. We first translate $\mathcal{A}$ into a Muller automaton, then complement this Muller automaton and then translate it back into a Büchi automaton. Let $\mathcal{A}'$ be the Büchi automaton resulting from Construction 5.1 and $\mathcal{A}''$ be the Büchi automaton resulting from aforementioned construction. Which of the following are true?

   ☐ There is a deterministic Büchi automaton $\mathcal{A}$ such that $\mathcal{A}'$ has less states than $\mathcal{A}''$.

   ☐ There is a deterministic Büchi automaton $\mathcal{A}$ such that $\mathcal{A}''$ has less states than $\mathcal{A}'$.