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Warm-up Questions 8
Discussions: June 17th, 2015

**Automata, Games, and Verification**

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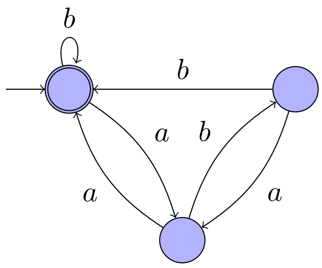
Please send a mail to `agv15@react.uni-saarland.de` if you can't make it to the discussion session.

1. Consider the following QPTL formula $\varphi$ and the Büchi automaton $\mathcal{A}$. Does $\mathcal{L}(\varphi) = \mathcal{L}(\mathcal{A})$ hold?

$$\varphi = \exists q.\ q \wedge \Box\Diamond(q \wedge (b \vee a \wedge \bigcirc a)) \ \wedge \ \Box((q \rightarrow b \wedge \bigcirc q \vee a \wedge \neg \bigcirc q) \wedge (\neg q \rightarrow \bigcirc q))$$




☐ Yes ☐ No

2. Which of the following are syntactically correct QPTL-formulas? ($a \in \mathrm{AP}$)

☐ $\exists p.\ ((p \wedge \bigcirc p) \ \wedge \neg\exists q.\ (a\ \mathcal{U}\, q))$ ☐ $\exists p.\ (a \wedge \exists q.\ \neg p \wedge q)$ ☐ $\Diamond\exists p.\ \Box(p \wedge a)$
☐ $\exists p.\ \bigcirc\exists q.\ q \wedge p \wedge a$ ☐ $\Diamond\Diamond\Box\Diamond\Box\Diamond\Diamond\Diamond\Box\Box\Diamond\Box\Box\Diamond\Box\Box\Diamond\Box\Box\Diamond\Box a$

3. Consider the LTL formula $\varphi = a\,\mathcal{U}\, b$. Which is the minimum number of existentially quantified variables, each QPTL formula $\psi$ with $\mathcal{L}(\varphi) = \mathcal{L}(\psi)$ contains?

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6 ☐ 7

4. Which of the following are true?

☐ Every Büchi-recognizable language is S1S definable.
☐ $S(\neg x) = S(y)$ is a syntactically correct S1S formula.
☐ In S1S we can test whether a second order variable $X$ has infinitly many members or not.

5. For every QPTL formula $\varphi$ there exists some $n \in \mathbb{N}$ and another QPTL formula

$$\varphi' = \exists x_1.\ \exists x_2.\ \ldots \exists x_n.\ \psi,$$

where $\psi$ is an LTL formula, such that $\mathcal{L}(\varphi') = \mathcal{L}(\varphi)$.

☐ True ☐ False

6. Which of the following QPTL-definable languages are non-empty?

☐ $\mathcal{L}(\forall p.\ (\Box\Diamond p \wedge \Box(p \rightarrow \Diamond q)) \rightarrow \Box\Diamond q)$
☐ $\mathcal{L}(\forall p.\ \exists q.\ \Box(q \leftrightarrow \Diamond\Box p))$
☐ $\mathcal{L}(\exists p, q.\ \Box((\neg a \wedge p \rightarrow \bigcirc q \wedge \bigcirc\bigcirc q) \wedge (\neg a \wedge q \rightarrow \bigcirc q \wedge \bigcirc\bigcirc p \vee \bigcirc p) \wedge \neg(p \wedge q) \wedge \Diamond\Box\neg p))$
☐ $\mathcal{L}(\exists p.\ \forall q. \Box(\Diamond q \rightarrow \Diamond p) \wedge a)$

7. Consider the S1S formulas $\varphi$ and $\psi$, given below. Which of the following is true?

$$\varphi = \forall X.\ \forall y.\ (y \in A \rightarrow y \in X) \wedge (y \in X \rightarrow S(y) \in X) \wedge (\exists z.\ y \neq z \wedge z \in A \wedge z \in X)$$
$$\psi = \exists X.\ \forall y.\ (y \in A \rightarrow y \notin X) \wedge (y \in X \rightarrow S(y) \in X)$$

☐ $\mathcal{L}(\varphi) \subseteq \mathcal{L}(\psi)$ ☐ $\mathcal{L}(\psi) \subseteq \mathcal{L}(\varphi)$ ☐ $\mathcal{L}(\psi) \cap \mathcal{L}(\varphi) = \emptyset$