Decision Procedures for Verification

Homework 4

Problem 1

Consider the following procedure basic presented in class.

```
procedure basic(phi, alpha) {
  if ([phi | alpha] is empty) return SAT ;
  if ([phi | alpha] contains an empty clause) return UNSAT
  Pick a letter p in [phi | alpha]
  if (basic(phi, alpha p) = SAT)
     return SAT
  else
     return basic(phi, alpha ~p)
}
```

(a) Prove that basic is terminating.
(b) Prove that $\Phi \mid \alpha$ is satisfiable iff basic(φ, α) returns SAT
(c) Using (b), show how to use basic in order to check whether a CNF Φ is satisfiable or not.

Problem 2

Is the following algorithm terminating? Why?

```
procedure basic2(phi, alpha) {
  if ([phi | alpha] is empty) return SAT ;
  if ([phi | alpha] contains an empty clause) return UNSAT
  Pick a letter p in phi
  if (basic2(phi, alpha p) = SAT)
     return SAT
  else
     return basic2(phi, alpha ~p)
}
```

Problem 3

Consider the resolution rule

$$\frac{\begin{array}{c} C \vee p \\ D \vee \neg p \end{array}}{C \vee D}$$

A *resolution derivation* of a CNF Φ is a sequence $C_1, \ldots, C_n$ of clauses such that, for each $i = 1, \ldots, n$, either $C_i \in \Phi$, or C_i has been obtained by means of the resolution rule from some C_j, C_k, with $j, k < i$.
A *resolution proof* of a CNF Φ is a resolution derication $C_1, \ldots, C_n$ such that C_n is the empty clause.
In this homework we prove that resolution is sound and complete.

(a) Let $C_1, \ldots, C_n$ be a resolution derivation of Φ. Assume that Φ is satisfiable. By induction on k, prove that $C_1 \wedge \cdots \wedge C_k$ is satisfiable, for every $k \leq n$.
(b) Using (a), prove that resolution is **sound**, that is, prove that if there exists a resolution proof of Φ, then Φ is unsatisfiable.
(c) Using the results of Problem 1, *and what was said in class*, prove that resolution is **complete**, that is, prove that if Φ is unsatisfiable, then there exists a resolution derivation of Φ.
(d) As it is described in this problem, is resolution terminating? Why?