Embedded Systems 5

Exam Dates / Registration

- **Midterm**, Thursday December 18, 2008, 16-18
- **Final**, Thursday February 12, 2009, 16-19

- Registration through **HISPOS** (open in approx. 1 week)
- (If HISPOS not applicable – Non-CS, Erasmus, etc –
  send email to finkbeiner@cs.uni-sb.de)

- No separate course sign-up
  BUT: Please indicate tutorial, matr nr, name, e-mail
  on **homework submissions.**

Petri Nets

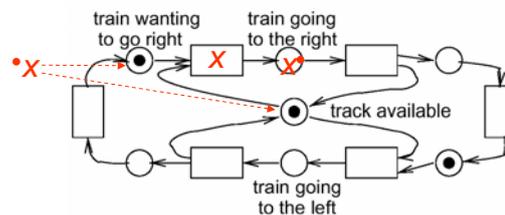**Def.:** $N=(C,E,F)$ is called a **Petri net**, iff the following holds
1. C and E are disjoint sets
2. $F \subseteq (C \times E) \cup (E \times C)$; is binary relation, („**flow relation**")

Def.: Let N be a net and let $x \in (C \cup E)$.
$\bullet x := \{y \mid y\,F\,x\}$ is called the set of **preconditions.**
$x\bullet := \{y \mid x\,F\,y\}$ is called the set of **postconditions.**
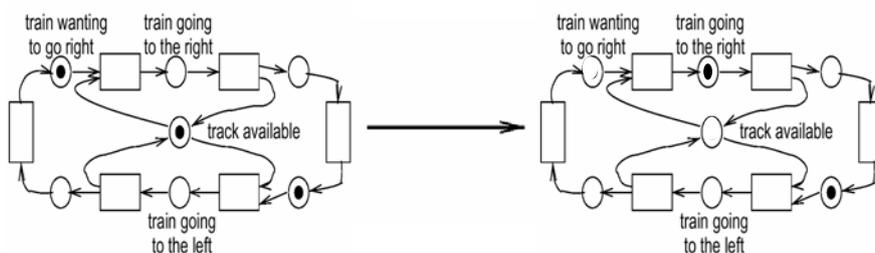
Competing Trains Example:

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**Boolean marking
and computing changes of markings**
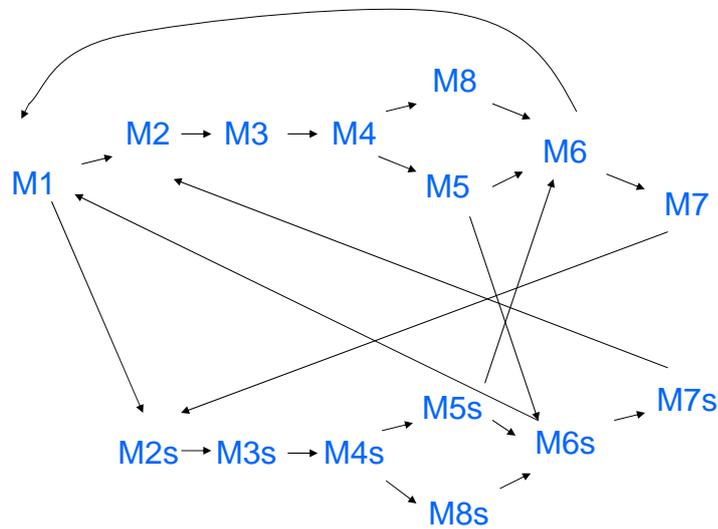
- A Boolean marking is a mapping $M: C \rightarrow \{0,1\}$.
- „Firing" events x generate new markings on each of the conditions c according to the following rules:

  a transition at x can be fired, iff $\bullet x$, *i.e.* all preconditions of x are marked and $x\bullet$ is not marked, after firing $\bullet x$ *is* unmarked and $x\bullet$ is marked
- $M \rightarrow M'$, iff M' results from M by firing exactly one transition

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Competing Trains Example is C/E net:

Condition/event nets (C/E nets)

Def.:
A Petri net $N=(C,E,F)$ together with a set of
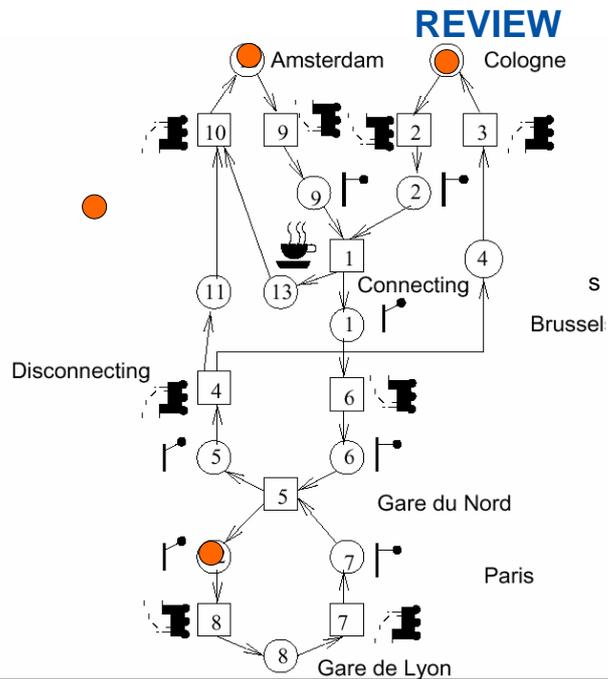markings M is called **condition/event net (C/E net)**, iff

- N is simple and has no isolated elements
- M is closed w.r.t. "firing" and "inverse firing"
- two markings in M can be transformed into each other by "firing" and "inverse firing"
- for each event $e \in E$, there exists a marking in M, that allows firing at e

Example Thalys trains: more complex

- Thalys trains between Cologne, Amsterdam, Brussels and Paris.
- Synchronization at Brussels and Paris

Amsterdam   Cologne

10   9   2   3

9   2

1   4

Connecting   s

1   Brussel

11   13

Disconnecting

4   6

5   6

5   Gare du Nord

7   Paris

8   7

8   Gare de Lyon

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Place/transition nets

Def.: (P, T, F, K, W, M_0) is called a **place/transition net (P/T net)** iff

1. $N=(P,T,F)$ is a **net** with places $p \in P$ and transitions $t \in T$
2. $K: P \to (N_0 \cup \{\omega\}) \setminus \{0\}$ denotes the **capacity** of places (ω symbolizes infinite capacity)
3. $W: F \to (N_0 \setminus \{0\})$ denotes the **weight of graph edges**
4. $M_0: P \to N_0 \cup \{\omega\}$ represents the **initial marking** of places

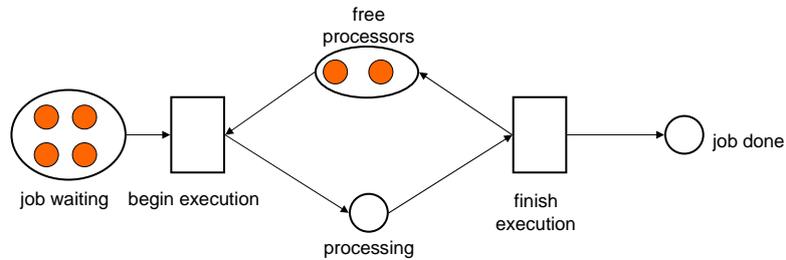M_0 W (Segment of some net)

t_j

t_j
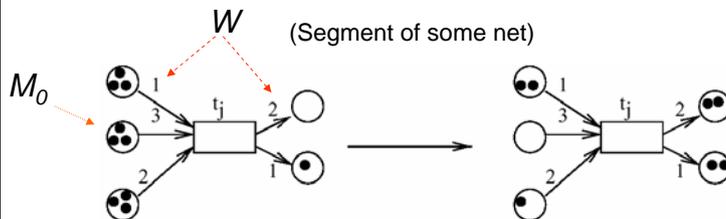
defaults:
$K = \omega$
$W = 1$

In the following: assume initial marking is finite, capacity ω.

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Example: Job processing system

free processors

job waiting   begin execution

processing

finish execution

job done

Computing changes of markings   REVIEW

- „Firing" transitions *t* generate new markings on each of the places *p* according to the following rules:

$$
M'(p) = \begin{cases}
M(p) - W(p,t), & \text{if } p \in {}^\bullet t \setminus t^\bullet \\
M(p) + W(t,p), & \text{if } p \in t^\bullet \setminus {}^\bullet t \\
M(p) - W(p,t) + W(t,p), & \text{if } p \in {}^\bullet t \cap t^\bullet \\
M(p) & \text{otherwise}
\end{cases}
$$
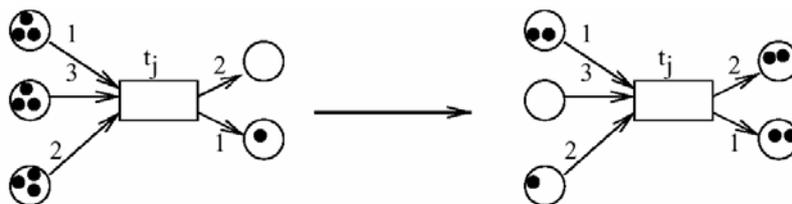
Activated transitions

- Transition t is „activated" iff

$$(\forall p \in {}^\bullet t : M(p) \geq W(p,t)) \wedge (\forall p \in t^\bullet : M(p) + W(t,p) \leq K(p))$$

Activated transitions can „take place" or „fire",
but don't have to.
The order in which activated transitions fire is not fixed
(it is non-deterministic).

Shorthand for changes of markings

Firing transition:

$$M'(p) = \begin{cases} M(p) - W(p,t), & \text{if } p \in {}^\bullet t \setminus t^\bullet \\ M(p) + W(t,p), & \text{if } p \in t^\bullet \setminus {}^\bullet t \\ M(p) - W(p,t) + W(t,p), & \text{if } p \in {}^\bullet t \cap t^\bullet \\ M(p) & \text{otherwise} \end{cases}$$
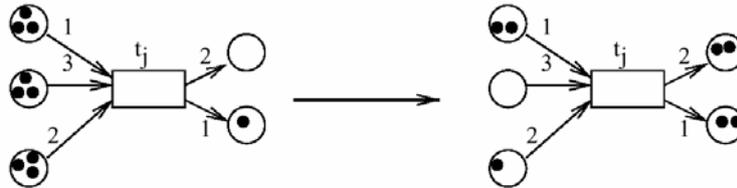
Let

$$\underline{t}(p) = \begin{cases} -W(p,t) \text{ if } p \in {}^\bullet t \setminus t^\bullet \\ +W(t,p) \text{ if } p \in t^\bullet \setminus {}^\bullet t \\ -W(p,t) + W(t,p) \text{ if } p \in t^\bullet \cap {}^\bullet t \\ 0 \end{cases}$$

$\Rightarrow$ $\qquad \forall p \in P:\ M'(p) = M(p) + \underline{t}(p)$

$\Rightarrow$ $\qquad\qquad M' = M + \underline{t}$ $\qquad\qquad$ +: vector add

Reachability relation

- M [t> M` iff $M´ = M + \underline{t}$
- M [ε> M` iff M=M`
- M [qt> M` iff ∃ M`` .  M [q> M`` and M`` [t> M`
- M [*> M' iff ∃ q . M [q> M`

- **Reachability set** R(M) = {M' | M [*> M' }
- **Reachability graph** RG(M):
  nodes R(M), edges { (M,t,M`) | M [t> M´ }

Unbounded Petri net
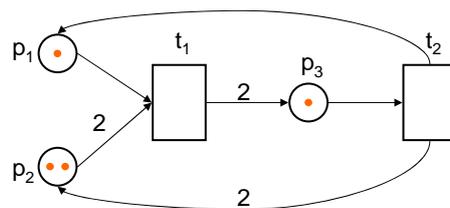
Boundedness

- A place is **safe** if it contains at most one token in all reachable markings.
- A net is **safe** if all places are safe.
- A place is *k***-bounded** if it contains at most k token in all reachable markings.
- A place is **bounded** if it is *k*-bounded for some *k*.
- A net is **bounded** if each place is bounded.

Boundedness

Theorem 1: A P/T net (with finite initial marking) is bounded iff its reachability set is finite.

Boundedness

Theorem 2: A P/T net is unbounded iff there exist two reachable markings M, M', such that
M[*>M' and M' > M.

Proof of Theorem 2

Lemma: Every infinite sequence of markings (M_i) contains a weakly monotonically growing infinite subsequence $(M`_i)$, i.e., for j<k, $M`_j \leq M`_k$.

Proof of Theorem 2

Algorithm for deciding boundedness

- Explore $RG(M_0)$ depth-first:
  - If there exists a marking M' on the stack such that M'<M, stop with result UNBOUNDED;
- If entire graph explored, return BOUNDED.

Weak Petri net computers

A P/T net with
   r distinguished input places (in_i),
   a finite number of internal places s_i,
   one extra output place (out),
   one extra start place (on), and
   one extra stop place (off)
   is called a **weak Petri net computer** for the function $f: N^r \rightarrow N$
   iff there exists for each $x \in N^r$ an initial marking M_x
   such that

- $M_x(on)=1$ and $M_x(in_i)=x_i$ for $1 \leq x_i \leq r$;
- $M_x(out)=M_x(off)=0$;
- $M_x(s_i)=0$;
- For all reachable markings $M \neq M_x$,
  $M(on)=0$ and $1 \leq M(off) \leq 1$ and $M(out) \leq f(x)$;
- For all reachable markings $M \neq M_x$, if $M(off)=1$ then M is dead;
- For all $0 \leq k \leq f(x)$, there exists a reachable marking M
  such that $M(out)=k$ and $M(off)=1$.
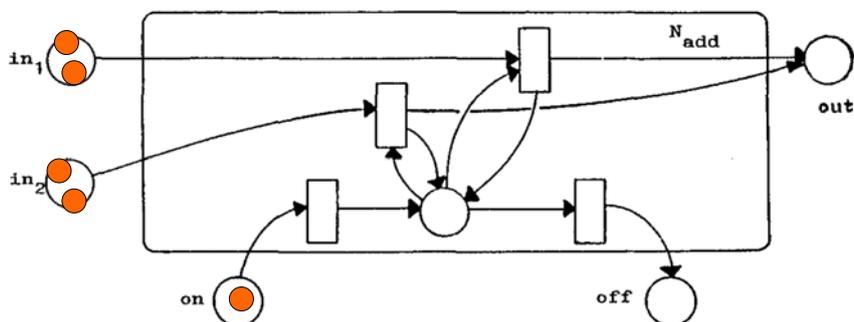
Addition

Source: Matthias Jantzen, Complexity of Place/Transition Nets (1986)

Multiplication
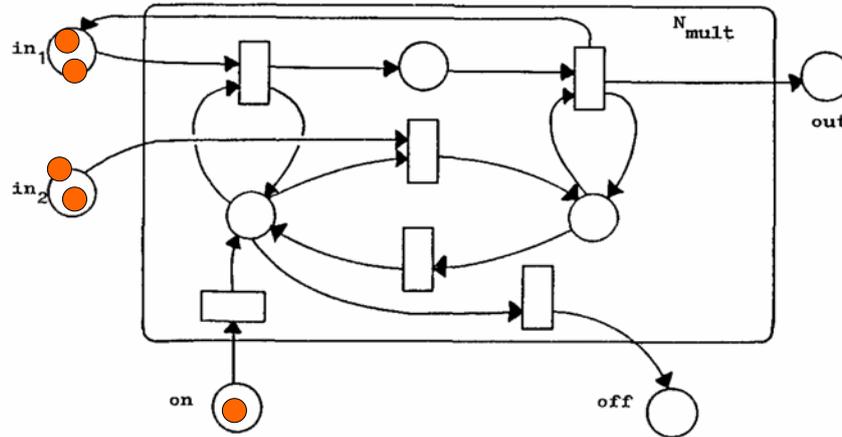
Source: Matthias Jantzen, Complexity of Place/Transition Nets (1986)

RG(M$_0$) is not primitive recursive

Proofidea:

Let A(n):N $\rightarrow$ N be defined by A(n)=A$_n$(2)

- A$_0$(m)=2m+1
- A$_{n+1}$(0)=1
- A$_{n+1}$(m+1)=A$_n$(A$_{n+1}$(m))

A(0)=5; A(1)=7; A(2)=15; A(3)=65535; …

A(n) majorizes the primitive recursive functions.

We can implement A(n) as a P/T net (that grows slowly with n).

Source: Matthias Jantzen, Complexity of Place/Transition Nets (1986)
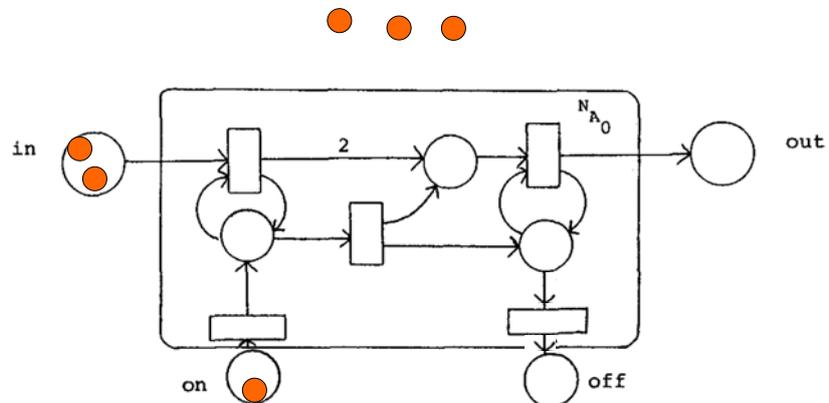
$A_0(m) = 2m+1$

g(m+1):=f(g(m)), g(0)=f(0)

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Deadlock

- A **dead marking (deadlock)** is a marking where no transition can fire.
- A Petri net is **deadlock-free** if no dead marking is reachable.

Liveness

- A **transition** t is **dead** at M if no marking M' is reachable from M such that t can fire in M'.
- A **transition** t is **live** at M if there is no marking M' reachable from M where t is dead.
- A **marking** is **live** if all transitions are live.
- A **P/T net** is **live** if the initial marking is live.

Observations:
- A live net is deadlock-free.
- No transition is live if the net is not deadlock-free.
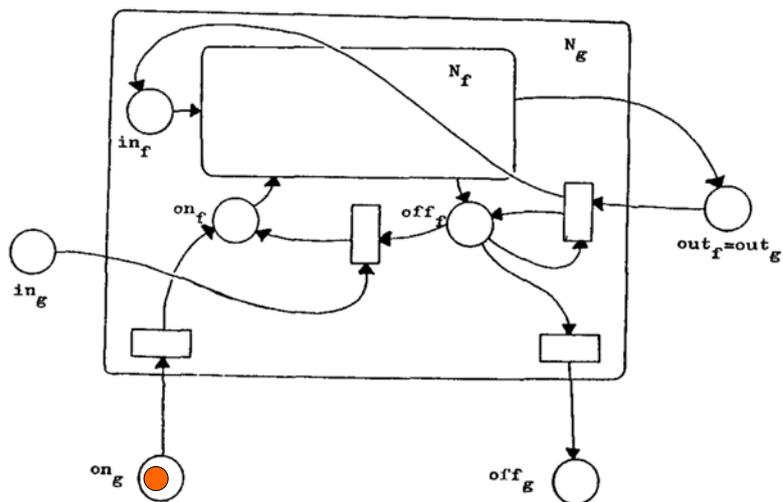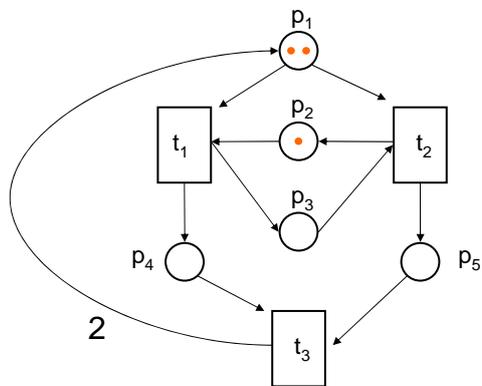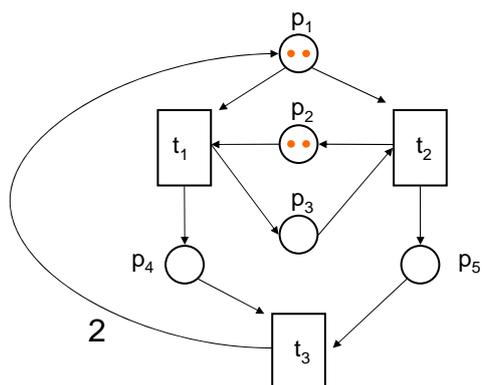
Reversiblity

- A P/T net is **reversible**
  iff for every $M \in R(M_0)$, $M [^*> M_0$

- **Theorem 3:** In a reversible net, a transition t is live
  iff t is not dead in M_0.

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Computation of Invariants

We are interested in subsets *R* of places whose number of labels remain invariant under transitions,

•e.g. the number of trains commuting between Amsterdam and Paris (Cologne and Paris) remains constant

Important for correctness proofs, e.g. the proof of liveness

Competing Trains Example:
Place Invariant 2,0,-2

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Shorthand for changes of markings   REVIEW

Firing transition:

$$M'(p) = \begin{cases} M(p) - W(p,t), & \text{if } p \in {}^\bullet t \setminus t^\bullet \\ M(p) + W(t,p), & \text{if } p \in t^\bullet \setminus {}^\bullet t \\ M(p) - W(p,t) + W(t,p), & \text{if } p \in {}^\bullet t \cap t^\bullet \\ M(p) & \text{otherwise} \end{cases}$$

Let

$$\underline{t}(p) = \begin{cases} -W(p,t) \text{ if } p \in {}^\bullet t \setminus t^\bullet \\ +W(t,p) \text{ if } p \in t^\bullet \setminus {}^\bullet t \\ -W(p,t) + W(t,p) \text{ if } p \in t^\bullet \cap {}^\bullet t \\ 0 \end{cases}$$

$\Rightarrow \qquad \forall p \in P: \ M'(p) = M(p) + \underline{t}(p)$
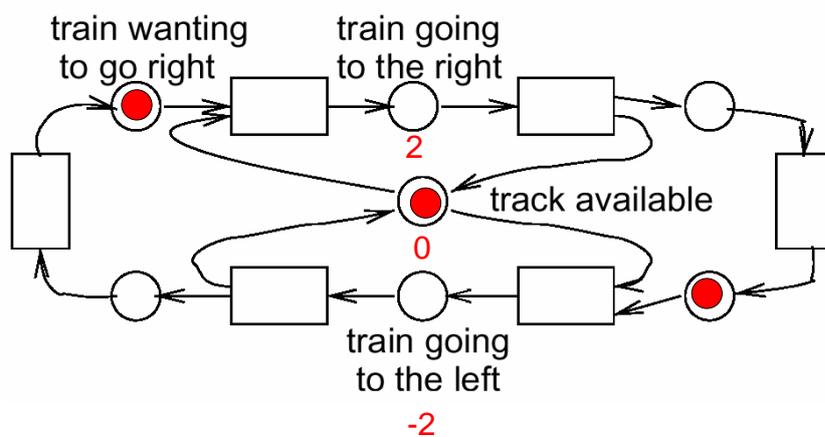
$\Rightarrow \qquad M' = M + \underline{t} \qquad\qquad +: \text{vector add}$

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Matrix $\underline{N}$ describing all changes of markings   **REVIEW**

$$\underline{t}(p) = \begin{cases} -W(p,t) \text{ if } p \in {}^\bullet t \setminus t^\bullet \\ +W(t,p) \text{ if } p \in t^\bullet \setminus {}^\bullet t \\ -W(p,t) + W(t,p) \text{ if } p \in t^\bullet \cap {}^\bullet t \\ 0 \end{cases}$$

Def.: Matrix $\underline{N}$ of net N is a mapping

$$\underline{N}: P \times T \to Z \text{ (integers)}$$

such that $\forall \ t \in T: \ \underline{N}(p,t) = \underline{t}(p)$
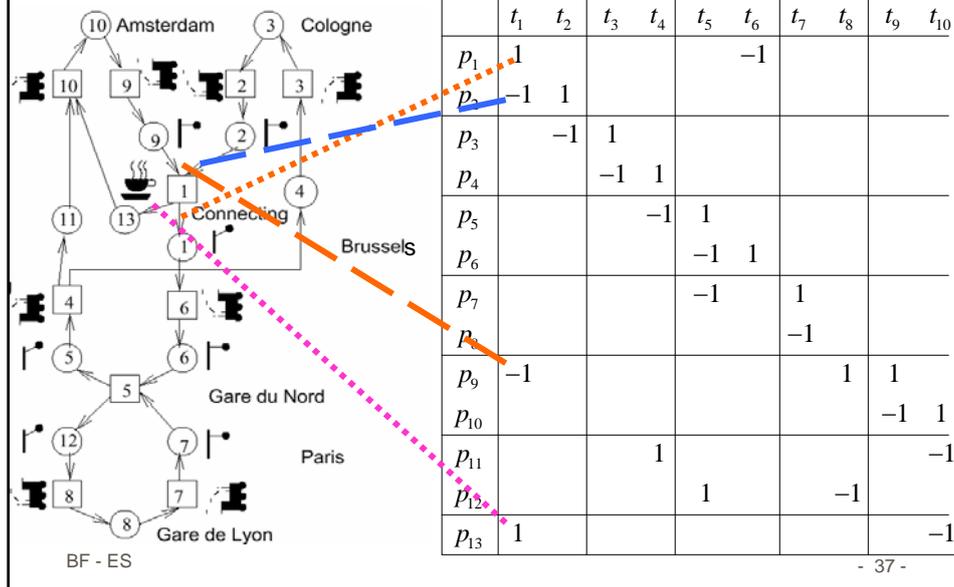
Component in column t and row p indicates the change of the marking of place p if transition t takes place.

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Example: $\underline{N}$ =

	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
p_1	1					-1				
p_2	-1	1								
p_3		-1	1							
p_4			-1	1						
p_5				-1	1					
p_6					-1	1				
p_7					-1		1			
p_8							-1			
p_9	-1							1	1	
p_{10}									-1	1
p_{11}				1						-1
p_{12}						1		-1		
p_{13}	1									-1

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Characteristic Vector

$$\sum_{p \in R} \underline{t}_j(p) = 0$$

Let:
$$\underline{c}_R(p) = \begin{cases} 1 \text{ if } p \in R \\ 0 \text{ if } p \notin R \end{cases}$$

$$\Rightarrow \quad \sum_{p \in R} \underline{t}_j(p) = \underline{t}_j \cdot \underline{c}_R = \sum_{p \in P} \underline{t}_j(p)\,\underline{c}_R(p) = 0$$

Scalar product

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Condition for place invariants

$$\sum_{p \in R} \underline{t}_j(p) = \underline{t}_j \cdot \underline{c}_R = \sum_{p \in P} \underline{t}_j(p) \, \underline{c}_R(p) = 0$$

Accumulated marking constant for **all** transitions if

$$\underline{t}_1 \cdot \underline{c}_R \quad = \quad 0$$

$$\dots \quad \dots \quad \dots$$

$$\underline{t}_n \cdot \underline{c}_R \quad = \quad 0$$

Equivalent to $\underline{N}^T \underline{c}_R = 0$ where $\underline{N}^T$ is the transposed of $\underline{N}$

More detailed view of computations

$$\begin{pmatrix} \underline{t}_1(p_1) \dots \underline{t}_1(p_n) \\ \underline{t}_2(p_1) \dots \underline{t}_2(p_n) \\ \dots \\ \underline{t}_m(p_1) \dots \underline{t}_m(p_n) \end{pmatrix} \begin{pmatrix} \underline{c}_R(p_1) \\ \underline{c}_R(p_2) \\ \dots \\ \underline{c}_R(p_n) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

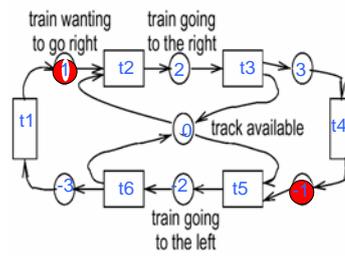System of linear equations.

Solution vectors must consist of zeros and ones.

Competing Trains Example

train wanting
to go right

train going
to the right

track available

train going
to the left