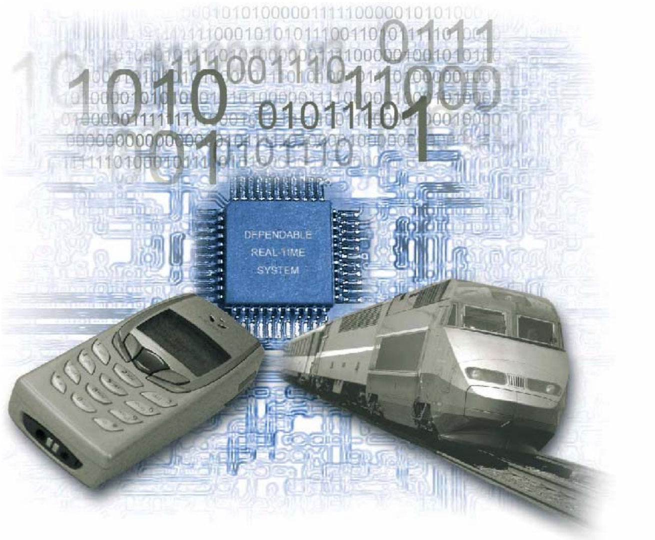
Embedded Systems 5



Exam Dates / Registration

- Current problem set: Please send mdl file to es08@react.cs.uni-sb.de

- **Midterm**, Thursday December 18, 2008, 16-18
- **Final**, Thursday February 12, 2009, 16-19

- Registration through **HISPOS** (open in approx. 1 week)
- (If HISPOS not applicable – Non-CS, Erasmus, etc – send email to finkbeiner@cs.uni-sb.de)

- No separate course sign-up
  BUT: Please indicate tutorial, matr nr, name, e-mail on **homework submissions.**

Petri Nets

REVIEW

Def.: $N=(C,E,F)$ is called a **Petri net**, iff the following holds

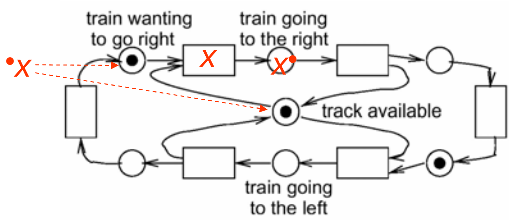
1. C and E are disjoint sets
2. $F \subseteq (C \times E) \cup (E \times C)$; is binary relation, („**flow relation**“)

Def.: Let N be a net and let $x \in (C \cup E)$.

$^\bullet x := \{y \mid y\ F\ x\}$ is called the set of **preconditions.**

$x^\bullet := \{y \mid x\ F\ y\}$ is called the set of **postconditions.**

Competing Trains Example:

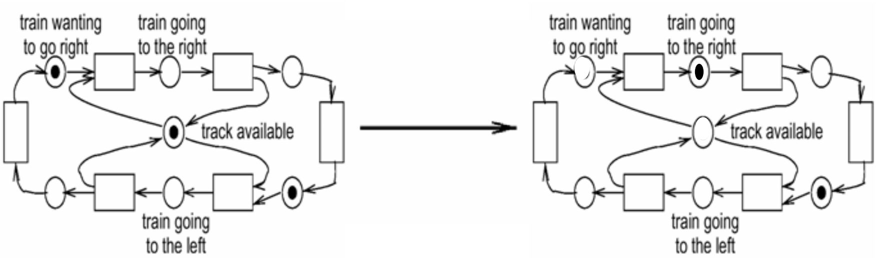


Boolean marking and computing changes of markings

REVIEW

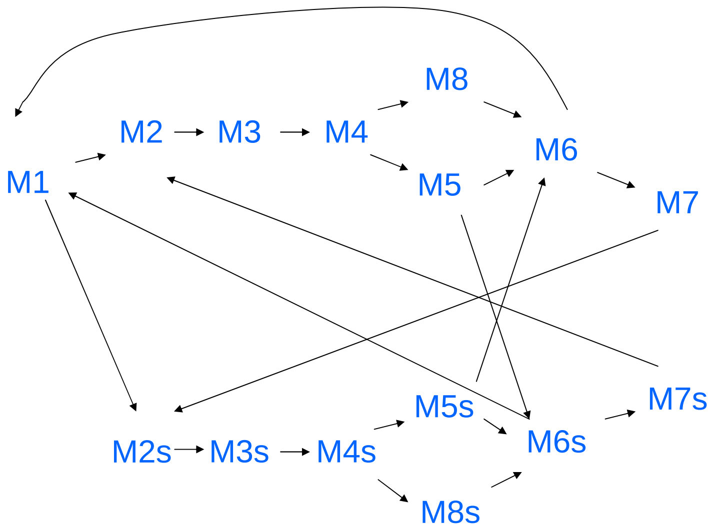
- A Boolean marking is a mapping M: C → { 0,1 }.
- „Firing“ events x generate new markings on each of the conditions c according to the following rules:

  a transition at x can be fired, iff $^\bullet x$, *i.e.* all preconditions of x are marked and $x^\bullet$ is not marked, after firing $^\bullet x$ *is* unmarked and $x^\bullet$ is marked
- M → M', iff M' results from M by firing exactly one transition



Competing Trains Example is C/E net:

REVIEW

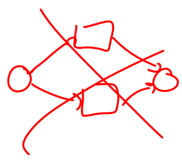


Condition/event nets (C/E nets)

REVIEW

Def.:
A Petri net $N=(C,E,F)$ together with a set of markings M is called **condition/event net (C/E net)**, iff

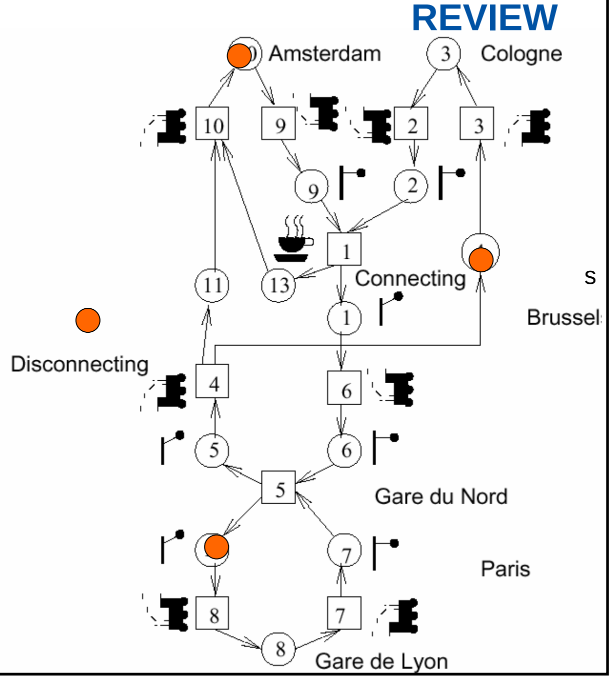
- N is simple and has no isolated elements
- M is closed w.r.t. "firing" and "inverse firing"
- two markings in M can be transformed into each other by "firing" and "inverse firing"
- for each event $e \in E$, there exists a marking in M, that allows firing at e



Example Thalys trains: more complex

REVIEW

- Thalys trains between Cologne, Amsterdam, Brussels and Paris.
- Synchronization at Brussels and Paris

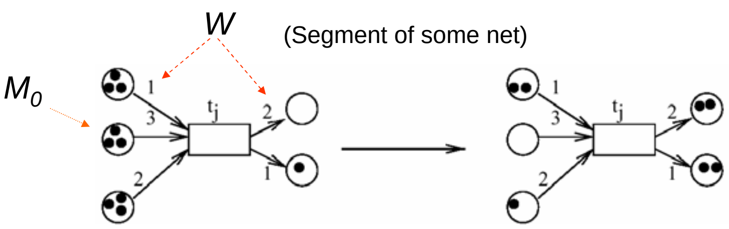


Place/transition nets

REVIEW

Def.: (P, T, F, K, W, M_0) is called a **place/transition net (P/T net)** iff

1. $N=(P,T,F)$ is a **net** with places $p \in P$ and transitions $t \in T$
2. $K: P \rightarrow (\mathbf{N}_0 \cup \{\omega\}) \setminus \{0\}$ denotes the **capacity** of places (ω symbolizes infinite capacity)
3. $W: F \rightarrow (\mathbf{N}_0 \setminus \{0\})$ denotes the **weight of graph edges**
4. $M_0: P \rightarrow \mathbf{N}_0 \cup \{\omega\}$ represents the **initial marking** of places

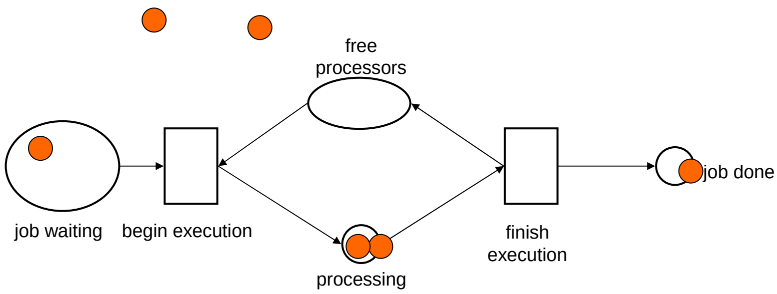


(Segment of some net)

defaults:
$K = \omega$
$W = 1$

In the following: assume initial marking is finite, capacity ω.

Example: Job processing system

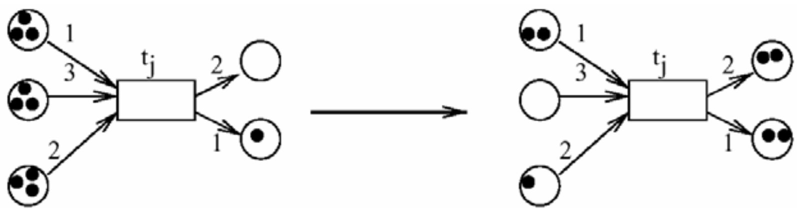


Computing changes of markings

REVIEW

- „Firing“ transitions *t* generate new markings on each of the places *p* according to the following rules:

$$M'(p) = \begin{cases} M(p) - W(p,t), & \text{if } p \in {}^{\bullet}t \setminus t^{\bullet} \\ M(p) + W(t,p), & \text{if } p \in t^{\bullet} \setminus {}^{\bullet}t \\ M(p) - W(p,t) + W(t,p), & \text{if } p \in {}^{\bullet}t \cap t^{\bullet} \\ M(p) & \text{otherwise} \end{cases}$$

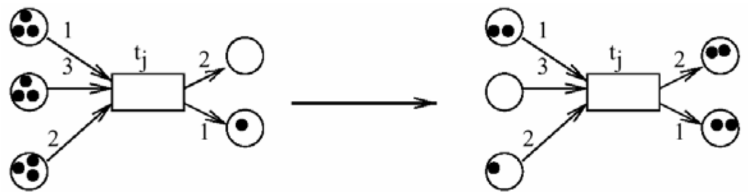


Activated transitions

REVIEW

- Transition *t* is „activated“ iff

$$(\forall p \in {}^{\bullet}t : M(p) \geq W(p,t)) \wedge (\forall p \in t^{\bullet} : M(p) + W(t,p) \leq K(p))$$



Activated transitions can „take place“ or „fire“,
but don‘t have to.
The order in which activated transitions fire is not fixed
(it is non-deterministic).

Shorthand for changes of markings

REVIEW

Firing transition:

$$M'(p) = \begin{cases} M(p) - W(p,t), & \text{if } p \in {}^{\bullet}t \setminus t^{\bullet} \\ M(p) + W(t,p), & \text{if } p \in t^{\bullet} \setminus {}^{\bullet}t \\ M(p) - W(p,t) + W(t,p), & \text{if } p \in {}^{\bullet}t \cap t^{\bullet} \\ M(p) & \text{otherwise} \end{cases}$$

Let

$$\underline{t}(p) = \begin{cases} -W(p,t) \text{ if } p \in {}^{\bullet}t \setminus t^{\bullet} \\ +W(t,p) \text{ if } p \in t^{\bullet} \setminus {}^{\bullet}t \\ -W(p,t) + W(t,p) \text{ if } p \in t^{\bullet} \cap {}^{\bullet}t \\ 0 \end{cases}$$

$\Rightarrow$ $\forall p \in P$: $M'(p) = M(p) + \underline{t}(p)$

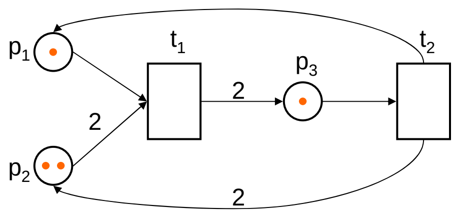
$\Rightarrow$ $M' = M + \underline{t}$ +: vector add

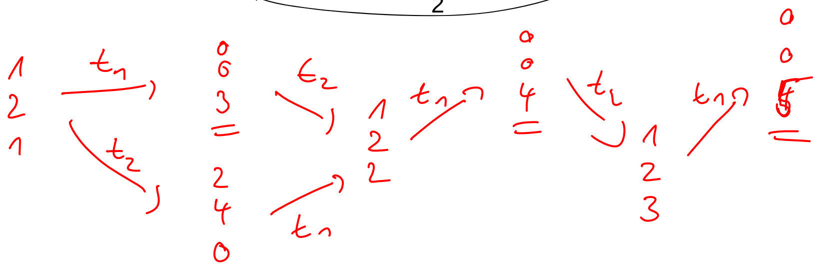
Reachability relation

- M [t> M` iff $M' = M + \underline{t}$
- M [ε> M` iff M=M`
- M [qt> M` iff ∃ M`` . M [q> M`` and M`` [t> M`
- M [*> M‘ iff ∃ q . M [q> M`

- **Reachability set** R(M) = {M‘ | M [*> M‘ }
- **Reachability graph** RG(M):
  nodes R(M), edges { (M,t,M`) | M [t> M´ }

Unbounded Petri net





Boundedness

- A place is **safe** if it contains at most one token in all reachable markings.
- A net is **safe** if all places are safe.
- A place is ***k*-bounded** if it contains at most *k* token in all reachable markings.
- A place is **bounded** if it is *k*-bounded for some *k*.
- A net is **bounded** if each place is bounded.

Boundedness

Theorem 1: A P/T net (with finite initial marking) is bounded iff its reachability set is finite.

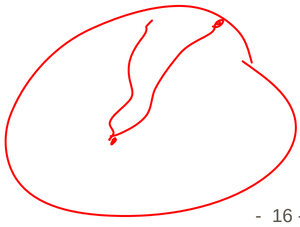
"$\Rightarrow$" Let k be the max bound

$|R(M_0)| \leq k^{|P|}$

"$\Leftarrow$" Let m_0 be the max marking in M_0

Let w be the max weight

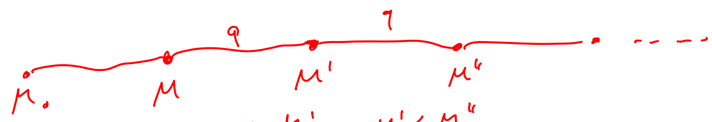
$k \leq m_0 + |R(M_0)| \cdot w$



Boundedness

Theorem 2: A P/T net is unbounded iff there exist two reachable markings M, M', such that M[*>M' and M' > M.

"$\Leftarrow$": · Since $M' > M$, $\exists p$ s.t. $M'(p) > M(p)$
· p is unbounded:



$M < M'$ $M' < M''$

· Since $M' > M$, q can fire from M' as well
· " $M'' > M'$, q can " " M'' " "

- - -

Proof of Theorem 2

Lemma: Every infinite sequence of markings (M_i) contains a weakly monotonically growing infinite subsequence ($M`_i$), i.e., for j<k, $M`_j \le M`_k$.

$\begin{pmatrix}1\\1\end{pmatrix}$ $\cancel{\begin{pmatrix}0\\2\end{pmatrix}}$ $\cancel{\begin{pmatrix}10\\0\end{pmatrix}}$ $\cancel{\begin{pmatrix}5\\2\end{pmatrix}}$ $\begin{pmatrix}11\\2\end{pmatrix}$ ...

1) Find a subsequence (M_i'') that is weakly monotonic in first place
2) Find a subsequence (M_i''') that is (also) weakly monotonic in second place

...

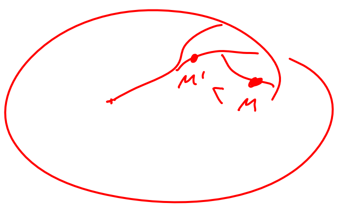
Proof of Theorem 2

"$\Rightarrow$"

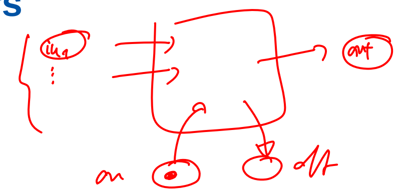
- Let G be the spanning tree of $RG(M_0)$
- G is infinite & finitely branching
  $\longrightarrow \exists$ infinite branch
- By lemma, $\exists$ weakly monotonically growing subsequence of markings, hence,
  $M_0 [*\rangle M_1 [*\rangle M_2$, $M_1 \leq M_2$
  $M_1 \neq M_2$

Algorithm for deciding boundedness

- Explore $RG(M_0)$ depth-first:
  - If there exists a marking M‘ on the stack such that M‘<M, stop with result UNBOUNDED;
- If entire graph explored, return BOUNDED.



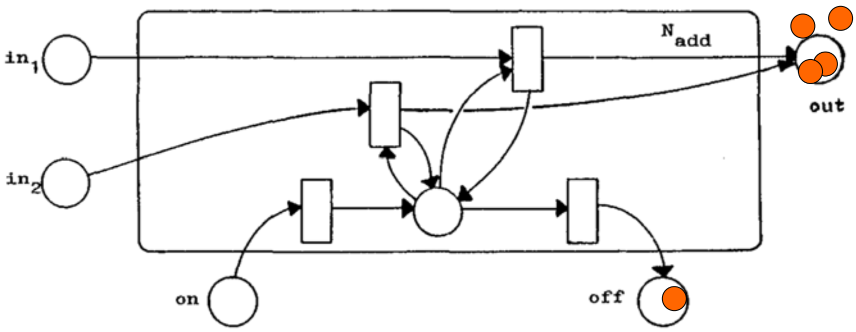
Weak Petri net computers



A P/T net with
r distinguished input places (in_i),
a finite number of internal places s_i,
one extra output place (out),
one extra start place (on), and
one extra stop place (off)
is called a **weak Petri net computer** for the function f: $N^r \rightarrow N$
iff there exists for each $x \in N^r$ an initial marking M_x
such that

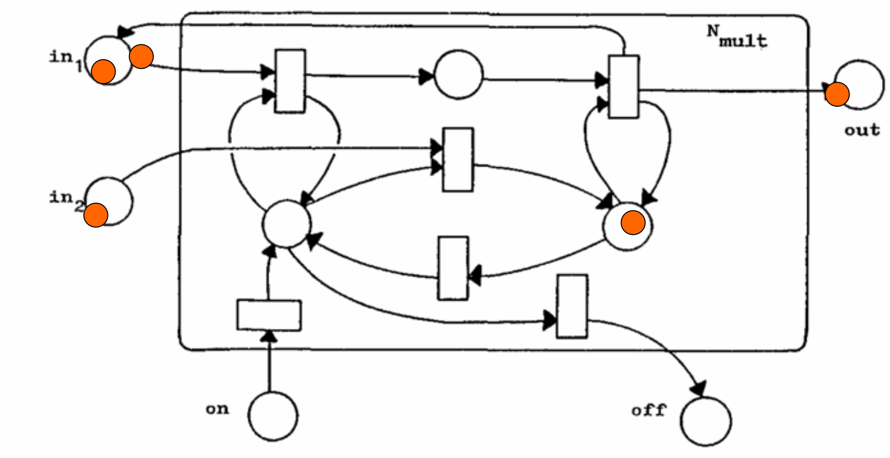
- $M_x(on)=1$ and $M_x(in_i)=\mathbf{x_i}$ for $1 \leq x_i \leq r$;
- $M_x(out)=M_x(off)$; = 0
- $M_x(s_i)=0$;
- For all reachable markings $M \neq M_x$, $M(on)=0$ and $0 \leq M(off) \leq 1$ and $M(out) \leq f(x)$;
- For all reachable markings $M \neq M_x$, if $M(off)=1$ then M is dead;
- For all $0 \leq k \leq f(x)$, there exists a reachable marking M such that $M(out)=k$ and $M(off)=1$.

Addition



Source: Matthias Jantzen, Complexity of Place/Transition Nets (1986)

Multiplication



Source: Matthias Jantzen, Complexity of Place/Transition Nets (1986)

RG(M_0) is not primitive recursive

Proofidea:

Let A(n):N → N be defined by A(n)=A_n(2)

- $A_0(m)=2m+1$
- $A_{n+1}(0)=1$
- $A_{n+1}(m+1)=A_n(A_{n+1}(m))$

$g = A_{n+1}$, $f = A_n$

$g(m+1) = \underbrace{f(f(\dots f}_{m+1}(f(0)))$

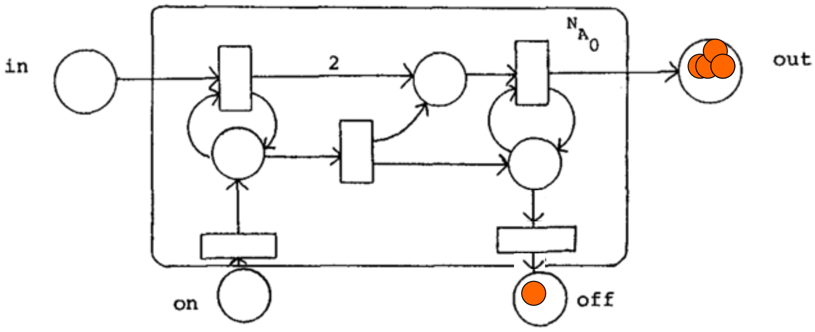
A(0)=5; A(1)=7; A(2)=15; A(3)=65535; …

A(n) majorizes the primitive recursive functions.

We can implement A(n) as a P/T net (that grows slowly with n).

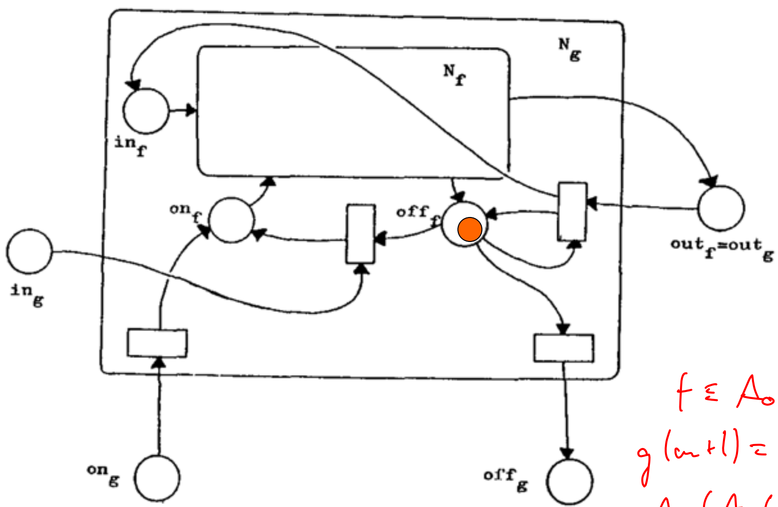
Source: Matthias Jantzen, Complexity of Place/Transition Nets (1986)

$A_0(m)=2m+1$



g(m+1):=f(g(m)), g(0)=f(0)

$\underbrace{f(f(\ldots}_{m} f(0) \ldots)$



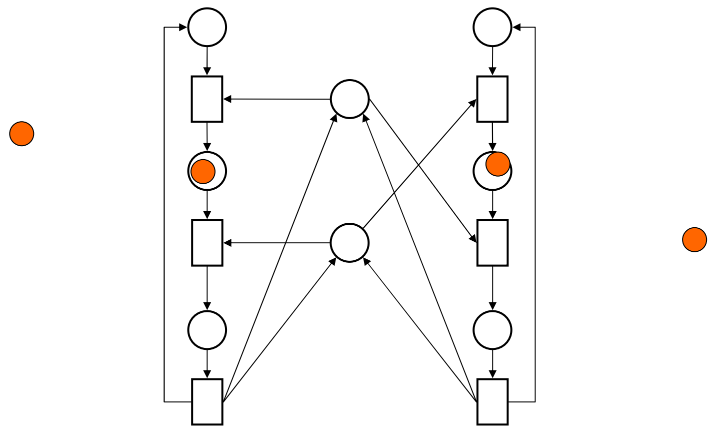
$f \in A_0$

$g(m+1) =$

$A_0(A_n(m))$

Deadlock

- A **dead marking (deadlock)** is a marking where no transition can fire.
- A Petri net is **deadlock-free** if no dead marking is reachable.



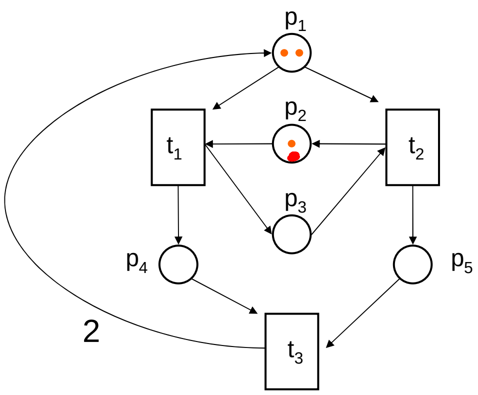
Liveness

- A **transition** t is **dead** at M if no marking M‘ is reachable from M such that t can fire in M‘.
- A **transition** t is **live** at M if there is no marking M‘ reachable from M where t is dead.
- A **marking** is **live** if all transitions are live.
- A **P/T net** is **live** if the initial marking is live.

Observations:

- A live net is deadlock-free.
- No transition is live if the net is not deadlock-free.

Example Liveness



(2 1 0 0 0) t1 t2 t3

$\downarrow t_1$

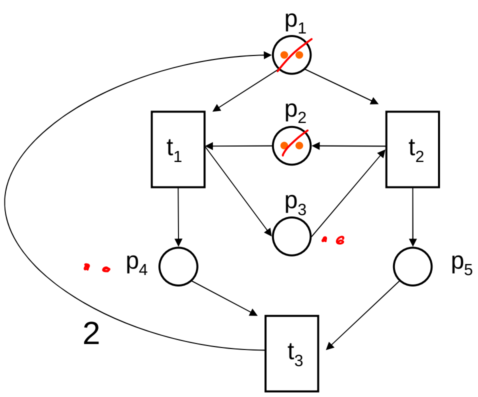
(1 0 1 1 0) t1 t2 t3

$\downarrow t_2$

(0 1 0 1 1) t3 t1 t2

t3

Example Liveness



(2, 2, 0, 0, 0)

$\downarrow t_1$

(1, 1, 1, 1, 0)

$\downarrow t_1$ t1

$\boxed{(0, 0, 2, 2, 0)}$

Reversiblity

- A P/T net is **reversible** iff for every $M \in R(M_0)$, M [*> M_0
- **Theorem 3:** In a reversible net, a transition t is live iff t is not dead in M_0.

"$\Rightarrow$" ✓

"$\Leftarrow$" : · t is not dead at M_0

$\Rightarrow \exists M'$, M_0 [*> M', t can fire in M'

· Let M be an arbitrary reachable marking

since net is reversible, M [*> M_0 [*> M'

$\Rightarrow$ t is not dead in M

$\Rightarrow$ t is live.

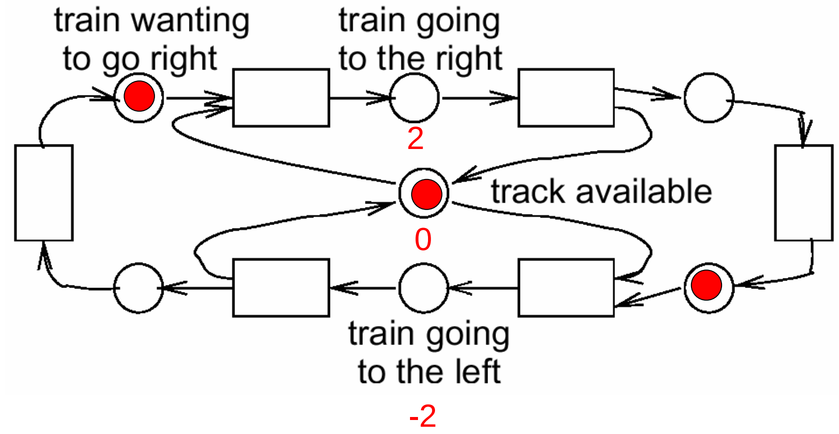
Computation of Invariants

We are interested in subsets *R* of places whose number of labels remain invariant under transitions,

•e.g. the number of trains commuting between Amsterdam and Paris (Cologne and Paris) remains constant

Important for correctness proofs, e.g. the proof of liveness

Competing Trains Example: Place Invariant 2,0,-2



Shorthand for changes of markings

REVIEW

Firing transition:

$$M'(p) = \begin{cases} M(p) - W(p,t), & \text{if } p \in {}^{\bullet}t \setminus t^{\bullet} \\ M(p) + W(t,p), & \text{if } p \in t^{\bullet} \setminus {}^{\bullet}t \\ M(p) - W(p,t) + W(t,p), & \text{if } p \in {}^{\bullet}t \cap t^{\bullet} \\ M(p) & \text{otherwise} \end{cases}$$

Let

$$\underline{t}(p) = \begin{cases} -W(p,t) \text{ if } p \in {}^{\bullet}t \setminus t^{\bullet} \\ +W(t,p) \text{ if } p \in t^{\bullet} \setminus {}^{\bullet}t \\ -W(p,t) + W(t,p) \text{ if } p \in t^{\bullet} \cap {}^{\bullet}t \\ 0 \end{cases}$$

$\Rightarrow$ $\forall p \in P$: $M'(p) = M(p) + \underline{t}(p)$

$\Rightarrow$ $M' = M + \underline{t}$ +: vector add

Matrix $\underline{N}$ describing all changes of markings

REVIEW

$$\underline{t}(p) = \begin{cases} -W(p,t) \text{ if } p \in {}^{\bullet}t \setminus t^{\bullet} \\ +W(t,p) \text{ if } p \in t^{\bullet} \setminus {}^{\bullet}t \\ -W(p,t) + W(t,p) \text{ if } p \in t^{\bullet} \cap {}^{\bullet}t \\ 0 \end{cases}$$

Def.: Matrix $\underline{N}$ of net N is a mapping

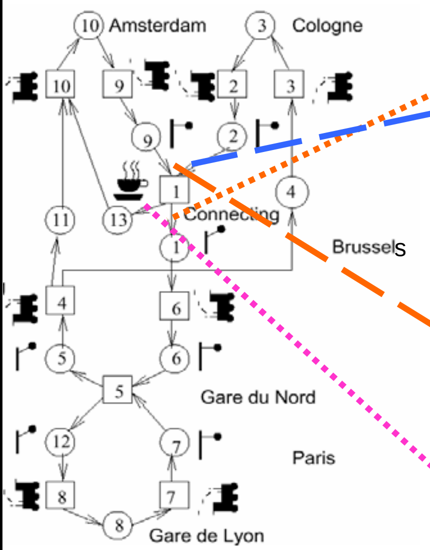
$\underline{N}: P \times T \to Z$ (integers)

such that $\forall\, t \in T$: $\underline{N}(p,t) = \underline{t}(p)$

Component in column t and row p indicates the change of the marking of place p if transition t takes place.

Example: $\underline{N}$ =

REVIEW



	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
p_1	1					–1				
p_2	–1	1								
p_3		–1	1							
p_4			–1	1						
p_5				–1	1					
p_6					–1	1				
p_7					–1		1			
p_8							–1			
p_9	–1							1	1	
p_{10}									–1	1
p_{11}				1						–1
p_{12}					1			–1		
p_{13}	1									–1

Characteristic Vector

$$\sum_{p \in R} \underline{t}_j(p) = 0$$

Let:

$$\underline{c}_R(p) = \begin{cases} 1 \text{ if } & p \in R \\ 0 \text{ if } & p \notin R \end{cases}$$

$$\Rightarrow \quad \sum_{p \in R} \underline{t}_j(p) = \underline{t}_j \cdot \underline{c}_R = \sum_{p \in P} \underline{t}_j(p)\, \underline{c}_R(p) = 0$$

Scalar product

Condition for place invariants

$$\sum_{p \in R} \underline{t}_j(p) = \underline{t}_j \cdot \underline{c}_R = \sum_{p \in P} \underline{t}_j(p)\, \underline{c}_R(p) = 0$$

Accumulated marking constant for **all** transitions if

$$\begin{array}{ccc} \underline{t}_1 \cdot \underline{c}_R & = & 0 \\ \dots & \dots & \dots \\ \underline{t}_n \cdot \underline{c}_R & = & 0 \end{array}$$

Equivalent to $\underline{N}^T\,\underline{c}_R = 0$ where $\underline{N}^T$ is the transposed of $\underline{N}$

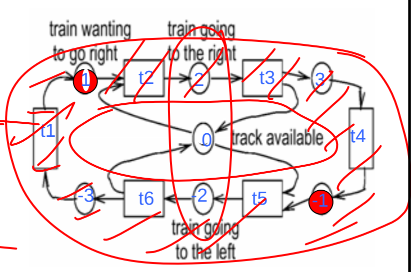
More detailed view of computations

$$\begin{pmatrix} \underline{t}_1(p_1) \dots \underline{t}_1(p_n) \\ \underline{t}_2(p_1) \dots \underline{t}_2(p_n) \\ \dots \\ \underline{t}_m(p_1) \dots \underline{t}_m(p_n) \end{pmatrix} \begin{pmatrix} \underline{c}_R(p_1) \\ \underline{c}_R(p_2) \\ \dots \\ \underline{c}_R(p_n) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

System of linear equations.

Solution vectors must consist of zeros and ones.

Competing Trains Example



	p_1	p_2	p_3	p_{-1}	p_{-2}	p_{-3}	p_0
t_1	+1					−1	
t_2	−1	+1					−1
t_3		−1	+1				+1
t_4			−1	+1			
t_5				−1	+1		−1
t_6					−1	+1	+1
A t_1	+1					−1	
B t_1+t_2		+1				−1	−1
C B+t_3			+1			−1	
D C+t_4				+1		−1	
D+t_5					+1	−1	−1

$c_0 = 0, \; \underline{c_{-3} = 1}$

$\Rightarrow \underline{c_{-2} = 1},$

$\underline{c_{-1} = 1}, \; \underline{c_1 = 1},$

$\underline{c_2 = 1}, \; c_3 = \underline{1}$

$\underline{c_0 = 1}, \; c_3 = 0$

$\Rightarrow \underline{c_{-2} = 1},$

$c_{-1} = 0, \; c_7 = 0,$

$\underline{c_2 = 1}, \; c_1 = 0$