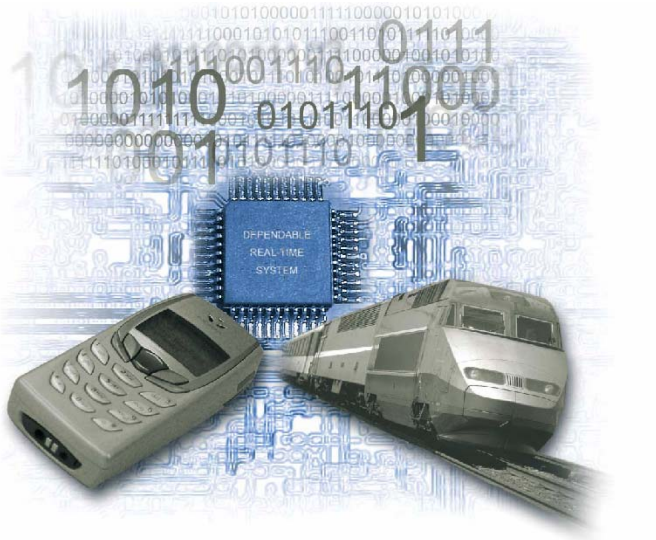
# Embedded Systems 7






# Models of computation for embedded systems

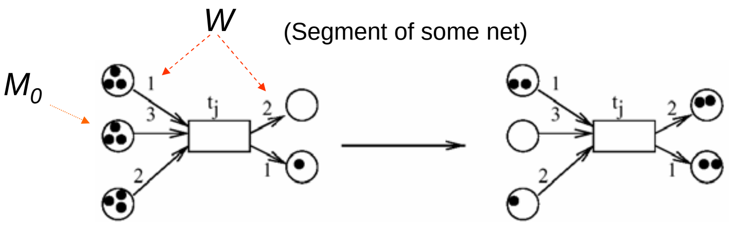
| Communication/ local computations | Shared memory | Message passing Synchronous | Asynchronous |
|---|---|---|---|
| **Communicating finite state machines** | StateCharts, StateFlow | | SDL, MSCs |
| **Data flow model** ⊂ **Computational graphs** | movement of | data | Kahn process networks, SDF<br>Petri nets |
| **Von Neumann model** | C, C++, Java (program counter) | C, C++, Java with libraries<br>CSP, ADA | |
| **Discrete event (DE) model** | VHDL, Simulink | Only experimental systems, e.g. distributed DE in Ptolemy | |

## Place/transition nets

**REVIEW**

**Def.:** $(P, T, F, K, W, M_0)$ is called a **place/transition net (P/T net)** iff

1. $N=(P,T,F)$ is a **net** with places $p \in P$ and transitions $t \in T$
2. $K: P \rightarrow (\mathbb{N}_0 \cup \{\omega\}) \setminus \{0\}$ denotes the **capacity** of places ($\omega$ symbolizes infinite capacity)
3. $W: F \rightarrow (\mathbb{N}_0 \setminus \{0\})$ denotes the **weight of graph edges**
4. $M_0: P \rightarrow \mathbb{N}_0 \cup \{\omega\}$ represents the **initial marking** of places



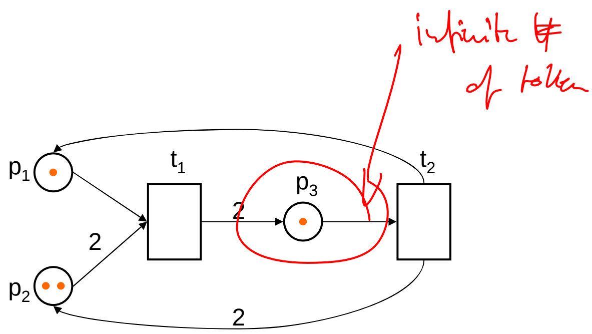

defaults:
K = $\omega$
W = 1

In the following: assume initial marking is finite, capacity $\omega$.



## Unbounded Petri net

**REVIEW**



## Invariants & boundedness

**REVIEW**

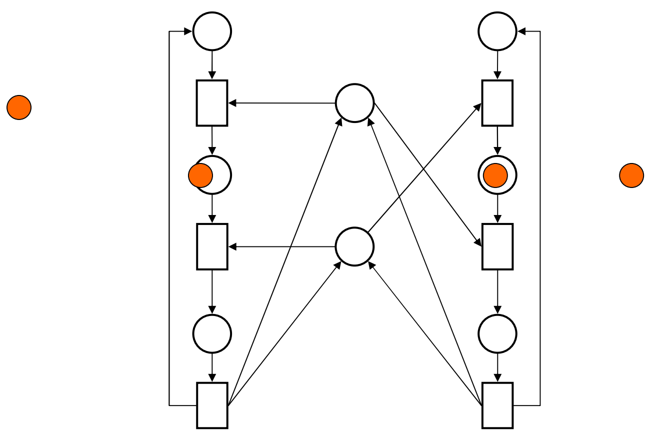
- A net is **covered** by place invariants
  iff every place is contained in some invariant.
- **Theorem 4:**
  **a)** If R is a place invariant and $p \in R$, then p is bounded.
  **b)** If a net is covered by place invariants then it is bounded.



## Deadlock

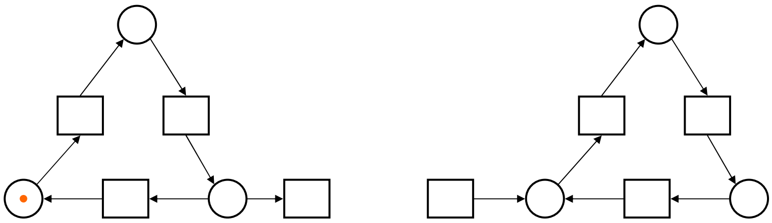
**REVIEW**

- A **dead marking (deadlock)** is a marking where no transition can fire.
- A Petri net is **deadlock-free** if no dead marking is reachable.



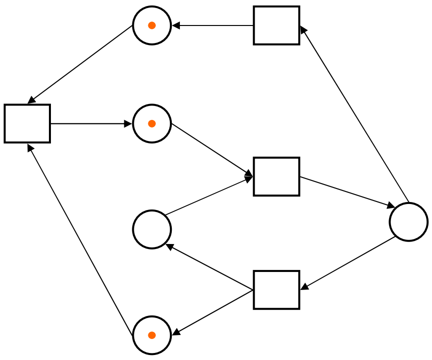
## Structural properties: deadlock-traps **REVIEW**

- A place set *S* is a **(static) deadlock** if every transition that adds token from S also removes token from *S*.
- A place set *S* is a **trap** if every transition that removes token from S also adds token to *S*.





## Empty structural deadlocks and marked traps **REVIEW**



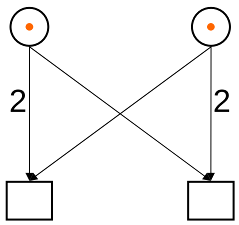
- Empty structural deadlocks are never re-marked;
- Marked traps are never emptied.

## Sufficiently marked places

**REVIEW**

A place is called sufficiently marked if there are enough token for one of the outgoing transitions:

- Define
  $W^-(p) = \min \{ W(p,t) \mid (p,t) \in F \}$ if there exists a $(p,t) \in F$
  and 0 otherwise
- Place $p$ is **sufficiently marked** in marking $M$, if $M(p) \geq W^-(p)$
- A set of places is sufficiently marked if it contains a sufficiently marked place.

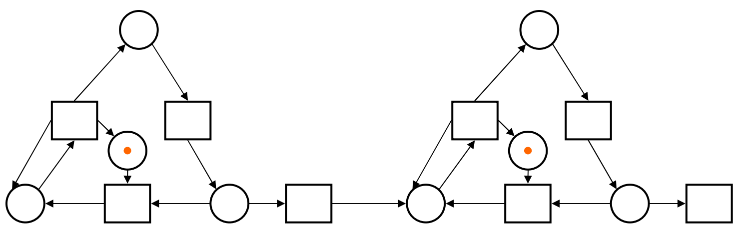





## Deadlock-Trap Property

**REVIEW**

- A P/T has the **deadlock-trap property**, if every (static) deadlock contains a trap that is sufficiently marked in $M_0$.



## Deadlock-Trap Property

**REVIEW**

**Theorem 5:**
Every homogeneous P/T net with non-blocking weights that has the deadlock-trap property is deadlock-free.

**Homogeneous**: For each place, all outgoing edges have the same weight.

**Non-blocking weights**: $W^+(p) \geq W^-(p)$

- $W^-(p)$= min { $W(p,t)$ | $(p,t) \in F$ } if there exists a $(p,t) \in F$ and 0 otherwise
- $W^+(p)$= min { $W(t,p)$ | $(t,p) \in F$ } if there exists a $(t,p) \in F$ and 0 otherwise

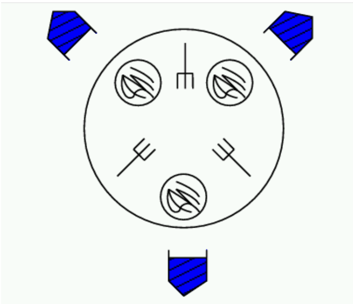


## Fairness

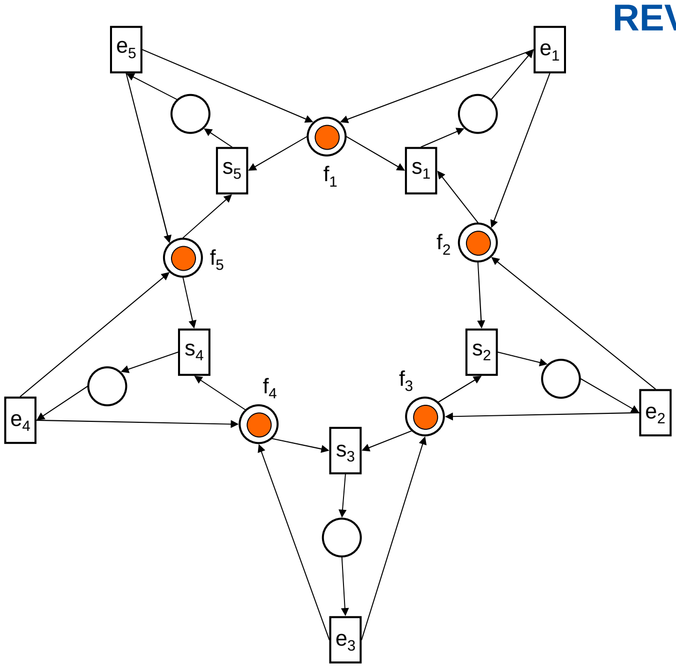
**REVIEW**

**Dining philosophers problem**

- *n>1* philosophers sitting at a round table;
- *n* forks,
- *n* plates with spaghetti;
- philosophers either thinking or eating spaghetti (using left and right fork).
- 2 forks needed!



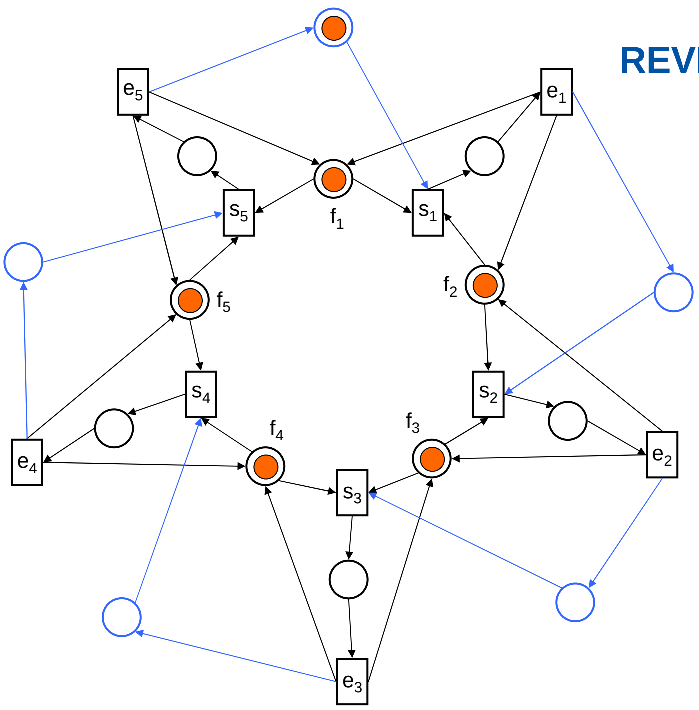
## REVIEW







## REVIEW

## Fairness

**REVIEW**

Let *N* be a Petri net and *w* an execution of *N*.

- *w* is **impartial** with respect to a set of transitions *T* iff every transition in *T* occurs infinitely often in *w*.
- *w* is **just** with respect to a set of transitions *T* iff every transition in *T* that is enabled in all except finitely many markings occurs infinitely often in *w*.
- *w* is **fair** with respect to a set of transitions *T* iff every transition in *T* that is enabled in infinitely many markings occurs infinitely often in *w*.

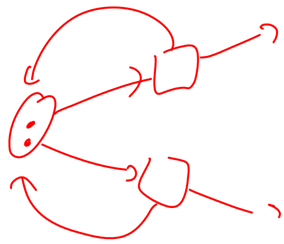
- *w* is **impartial** ⇒ *w* is **fair**
- *w* is **fair** ⇒ *w* is **just**



## Persistent nets

**REVIEW**

**Theorem 7:** If the net is persistent, then every just execution is fair.



## State Fairness

**REVIEW**

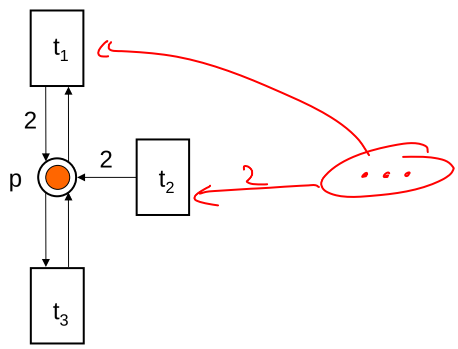
**Theorem 8:** Let *N* be a bounded net, *t* a live transition, and *w* a state-fair execution of *N*. Then *t* occurs infinitely often in *w*.



## Extensions: finite capacities

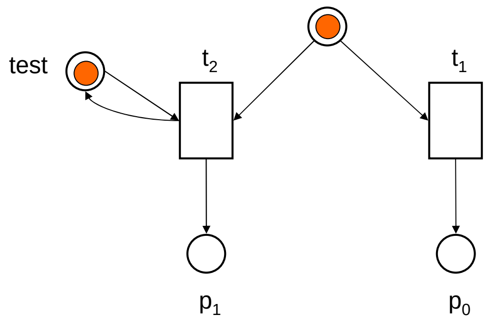
**REVIEW**

- K(p)=4



## Extensions: Petri nets with priorities REVIEW

- $t_1 \langle t_2$ : $t_2$ has higher priority than $t_1$.

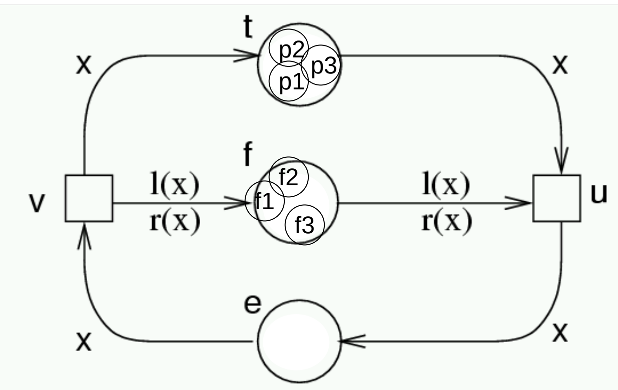



- Petri nets with priorities are Turing-complete.



## Predicate/transition model of the dining philosophers problem REVIEW

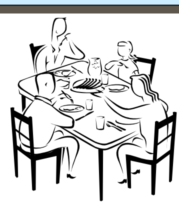
- Let x be one of the philosophers,
- let l(x) be the left fork of x,
- let r(x) be the right fork of x.




Token: individuals.

Semantics can be defined by replacing net by equivalent condition/event net.

Model can be extended to arbitrary numbers.



## Summary Petri nets

**Pros:**

- Appropriate for distributed applications,
- Well-known theory for formally proving properties,
- Initially theoretical topic, but now widely adapted in practice due to increasing number of distributed applications.

**Cons** (for the nets presented) **:**

- problems with modeling timing,
- no programming elements,
- no hierarchy.

**Extensions:**

- Enormous amounts of efforts on removing limitations.



## Data Flow Models

## Data flow modeling

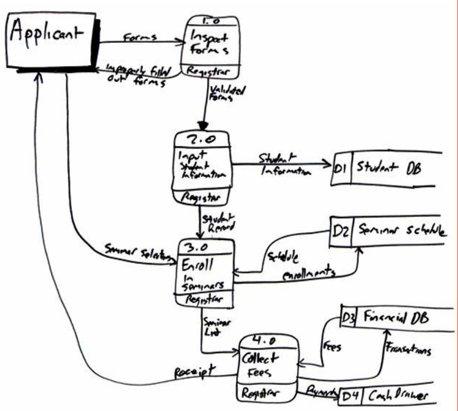
- **Def.**: The process of identifying, modeling and documenting how data moves around an information system.

  Data flow modeling examines
  - *processes* (activities that transform data from one form to another),
  - *data stores* (the holding areas for data),
  - *external entities* (what sends data into a system or receives data from a system, and
  - *data flows* (routes by which data can flow).



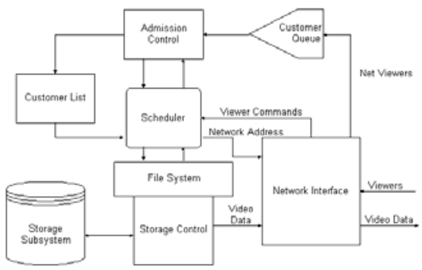
## Data flow as a “natural” model of applications

Registering for courses



http://www.agilemodeling.com/artifacts/dataFlowDiagram.htm

Video on demand system

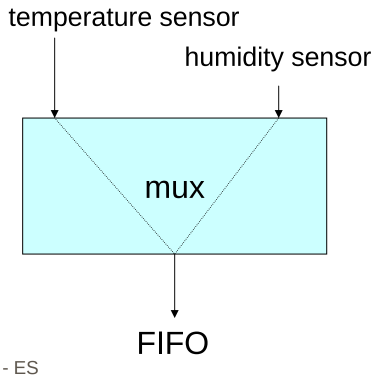



**www.ece.ubc.ca/~irenek/techpaps/vod/vod.html**

## Process networks

Many applications can be specified in the form of a set of communicating processes.

**Example:** system with two sensors:




Alternating read

**loop**
read_temp; read_humidity
**until false;**

of the two sensors
not the right approach.



## The case for multi-process modeling in imperative languages

```
MODULE main;
 TYPE some_channel =
          (temperature, humidity);
      some_sample : RECORD
                    value : integer;
                    line  : some_channel
                   END;
 PROCESS get_temperature;
 VAR sample : some_sample;
 BEGIN
  LOOP
   sample.value := new_temperature;
   IF sample.value > 30 THEN ....
   sample.line := temperature;
   to_fifo(sample);
  END
 END get_temperature;
```

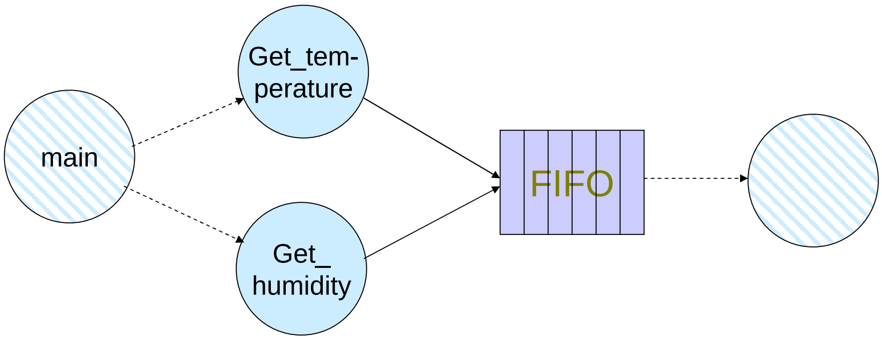
```
PROCESS get_humidity;
  VAR sample : some_sample;
  BEGIN
   LOOP
    sample.value := new_humidity;
    sample.line := humidity;
    to_fifo(sample);
   END
  END  get_humidity;

 BEGIN
  get_temperature; get_humidity;
 END;
```

- Blocking calls new_temperature, new_humidity
- **Structure clearer than alternating checks for new values in a single process**

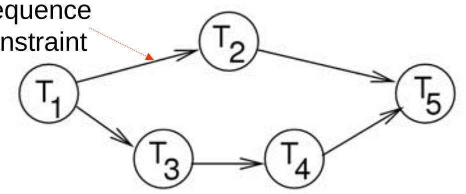
How to model dependencies between tasks/processes?

## Dependences between processes/tasks






## Task graphs

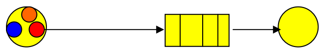



Nodes are assumed to be a „program" described in some programming language, e.g. C or Java.

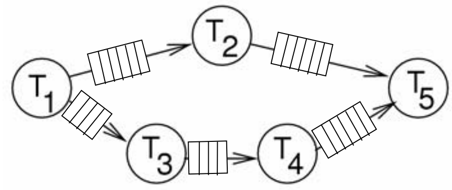
- **Def.:** A **dependence graph** is a directed graph $G=(V,E)$ in which $E \subseteq V \times V$ is a partial order.
- If $(v1, v2) \in E$, then $v1$ is called an **immediate predecessor** of $v2$ and $v2$ is called an **immediate successor** of $v1$.
- Suppose $E^*$ is the transitive closure of $E$.
  If $(v1, v2) \in E^*$, then $v1$ is called a **predecessor** of $v2$ and $v2$ is called a **successor** of $v1$.

## Reference model for data flow:
### Kahn process networks (1974)

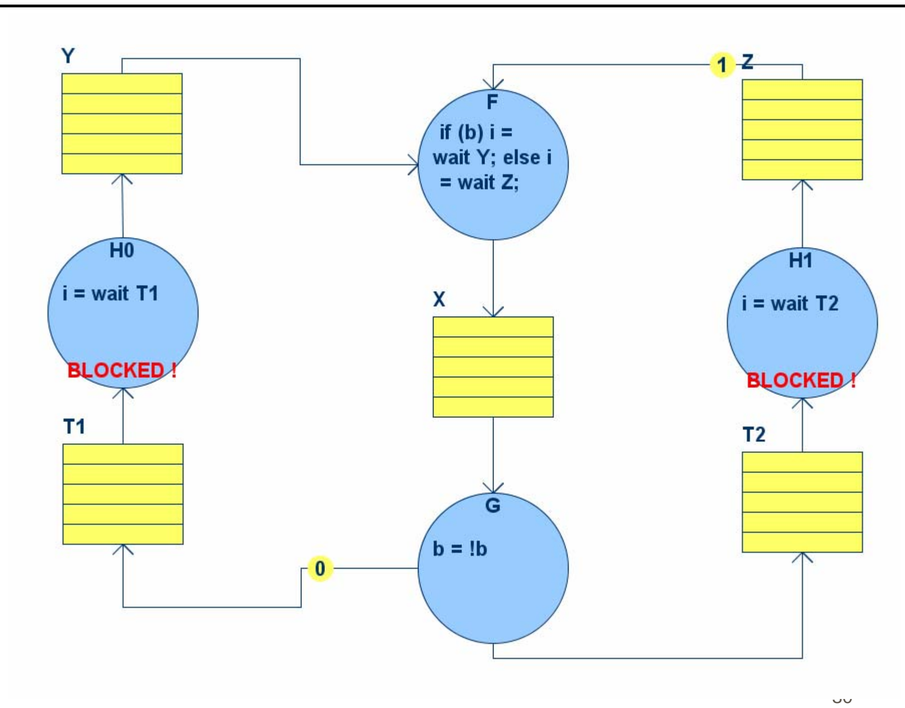
For asynchronous message passing:
communication between tasks is buffered



Special case: Kahn process networks:
executable task graphs;
Communication via infinitely large FIFOs





## Properties of Kahn process networks (1)

- Each node corresponds to one program/task;
- Communication is only via channels;
- Channels include FIFOs as large as needed;
- Channels transmit information within an unpredictable but finite amount of time;
- Mapping from ≥1 input seq. to ≥1 output sequence;
- In general, execution times are unknown;
- Send operations are non-blocking, reads are blocking.
- One producer and one consumer;
  i.e. there is only one sender per channel;





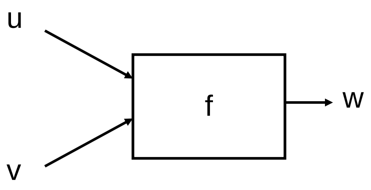
## Properties of Kahn process networks (2)

- There is only one sender per channel.
- A process cannot check whether data is available before attempting a read.
- A process cannot wait for data for more than one port at a time.
- Therefore, the order of reads depends only on data, not on the arrival time.
- Therefore, Kahn process networks are deterministic (!); for a given input, the result will always the same, regardless of the speed of the nodes.

This is the key beauty of KPNs!

## A Kahn Process

```
process f(in int u, in int v, out int w)
{
  int i; bool b = true;
  for (;;) {
    i = b ? wait(u) : wait(w);
    printf("%i\n", i);
    send(i, w);
    b = !b;
  }
}
```




Process alternately reads from u and v, prints the data value, and writes it to w

Source: Gilles Kahn, The Semantics of a Simple Language for Parallel Programming (1974)



## A Kahn Process

```
process f(in int u, in int v, out int w)
{
  int i; bool b = true;
  for (;;) {
    i = b ? wait(u) : wait(w);
    printf("%i\n", i);
    send(i, w);
    b = !b;
  }
}
```

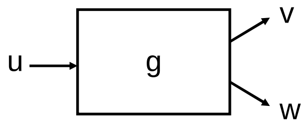
wait() returns the next token in an input FIFO, blocking if it's empty

send() writes a data value on an output FIFO

Source: Gilles Kahn, The Semantics of a Simple Language for Parallel Programming (1974)

## A Kahn Process

```
process g(in int u, out int v, out int w)
{
  int i; bool b = true;
  for(;;) {
    i = wait(u);
    if (b) send(i, v); else send(i, w);
    b = !b;
  }
}
```



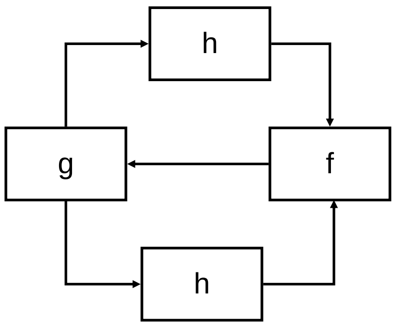

Process reads from u and alternately copies it to v and w



## A Kahn System

- Prints an alternating sequence of 0's and 1's

Emits a 1 then copies input to output




Emits a 0 then copies input to output

## Definition: Kahn networks

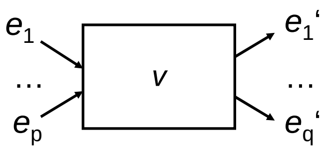
A **Kahn process network** is a directed graph $(V,E)$, where

- $V$ is a set of **processes**,
- $E \subseteq V \times V$ is a set of **edges**,
- associated with each edge $e$ is a **domain** $D_e$
- $D^\omega$: finite of countably infinite sequences over $D$

  $D^\omega$ is a complete partial order where
  $X \leq Y$ iff $X$ is an initial segment of $Y$



## Definition: Kahn networks




- associated with each process $v \in V$ with incoming edges $e_1, \ldots, e_p$ and outgoing edges $e_1', \ldots, e_q'$
  is a continuous **function**
  $f_v: D_{e_1}^\omega \times \ldots \times D_{e_p}^\omega \rightarrow D_{e_1}^\omega \times \ldots \times D_{e_q}^\omega$

(A function $f: A \rightarrow B$ is **continuous** if $f(\lim_A a) = \lim_B f(a)$ )

## Semantics: Kahn networks

A process network defines for each edge $e \in E$ a **unique** sequence $X_e$.

$X_e$ is the least fixed point of the equations

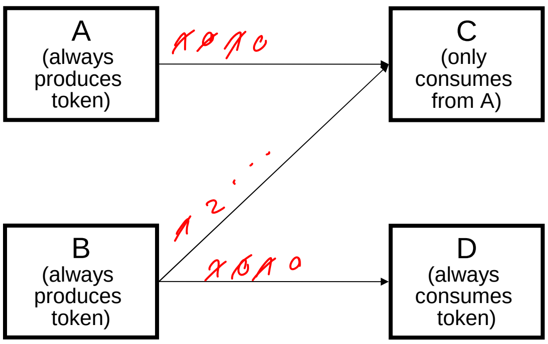
$$(X_{e_1'}, \ldots, X_{e_q'}) = f_v(X_{e_1}, \ldots, X_{e_q})$$

for all $v \in V$.

Result is independent of scheduling!

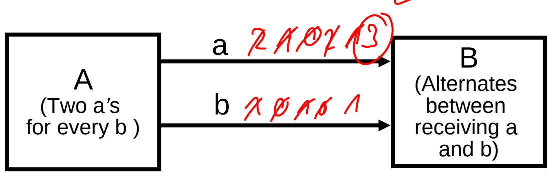


## Scheduling Kahn Networks




Problem: run processes with finite buffer

## Scheduling may be impossible



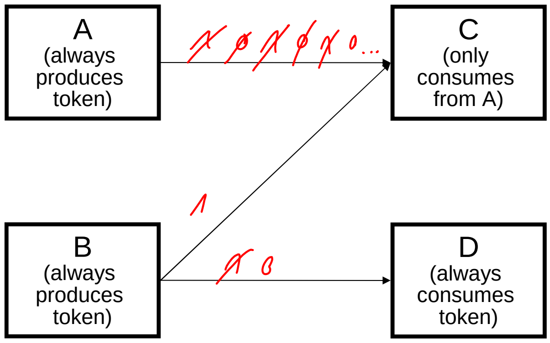



## Parks' Scheduling Algorithm (1995)

- Set a capacity on each channel
- Block a write if the channel is full
- Repeat
  - Run until deadlock occurs
  - If there are no blocking writes → terminate
  - Among the channels that block writes,
    select the channel with least capacity
    and increase capacity until producer can fire.

## Example

Bound = 1






## Parks' Scheduling Algorithm

- Whether a Kahn network can execute in bounded memory is undecidable
- Parks' algorithm does not violate this
- It will run in bounded memory if possible, and use unbounded memory if necessary

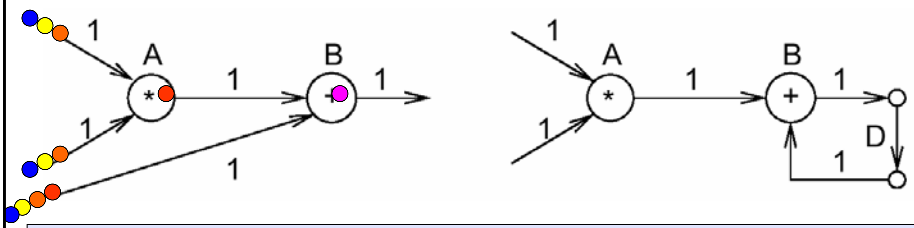
non-terminating execution > bounded execution > complete execution

**Disadvantages:**

- Requires dynamic memory allocation
- Does not guarantee minimum memory usage
- Scheduling choices may affect memory usage
- Data-dependent decisions may affect memory usage
- Relatively costly scheduling technique
- Detecting deadlock may be difficult

## Synchronous data flow (SDF)

- Asynchronous message passing=
  tasks do not have to wait until output is accepted.
- Synchronous data flow =
  all tokens are consumed at the same time.




SDF model allows static scheduling of token production and consumption.
In the general case, buffers may be needed at edges.



## SDF: restriction of Kahn networks

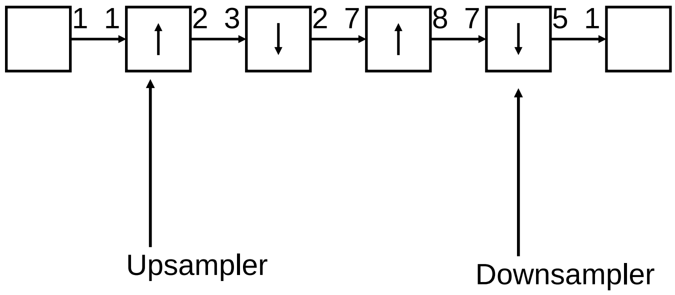
An **SDF graph** is a tuple (V, E, cons, prod, d) where

- V is a set of nodes (activities)
- E is a set of edges (buffers)
- cons: E $\rightarrow$ N number of tokens consumed
- prod: E $\rightarrow$ N number of tokens produced
- d: E $\rightarrow$ N number of initial tokens

  d: „delay“ (sample offset between input and output)

## Multi-rate SDF System

- DAT-to-CD rate converter
- Converts a 44.1 kHz sampling rate to 48 kHz



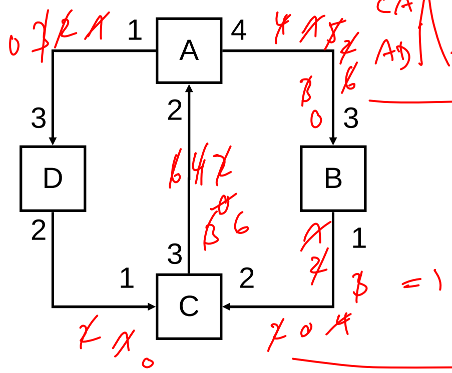



## SDF Scheduling Algorithm Lee/Messerschmitt 1987

1. Establish **relative execution rates**
   - Generate balance equations
   - Solve for smallest positive integer vector **c**
2. Determine **periodic schedule**
   - Form an arbitrarily ordered list of all nodes in the system
   - Repeat:
     - For each node in the list, schedule it if it is runnable, trying each node once
     - If each node has been scheduled $\mathbf{c}_n$ times, stop.
     - If no node can be scheduled, indicate deadlock.

Source: Lee/Messerschmitt, Synchronous Data Flow (1987)

## Example 1




d(CA)=6

$c_C = 2 c_D$

$c_B = 4 c_D$

$c_A = 3 c_D$

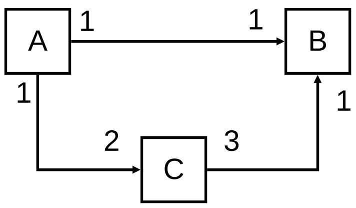
$\Rightarrow c_D = 1,\ c_A = 3,\ c_B = 4,\ c_C = 2$

Schedule: A B A B A B B D C C



## Example 2

$3 c_C = c_B$




$c_A = c_B$

$2 c_C = c_A$

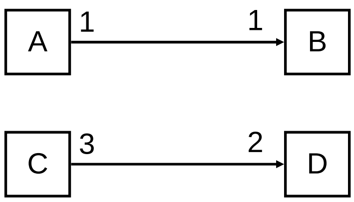
$c_B = \frac{3}{2} c_A$

$c_A = 0,\ c_B = 0,\ c_C = 0$

inconsistent!

→ no schedule.

## Example 3




$$c_D \; (-3 \;\; 2) \cdot c = 0$$

$$-3c_C + 2c_D = 0$$

$$2c_D = 3c_C$$

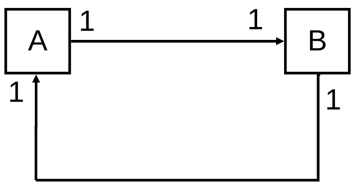
$$c_B = c_A$$

$$3c_C = 2c_D$$

relative rate between A and C is undefined!



## Example 4




$$c_A = c_B$$

$$c_A = 1$$

$$c_B = 1$$

deadlock!

(missing delays)

## Observations

- Consistent, connected systems have one-dimensional solution
- Disconnected systems have higher-dimensional solution
- Inconsistent systems have 0-schedule; otherwise infinite accumulation of tokens
- Systems may have multiple schedules



## Summary dataflow

- Communication exclusively through FIFOs
- Kahn process networks
  - blocking read, nonblocking write
  - deterministic
  - schedulability undecidable
  - Parks‘ scheduling algorithm
- SDF
  - fixed token consumption/production
  - compile-time scheduling: balance equations