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Warm-up Questions 9
Discussions: June 24th, 2015

**Automata, Games, and Verification**

Please send a mail to `agv15@react.uni-saarland.de` if you can't make it to the discussion session.

1. Which of the following S1S-formulas are $S1S_0$ formulas?

   ☐ $\exists x.\, \neg(x \in Y \vee x = S(0))$ ☐ $\exists X.\, y \in X \vee y = S(y) \vee 0 \in X$
   ☐ $\exists x.\, (x \in X \wedge \exists x.\, x = y)$ ☐ $\exists x.\, S(S(S(S(x)))) = 0$
   ☐ $\neg(x \vee \neg(y \vee \neg(\exists Y.\, x = S(y))))$ ☐ $0 \in X \vee S(0) \in X \vee S(S(0)) \in X$

2. Consider the Büchi automaton below. Which of the following S1S formulas define the same language?


   ☐ $0 \in X$ ☐ $x \in X$ ☐ $x = 0$ ☐ $\exists x. x \in X$ ☐ $x = x$ ☐ $x = S(x)$

3. Let $\varphi$ be a S1S formula over the first-order variables $V_1$ and the second-order variables $V_2$ and let $V_1' \subseteq V_1$ and $V_2' \subseteq V_2$ be the corresponding free first- and second-order variables, respectively. Furthermore, let $y, z$ and $z'$ be fresh first-order variables, i.e., $\{y, z, z'\} \cap V_1 = \emptyset$. Select a minimal subset of the following rules, which suffices to eliminate all free first-order variables from $\varphi$ without changing the language?

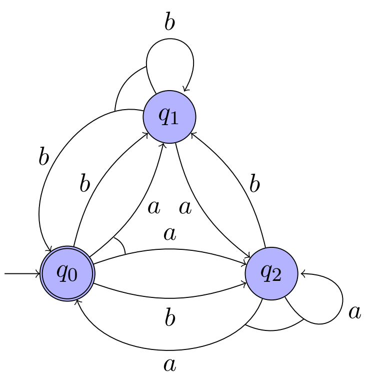
   For each free first-order variable $X \in V_1'$ of $\varphi$ do:

   ☐ Replace each $X$ in $\varphi$ by the first-order variable $y$ and add $y$ to $V_1$.
   ☐ Replace each sub-formula $X = t$ and $t = X$ in $\varphi$ by $t \in X$ for any term $t$.
   ☐ Interpret $X$ as second-order variable, i.e. move $X$ from $V_1$ ($V_1'$) to $V_2$ ($V_2'$).
   ☐ Replace each sub-formula $t = S(X)$ and $S(X) = t$ in $\varphi$ by $S(t) \in X$ for any term $t$.
   ☐ Convert $\varphi$ to an equivalent $S1S_0$ formula.
   ☐ Apply the rule $\psi \;\mapsto\; \psi \wedge \exists z.\, z \in X \wedge \forall z'.\, z' \in X \rightarrow z' = z$ and add $z, z'$ to $V_1$.
   ☐ Replace each sub-formula $X \in Y$ in $\varphi$ by $X \subseteq Y$.
   ☐ Apply the rule $\psi \;\mapsto\; \exists y.\, y \in X \wedge \forall z.\, z \in X \rightarrow y = z \wedge \psi$ and add $y, z$ to $V_1$.

4. Every deterministic Büchi automaton is an alternating Büchi automaton.

   ☐ True ☐ False

5. Consider the following alternating Büchi automaton $\mathcal{A}$. Does there exist a word $\alpha \in \{a, b\}^\omega$ that has an accepting run tree on $\mathcal{A}$?




   ☐ Yes ☐ No